

# **Distributed algorithms**

## **The Leader Election Protocol (IEEE 1394)**

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Distributed Systems  
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# This Session

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- Background :-)
  - An informal presentation of the protocol :-)
  - Step by step formal design :-|
  - Short Conclusion. :-)
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# IEEE 1394 High Performance Serial Bus (FireWire)

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- It is an **international standard**
  - There exists a **widespread commercial interest** in its correctness
  - Sun, Apple, Philips, Microsoft, Sony, etc **involved in its development**
  - Made of **three layers** (physical, link, transaction)
  - The protocol under study is the **Tree Identify Protocol**
  - Situated in the **Bus Reset phase** of the physical layer
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# The Problem (1)

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- The bus is used to transport digitized **video and audio signals**
  - It is **“hot-pluggable”**
  - Devices and peripherals can be **added and removed at any time**
  - Such changes are followed by a **bus reset**
  - The **leader election** takes place after a bus reset in the network
  - A leader needs to be chosen to act as the **manager of the bus**
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# The Problem (2)

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- After a bus reset: all nodes in the network have **equal status**
  - A node **only knows** to which nodes it is **directly connected**
  - The network is **connected**
  - The network is **acyclic**
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# References (1)

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## BASIC

- IEEE. *IEEE Standard for a High Performance Serial Bus. Std 1394-1995.* 1995
  - IEEE. *IEEE Standard for a High Performance Serial Bus (supplement). Std 1394a-2000.* 2000
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## References (2)

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### GENERAL

- N. Lynch. *Distributed Algorithms*. Morgan Kaufmann. 1996
  - R. G. Gallager et al. *A Distributed Algorithm for Minimum Weight Spanning Trees*. IEEE Trans. on Prog. Lang. and Systems. 1983.
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# References (3)

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## MODEL CHECKING

- D.P.L. Simons et al. *Mechanical Verification of the IEE 1394a Root Contention Protocol using Uppaal2* Springer International Journal of Software Tools for Technology Transfer. 2001
  - H. Toetenel et al. *Parametric verification of the IEEE 1394a Root Contention Protocol using LPMC* Proceedings of the 7th International Conference on Real-time Computing Systems and Applications. IEEE Computer Society Press. 2000
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## References (4)

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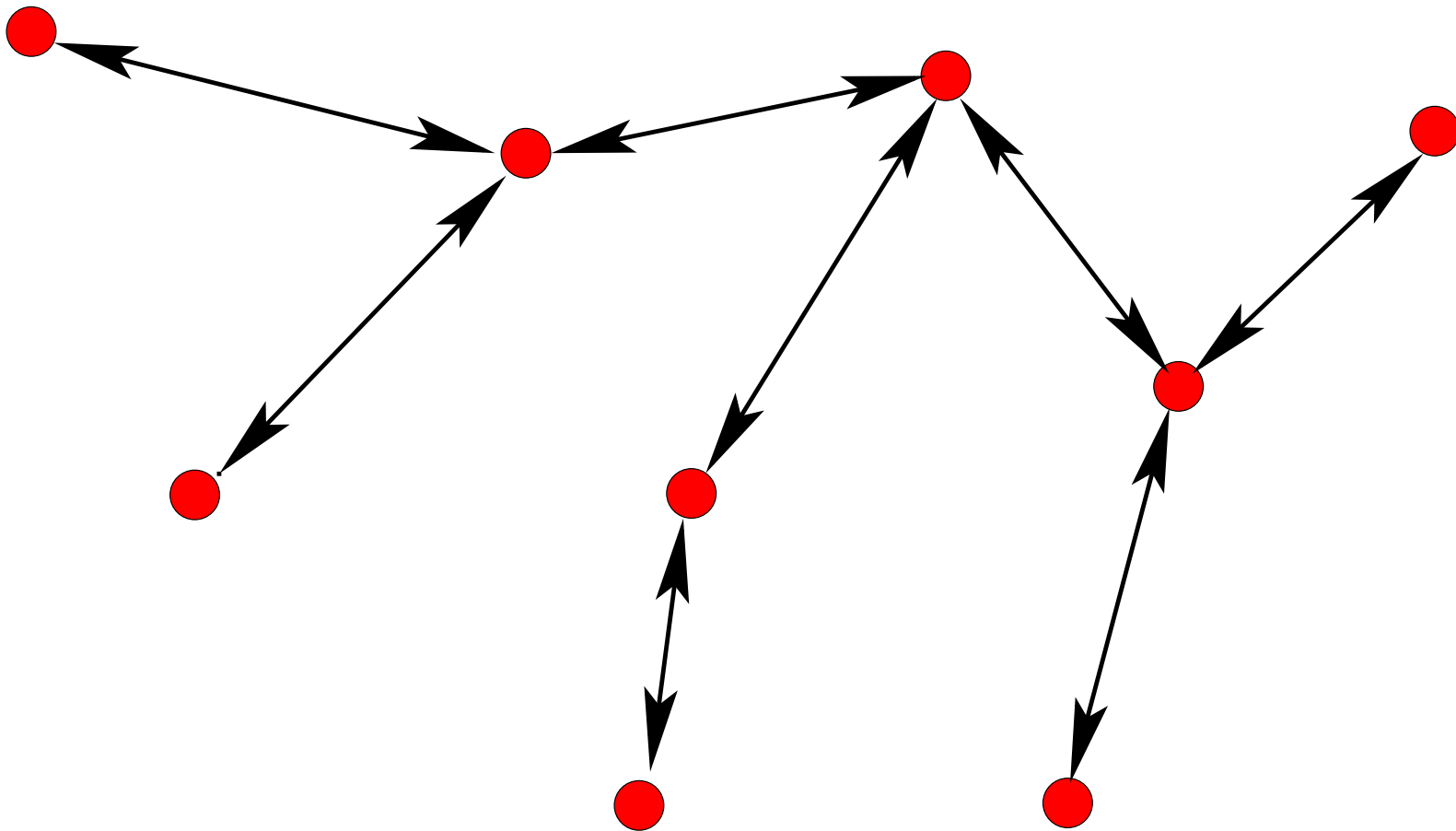
### THEOREM PROVING

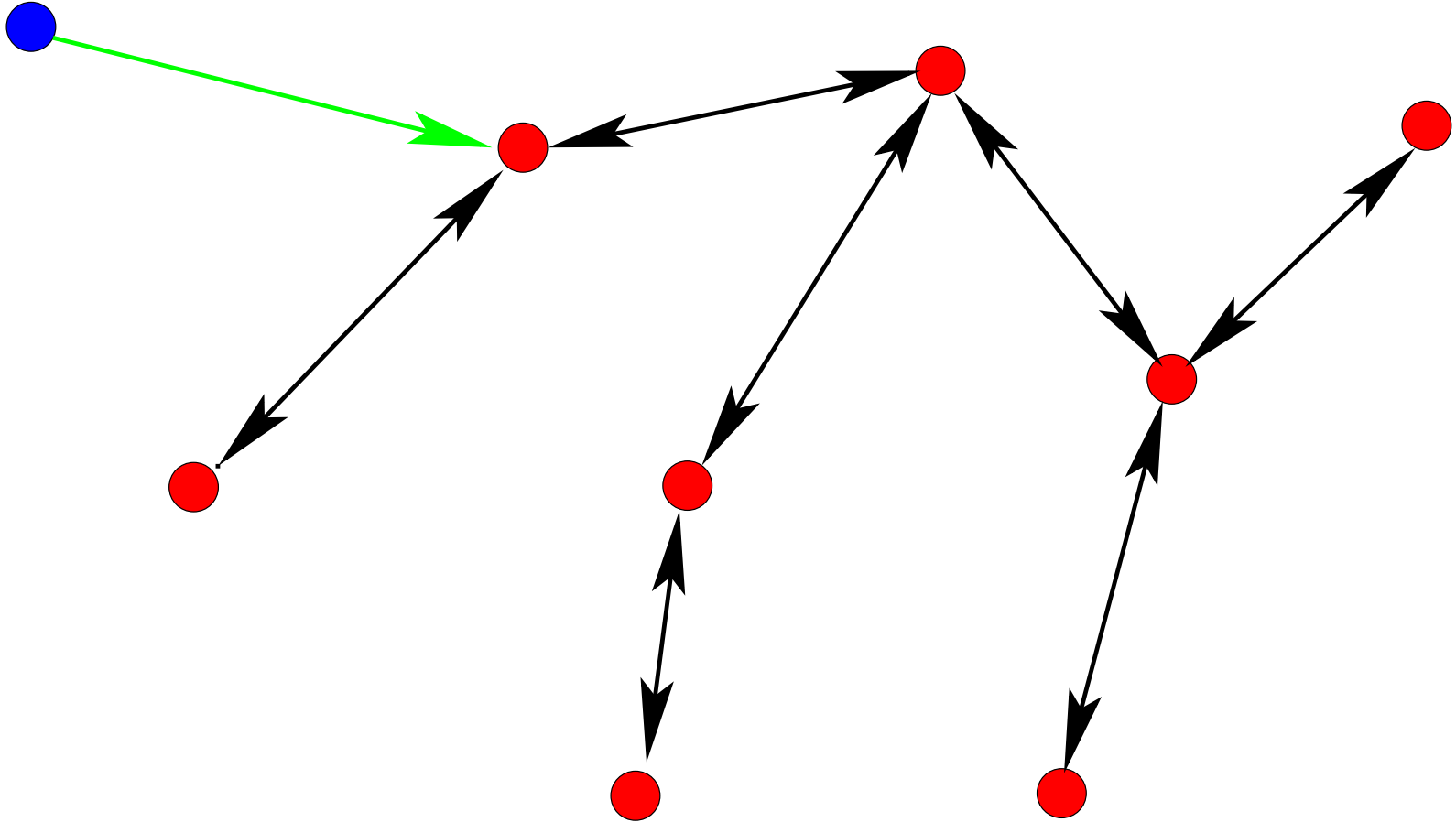
- M. Devillers et al. *Verification of the Leader Election: Formal Method Applied to IEEE 1394*. Formal Methods in System Design. 2000
  - J.R. Abrial et al. *A Mechanically Proved and Incremental Development of IEEE 1394*. To be published 2002
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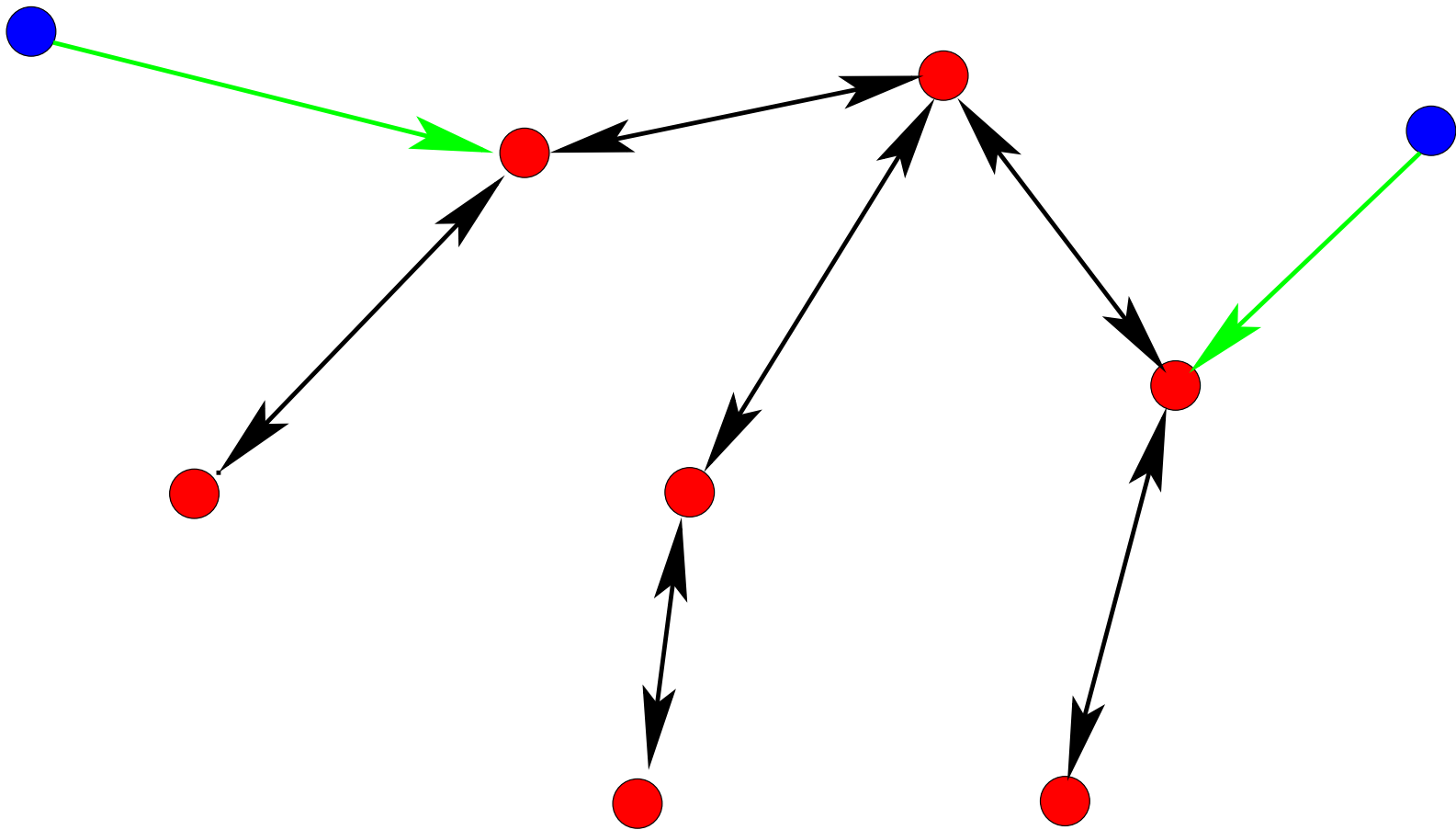
# Informal Abstract Properties of the Protocol

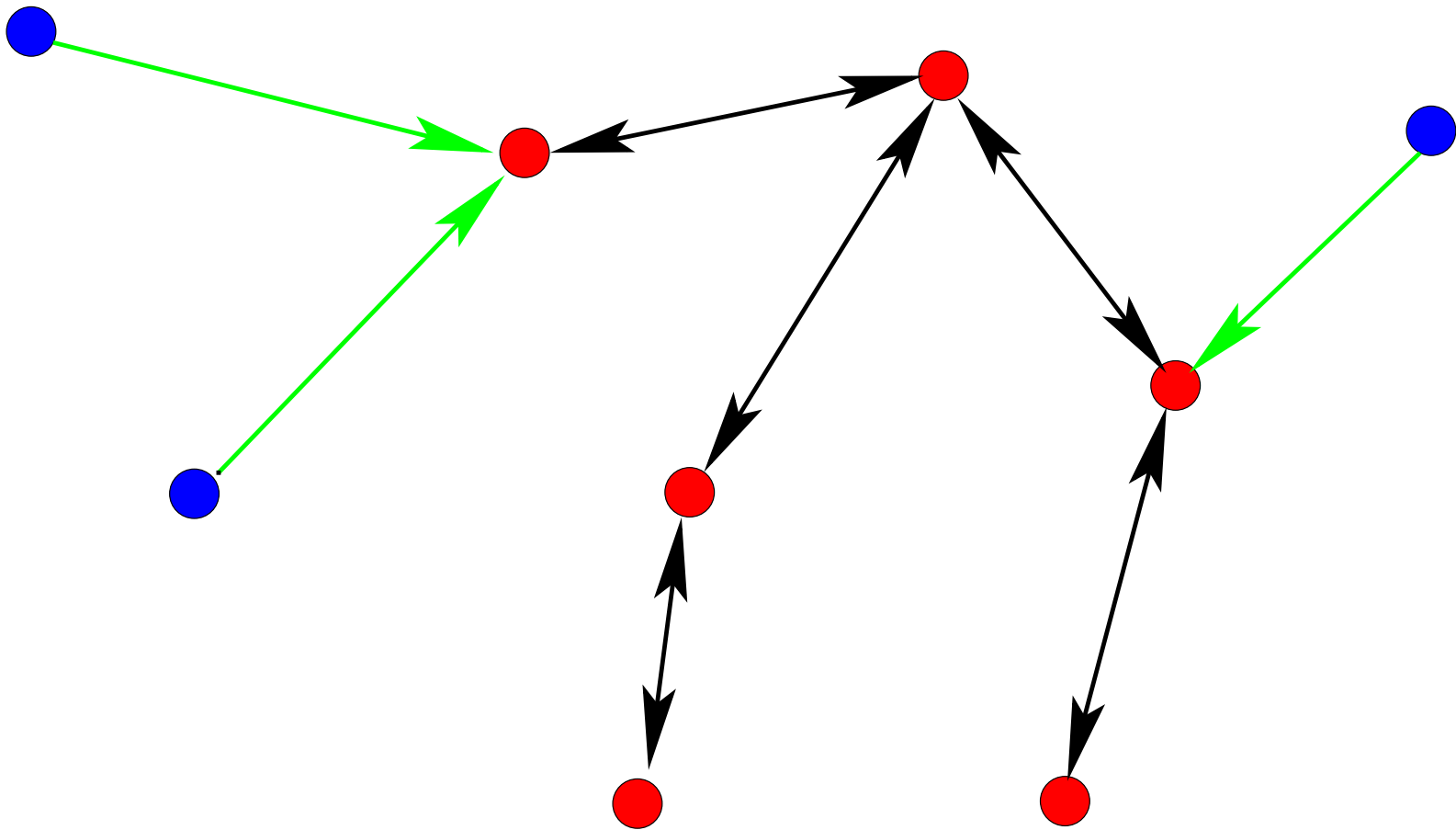
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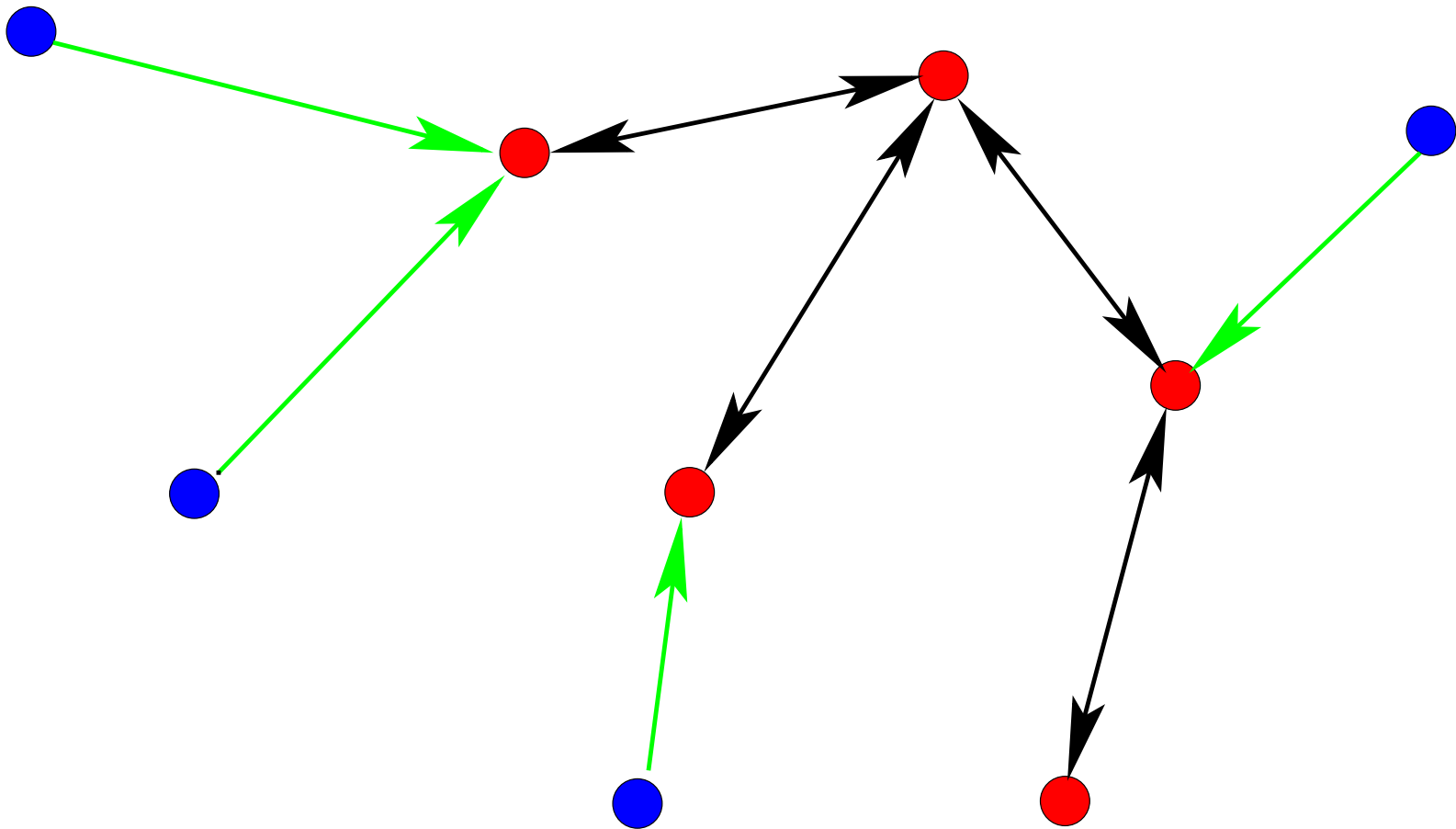
- We are given a **connected** and **acyclic** network of nodes
  - Nodes are linked by **bidirectional channels**
  - We want to have one node being elected **the leader** in a finite time
  - This is to be done in a **distributed** and **non-deterministic** way
  - Next are two distinct **abstract animations** of the protocol
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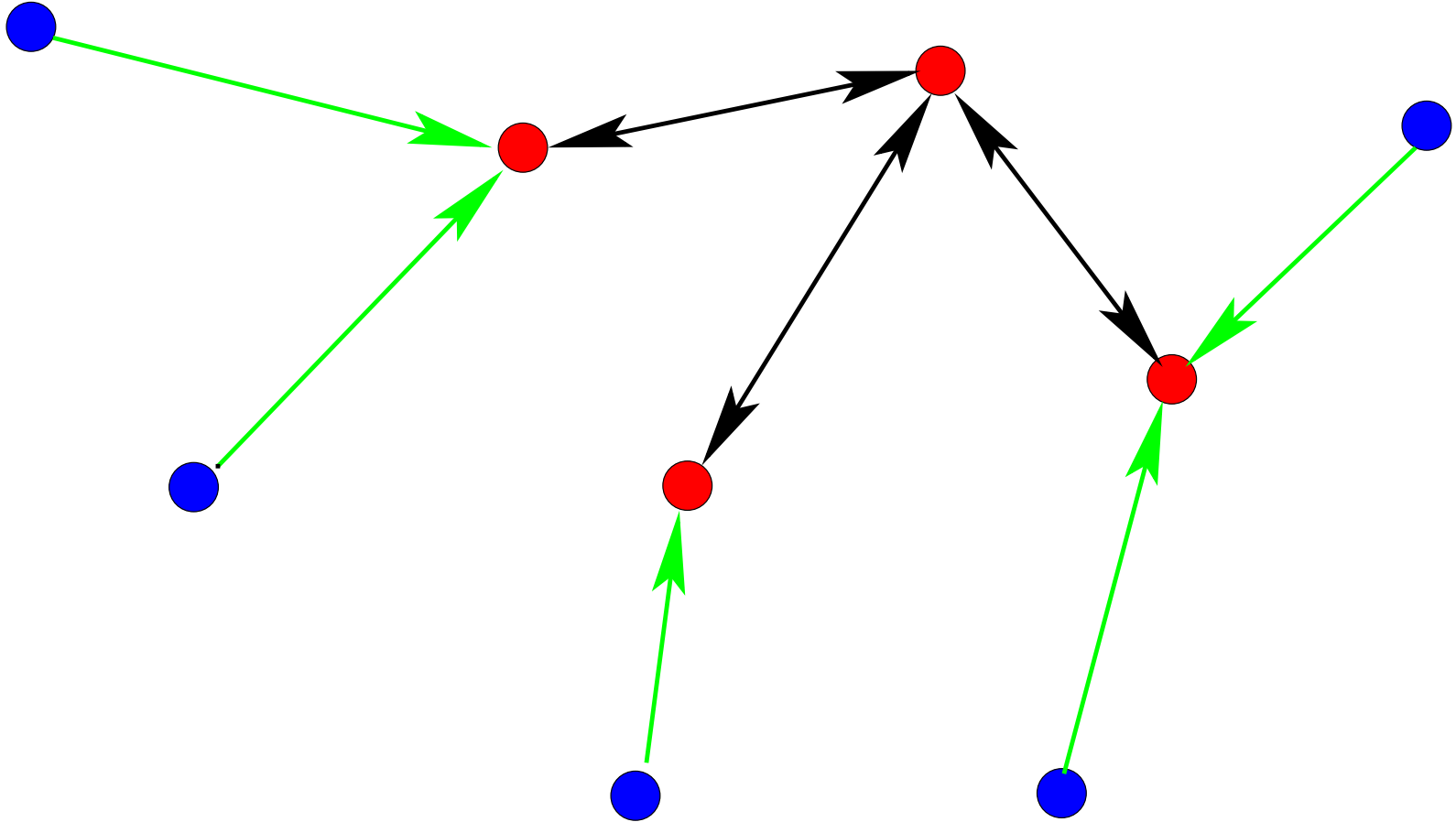




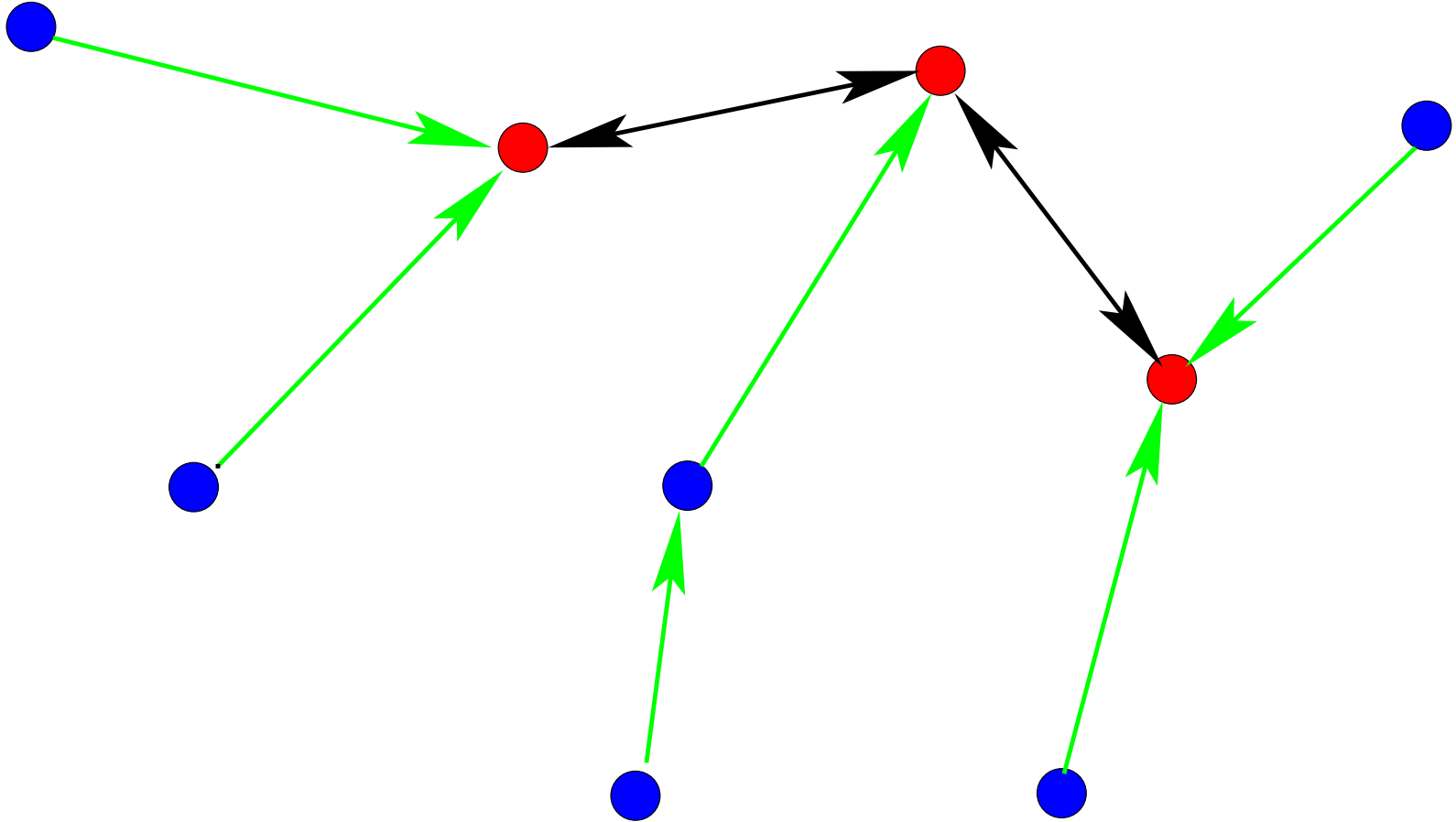


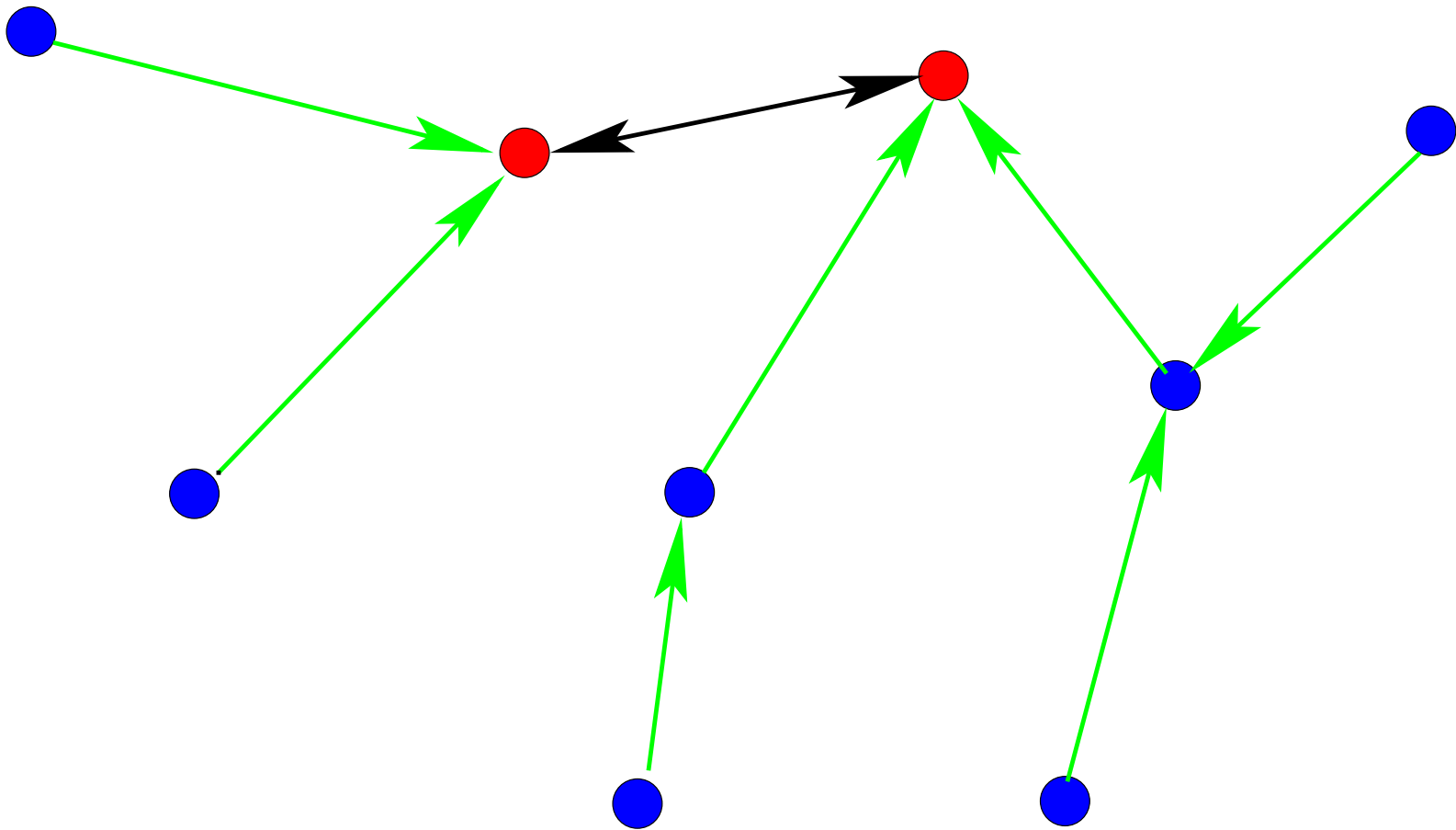


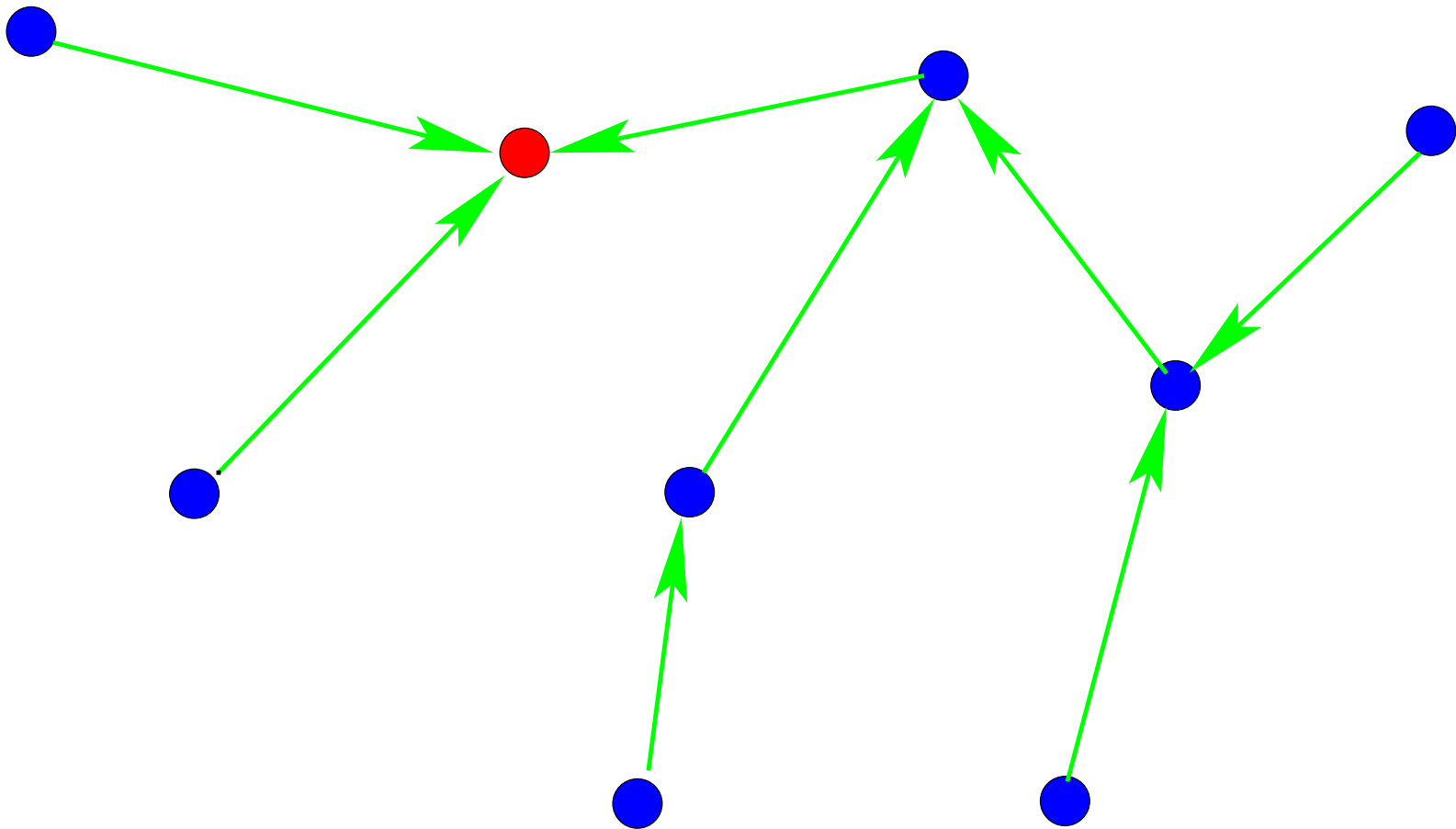


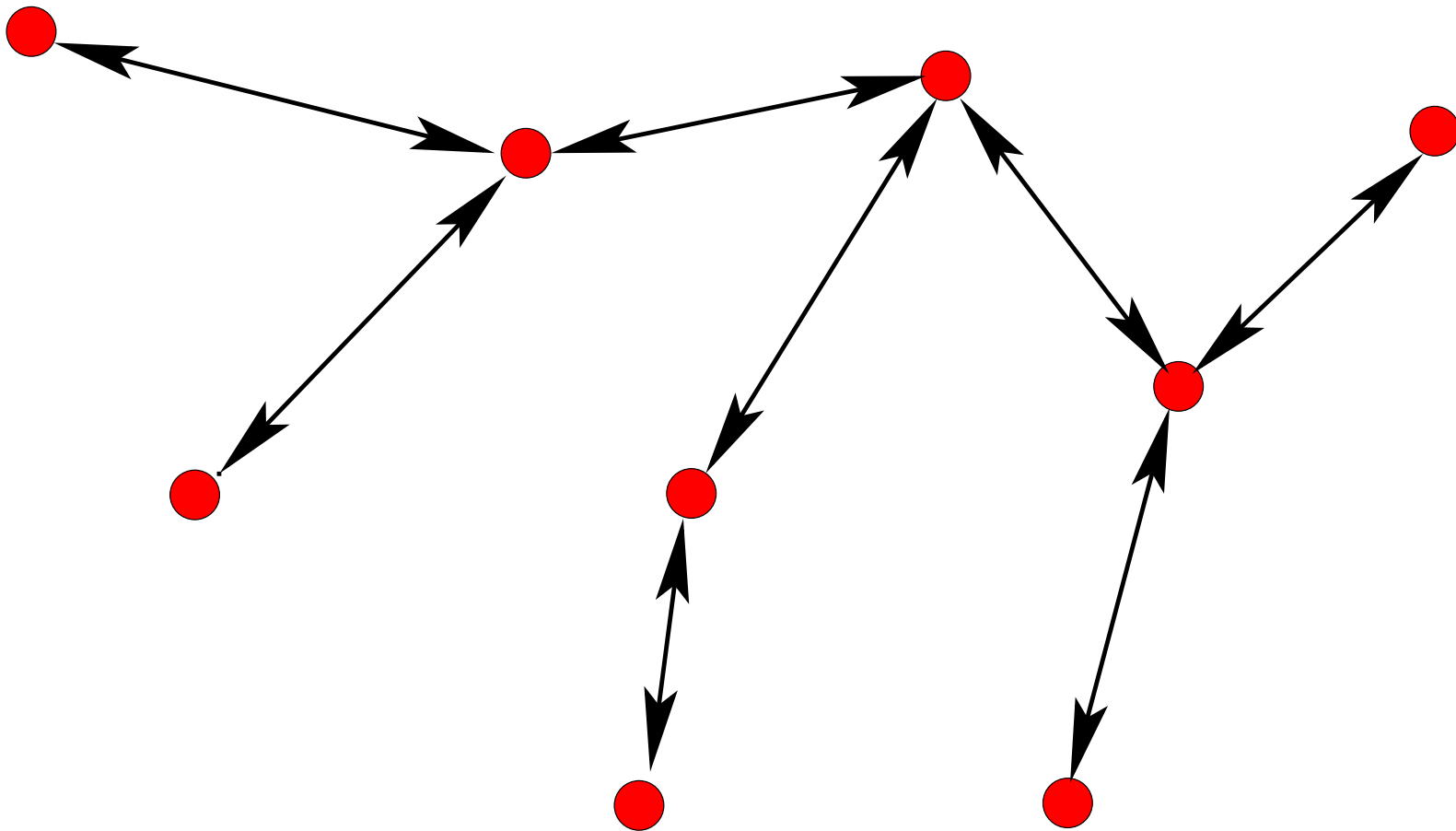


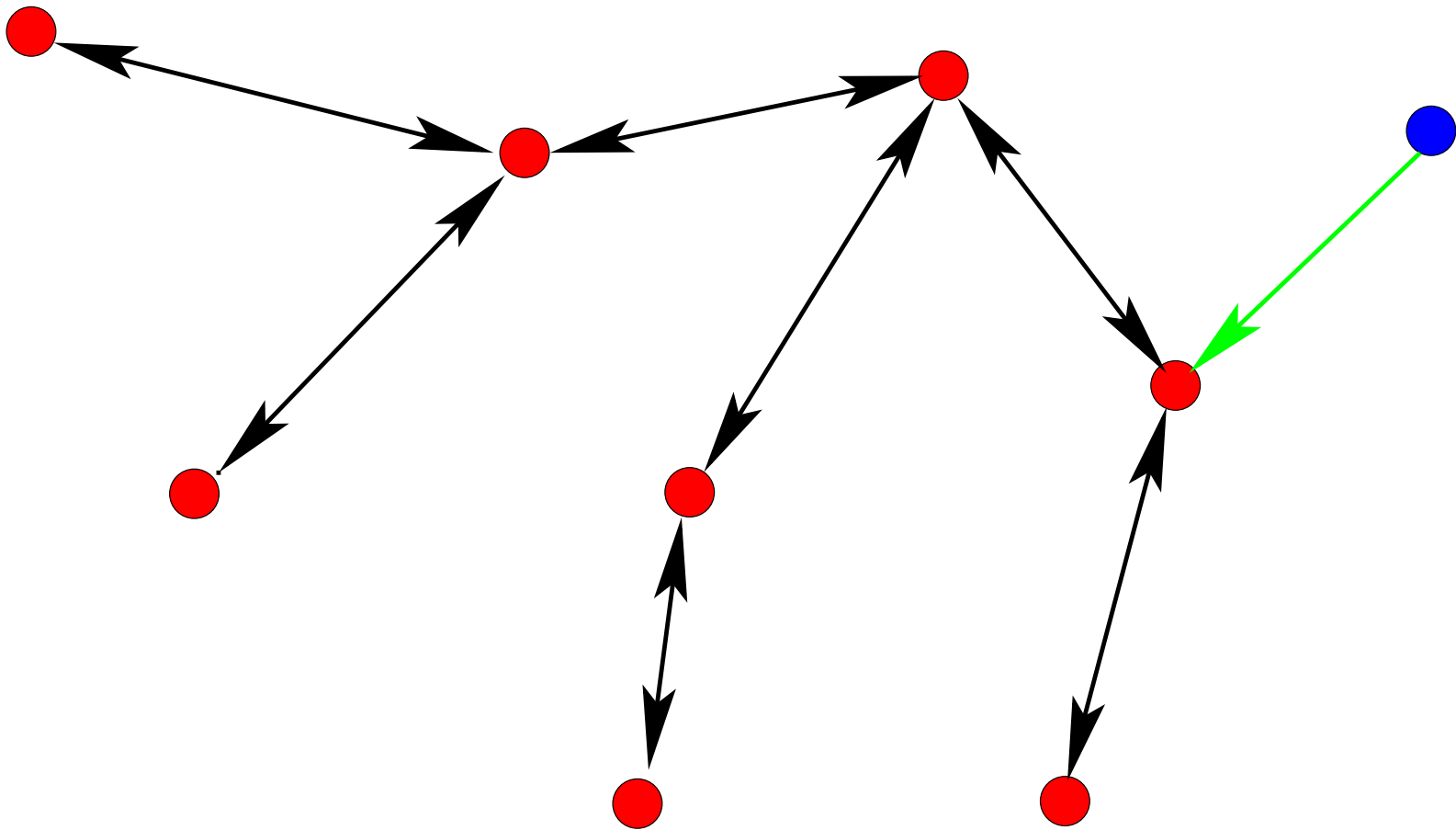


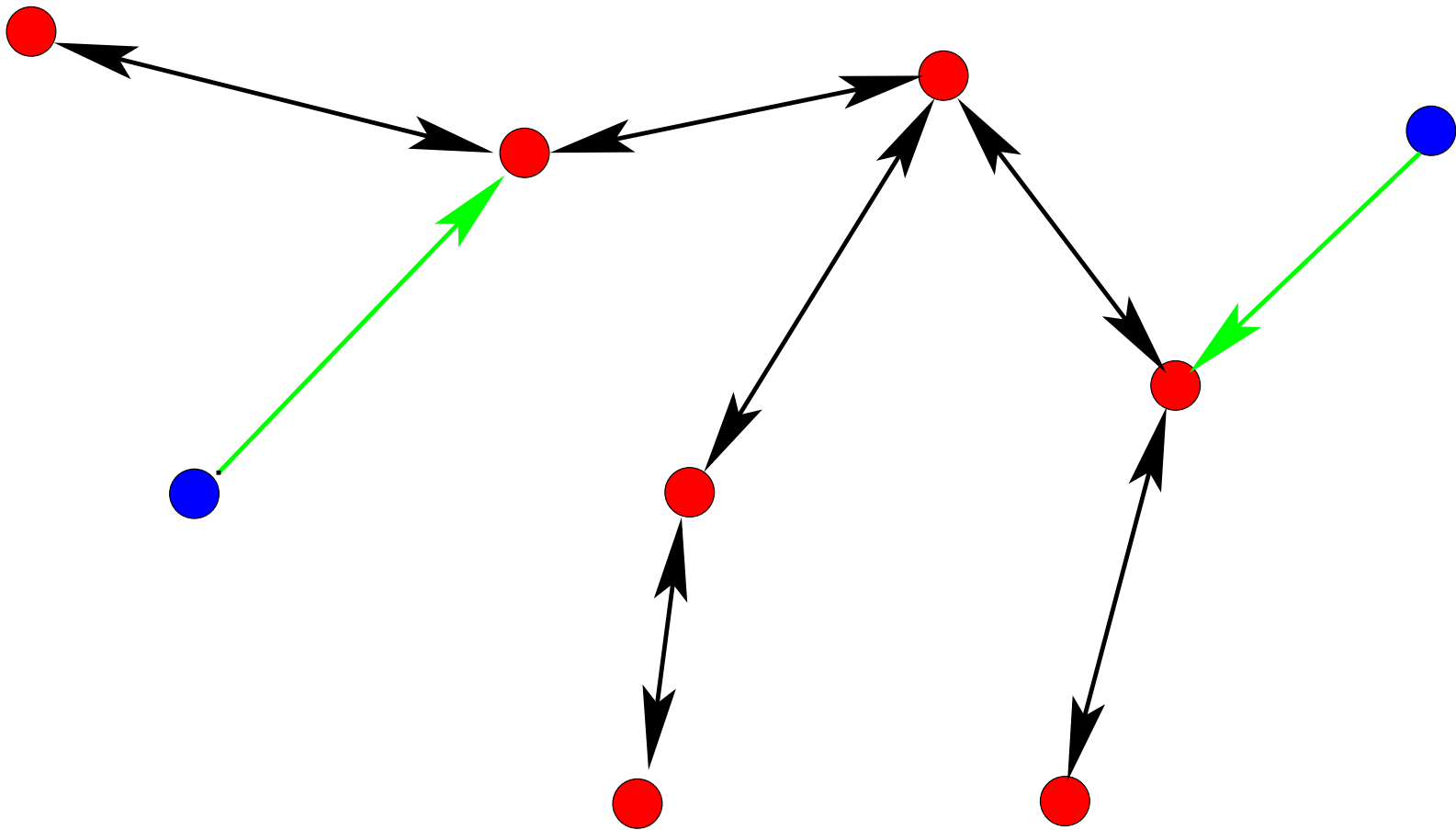


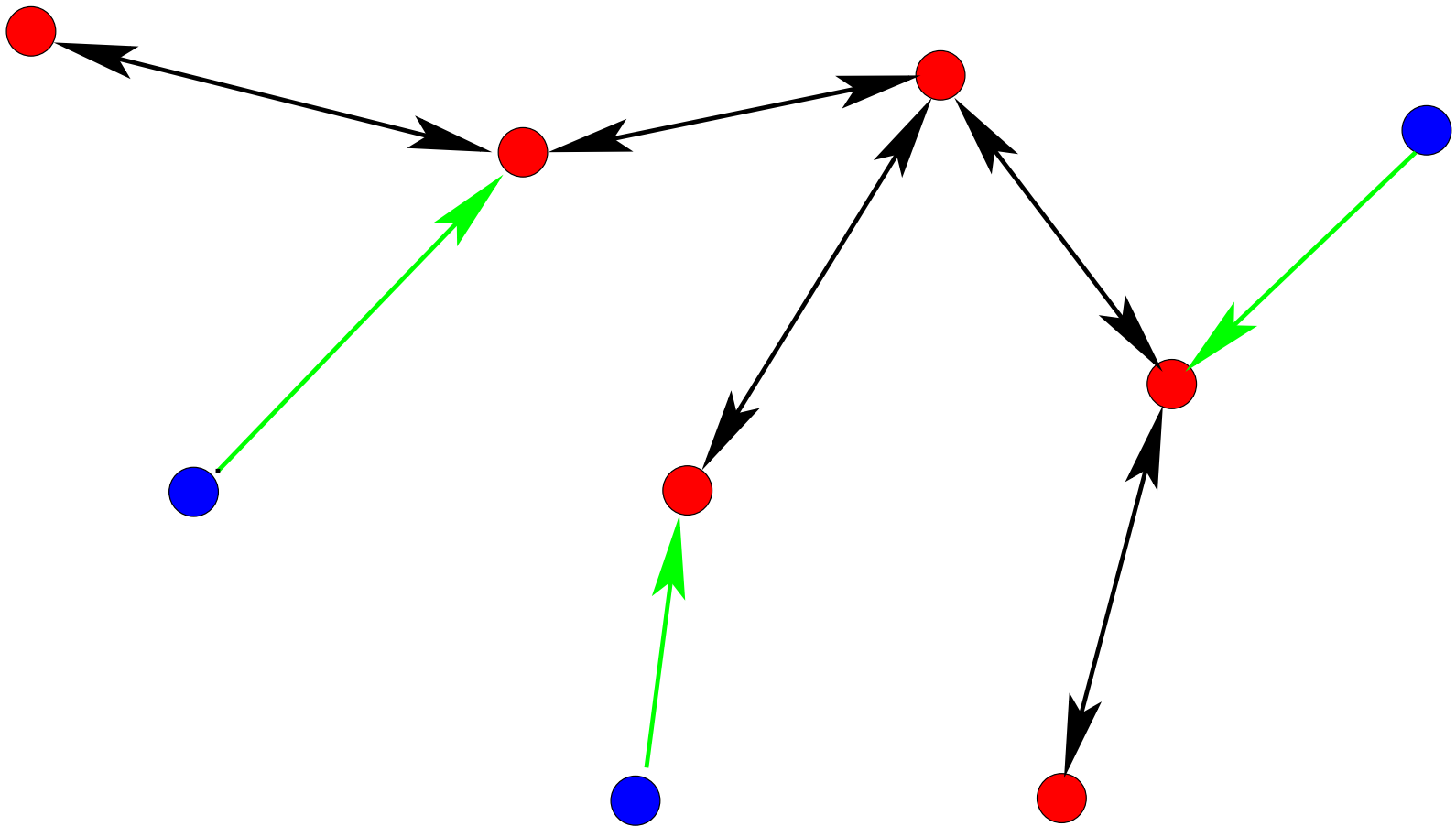


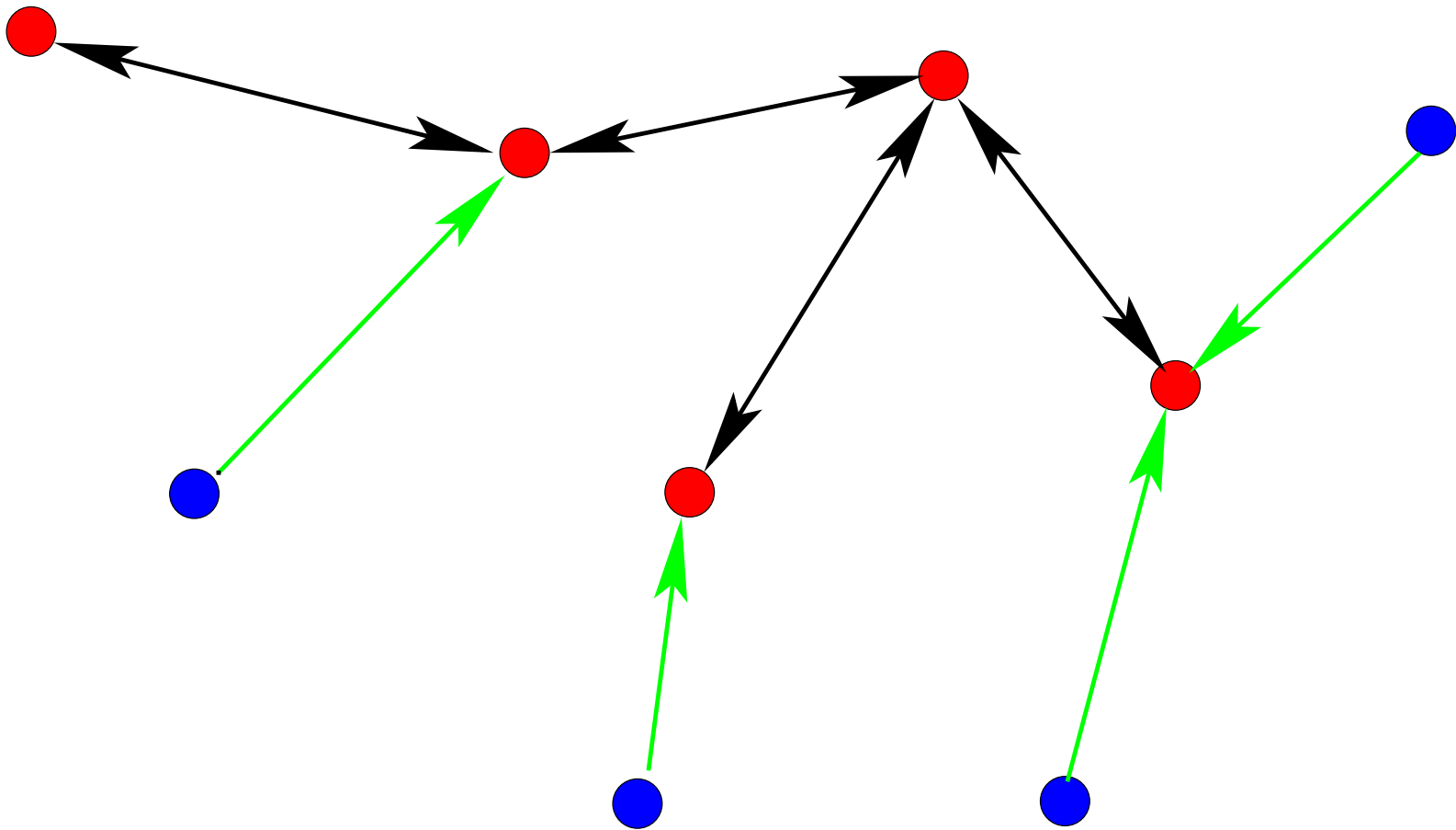




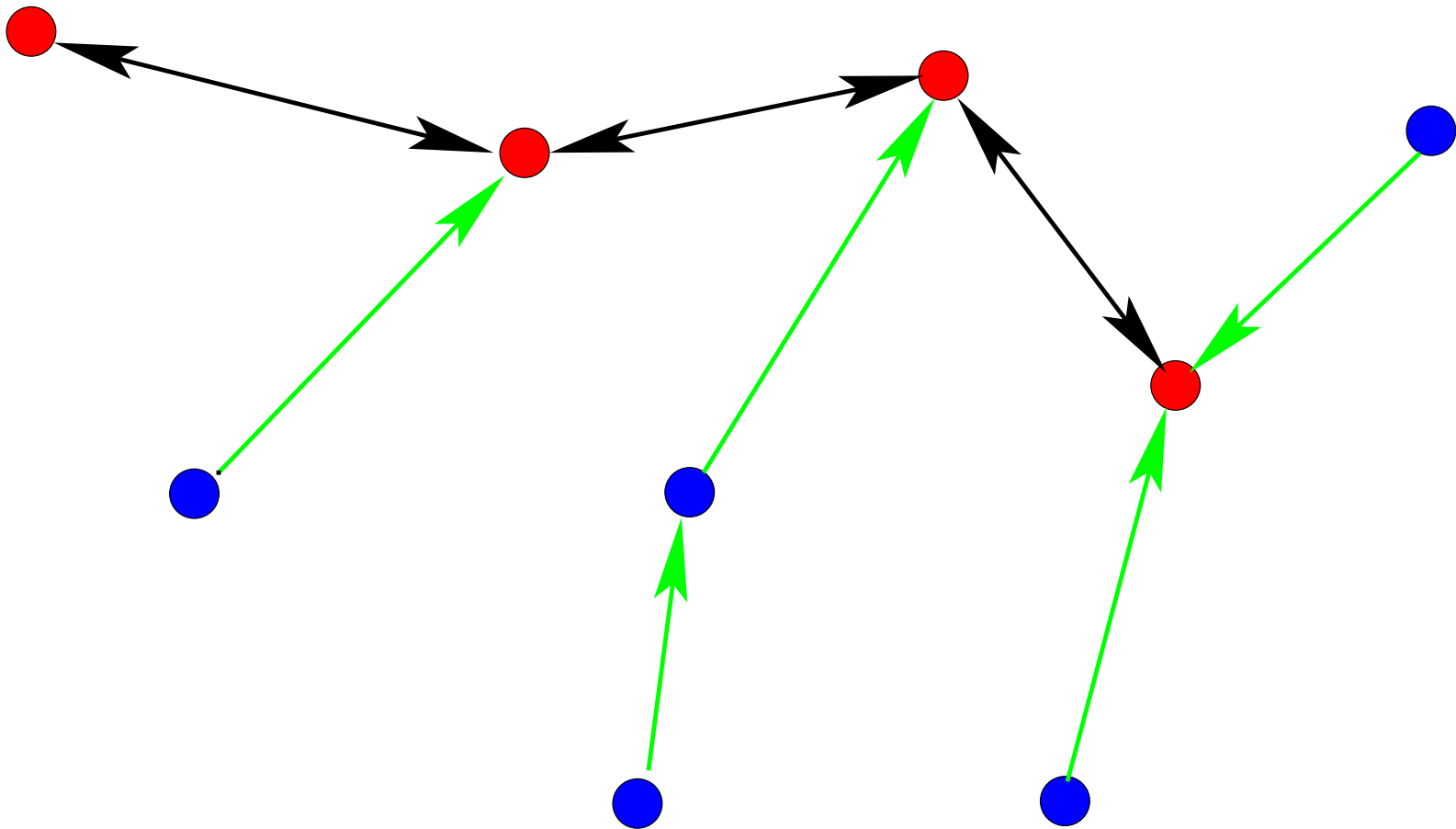


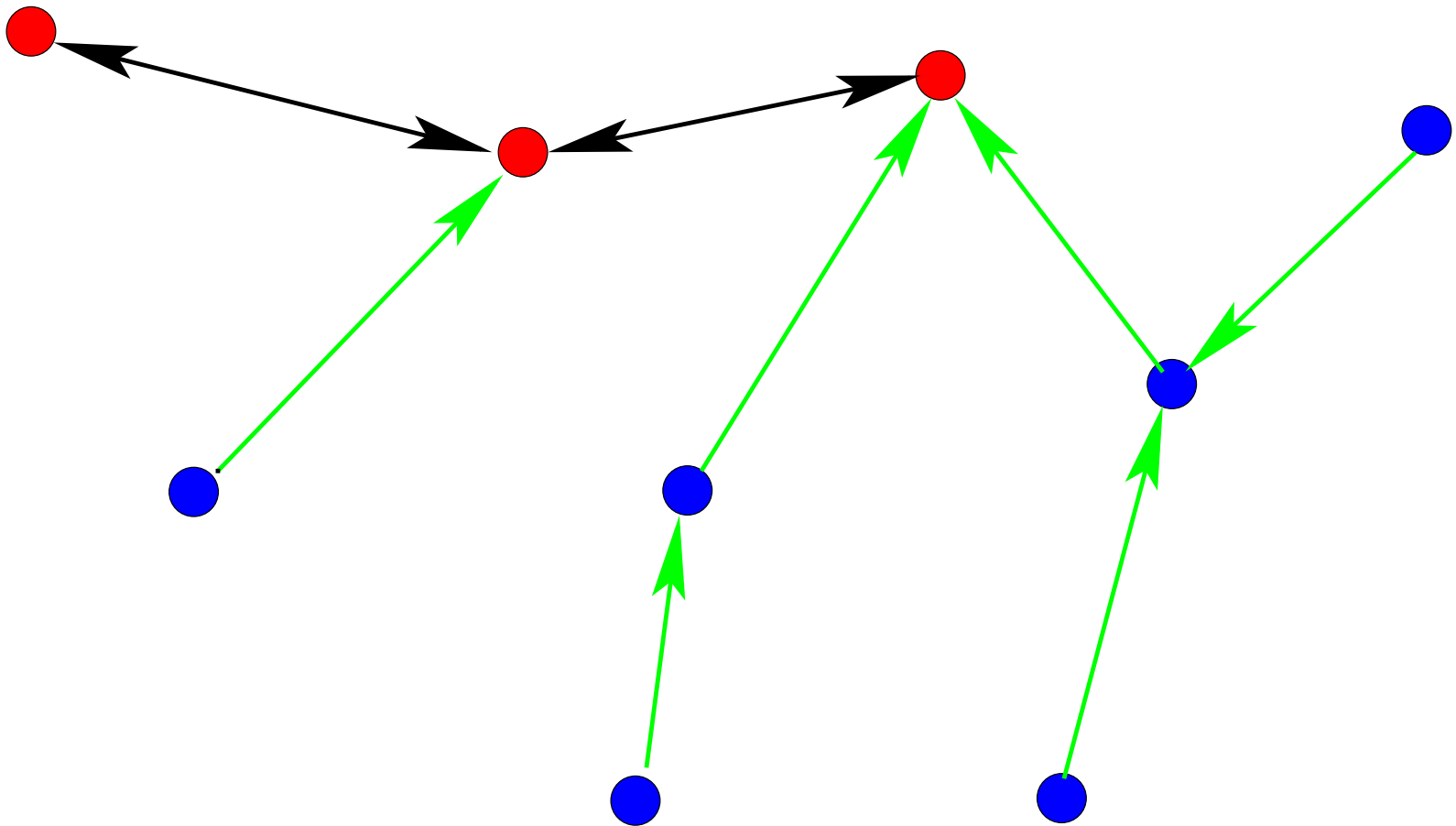


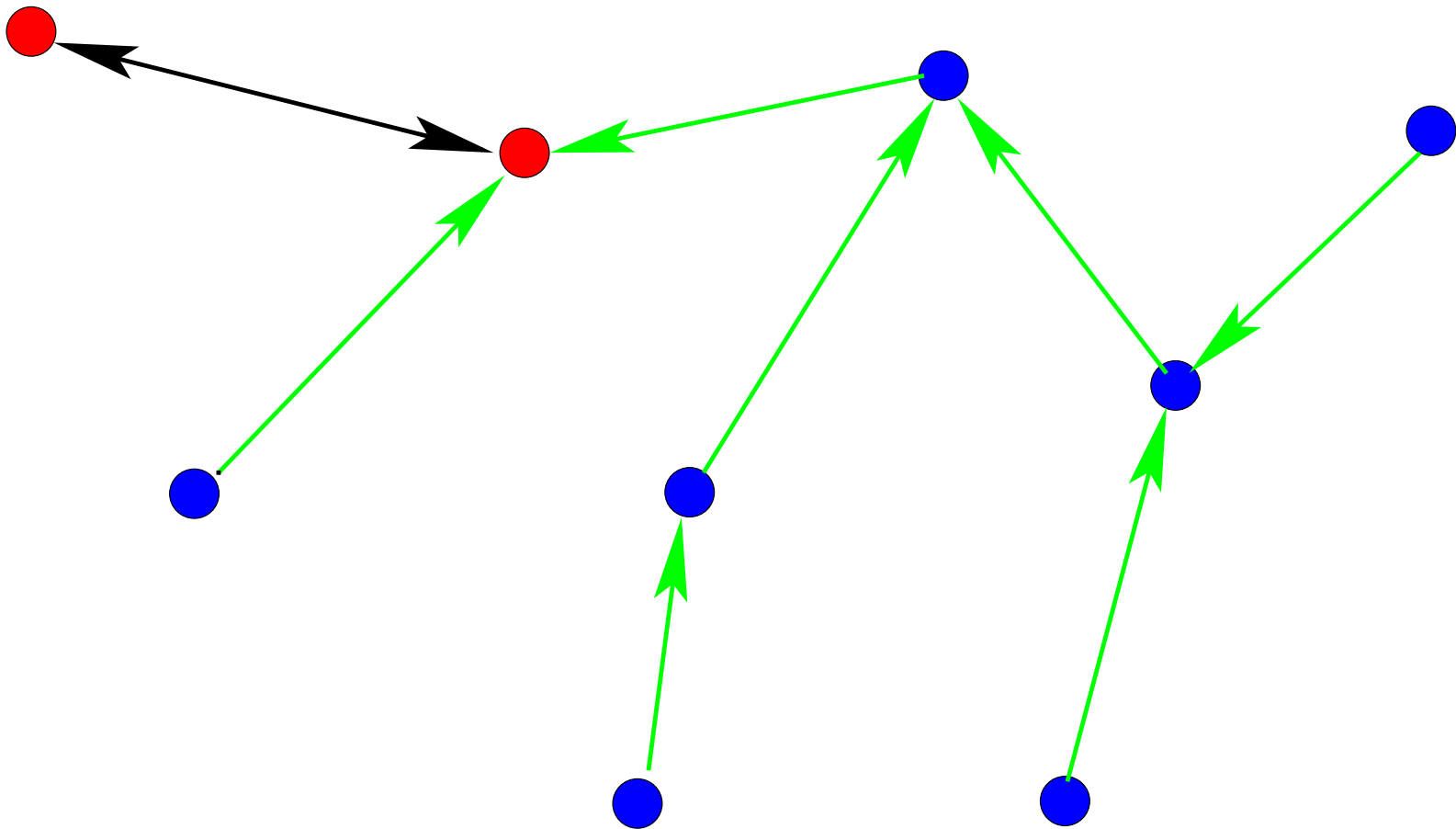


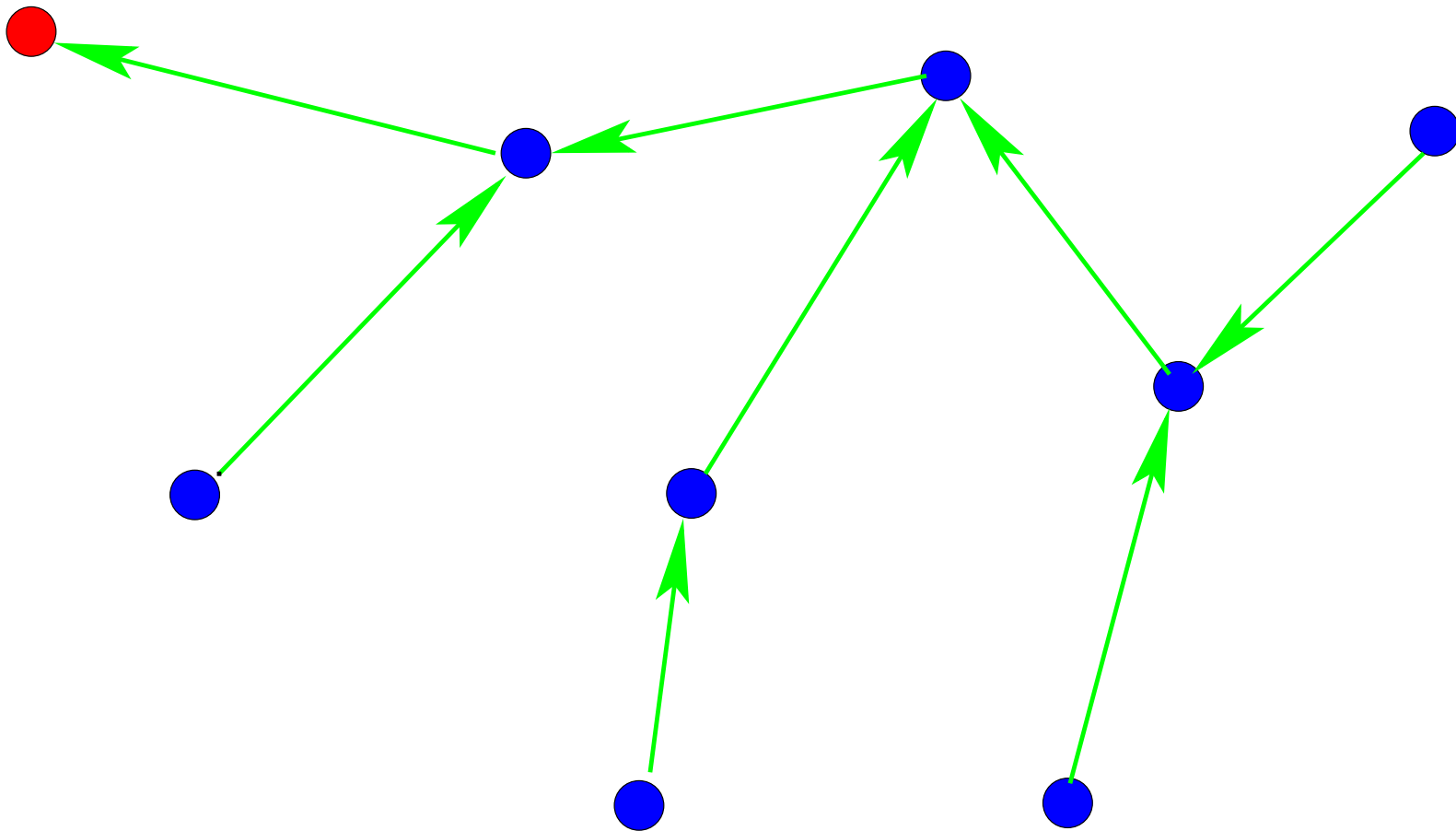








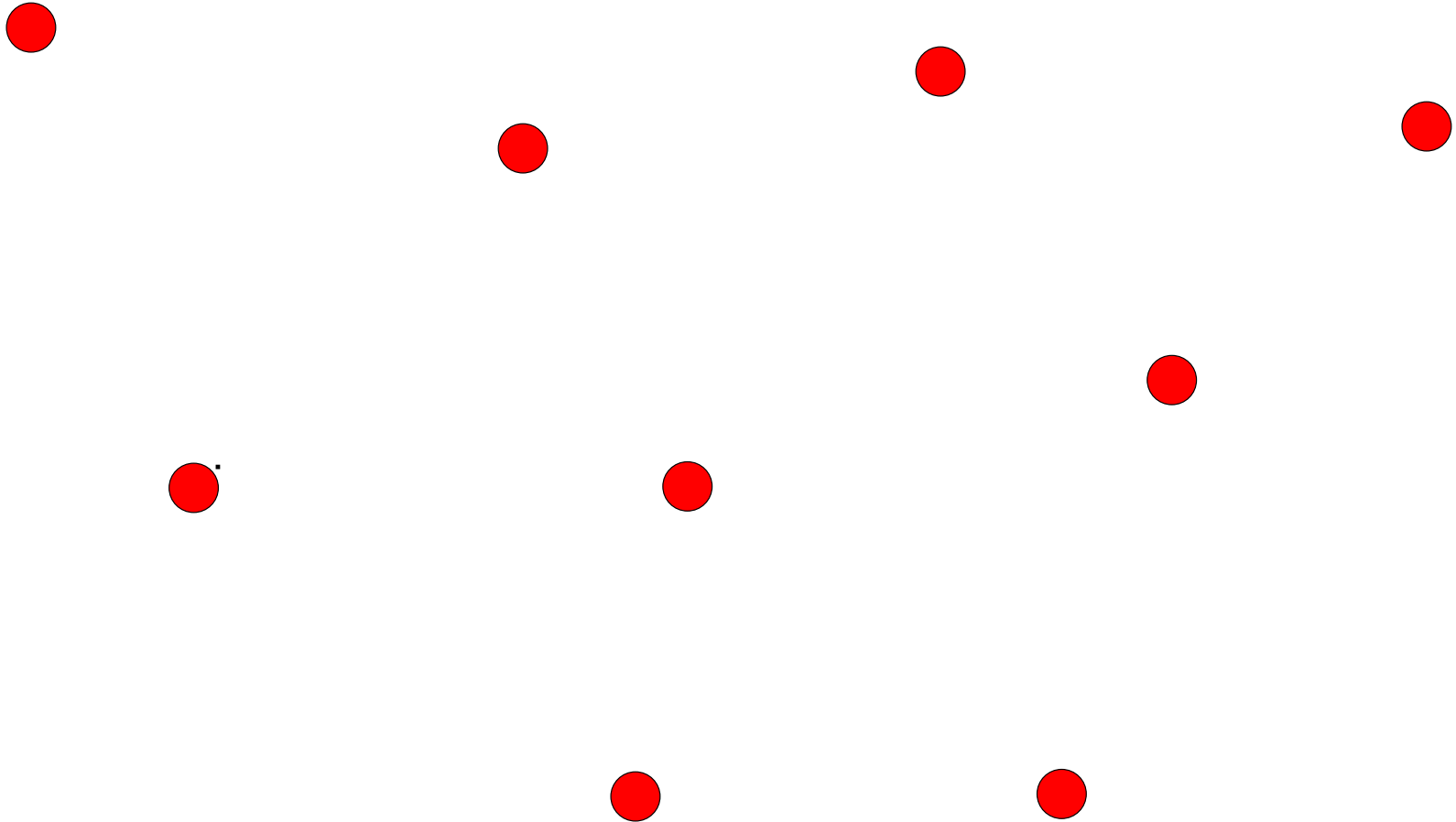




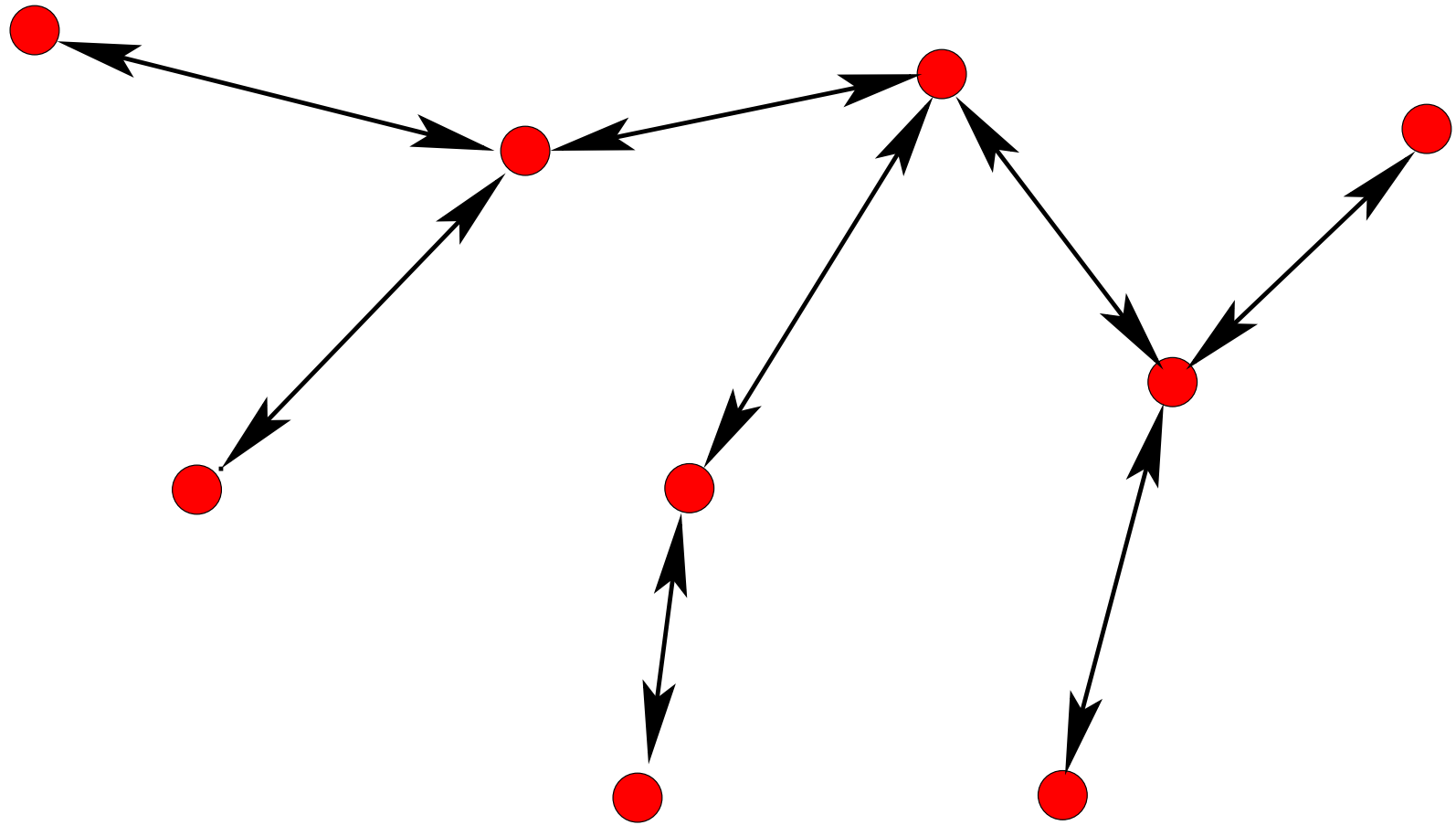
# Summary of Development Process

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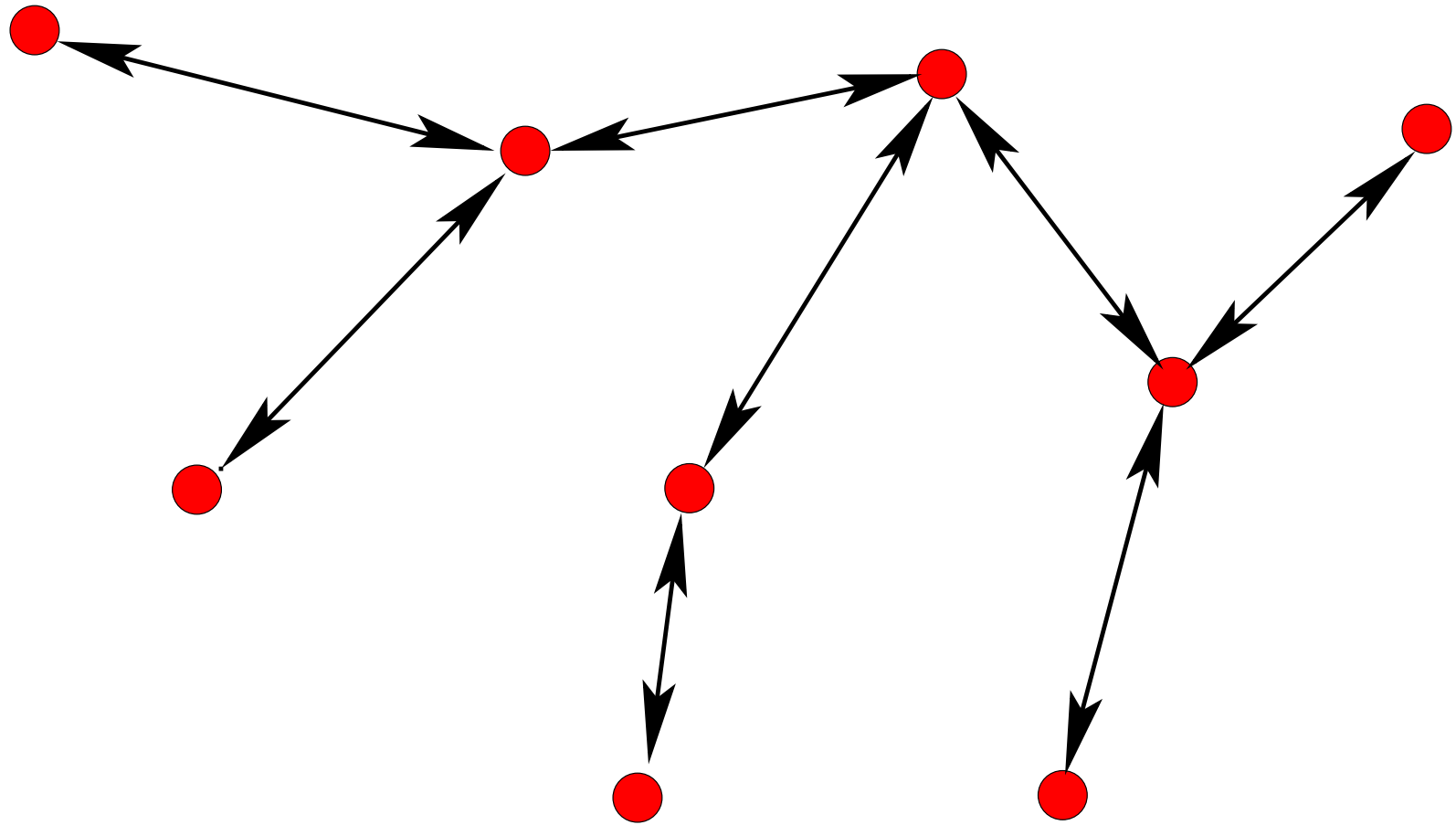
- Formal **definition** and **properties** of the network
  - A **one-shot** abstract model of the protocol
  - Presenting a (still abstract) loop-like **centralized solution**
  - Introducing **message passing** between the nodes (**delays**)
  - Modifying the data structure in order to **distribute the protocol**
-



Let  $ND$  be a set of nodes (with at least 2 nodes)



Let  $gr$  be a graph built and defined on  $ND$



gr is a symmetric and irreflexive graph



*gr* is a graph built on  $ND$

$$gr \subseteq ND \times ND$$

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*gr* is defined on  $ND$

$$\text{dom}(gr) = ND$$

$gr$  is a graph built on  $ND$

$$gr \subseteq ND \times ND$$

$gr$  is defined on  $ND$

$$\text{dom}(gr) = ND$$

$gr$  is symmetric

$$gr = gr^{-1}$$

*gr* is a graph built on  $ND$

$$gr \subseteq ND \times ND$$

*gr* is defined on  $ND$

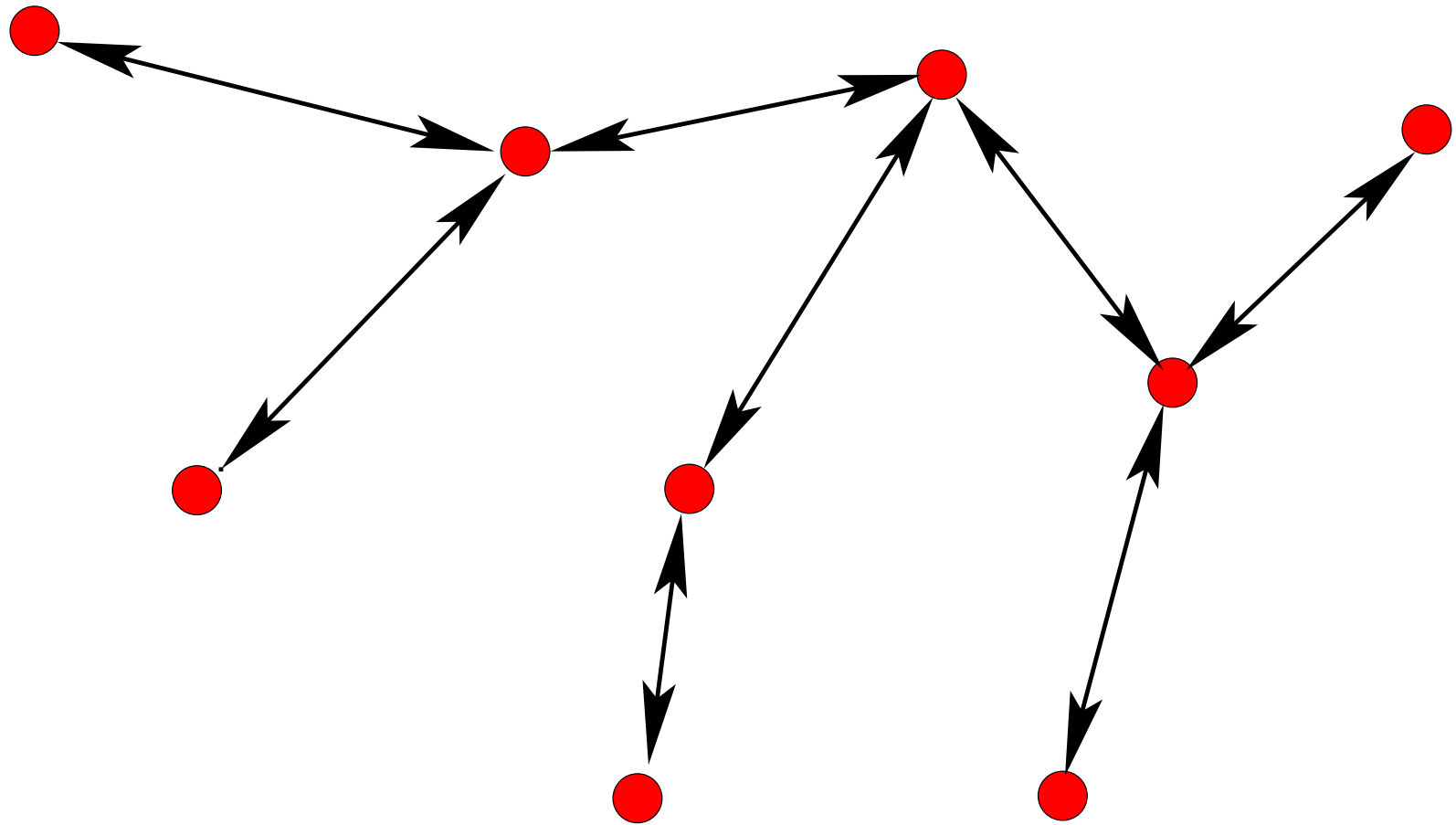
$$\text{dom}(gr) = ND$$

*gr* is symmetric

$$gr = gr^{-1}$$

*gr* is irreflexive

$$\text{id}(ND) \cap gr = \emptyset$$



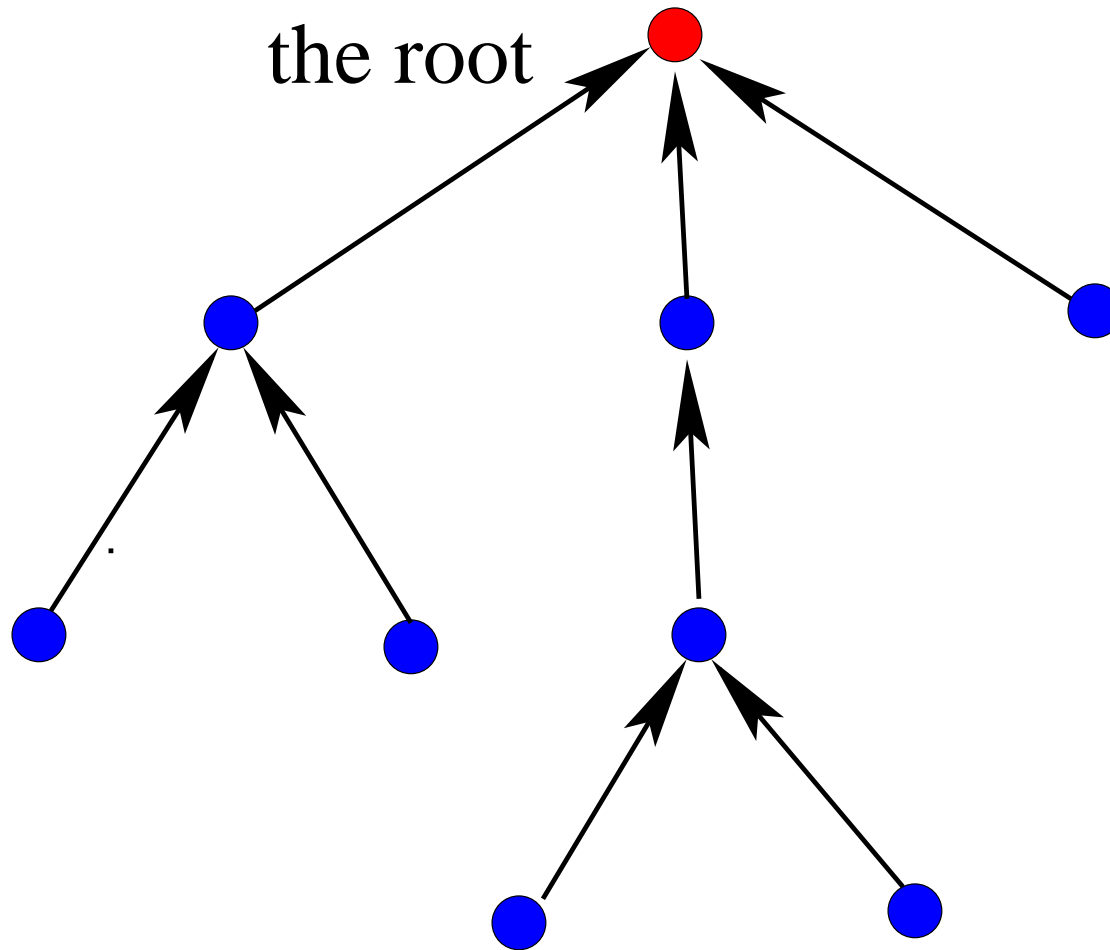
gr is connected and acyclic

# A Little Detour Through Trees

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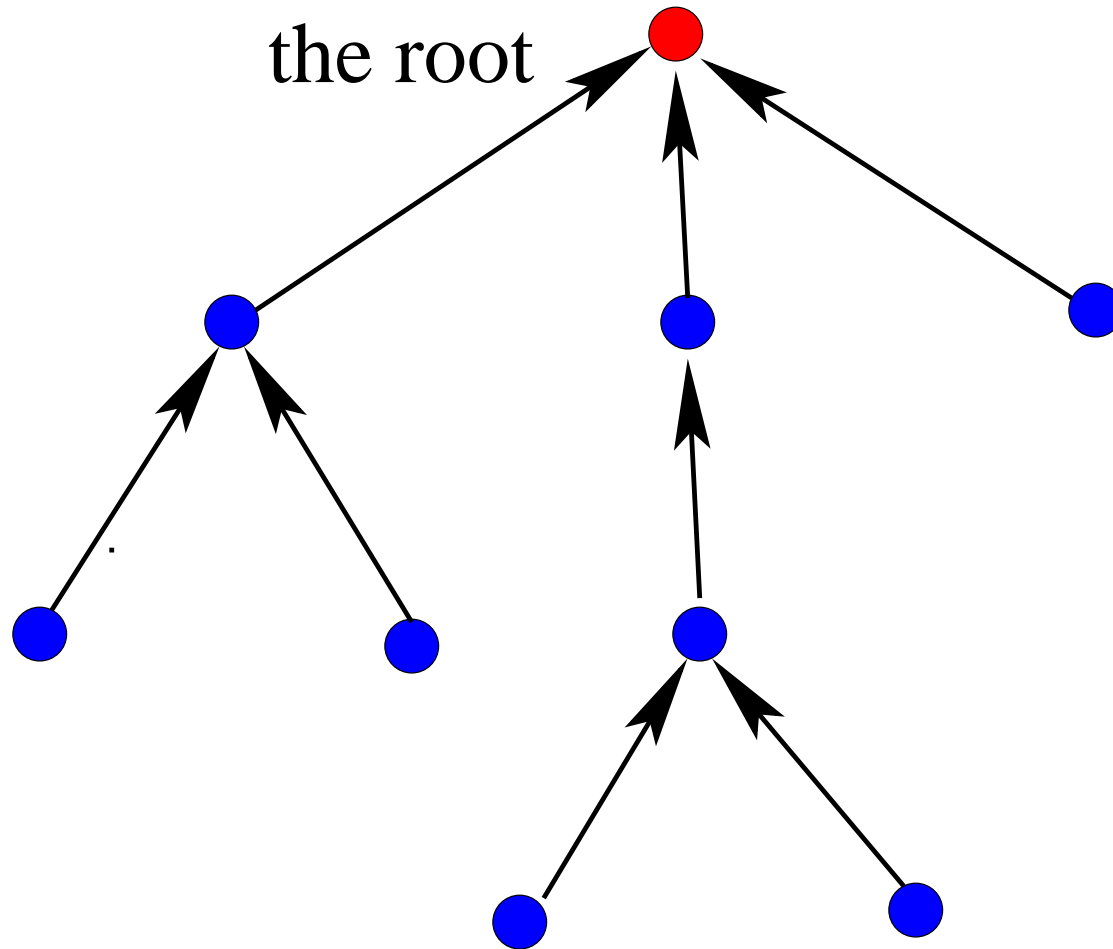
- A tree is a **special graph**
- A tree has a **root**
- A tree has a, so-called, **father function**
- A tree is **acyclic**
- A tree is **connected from the root**



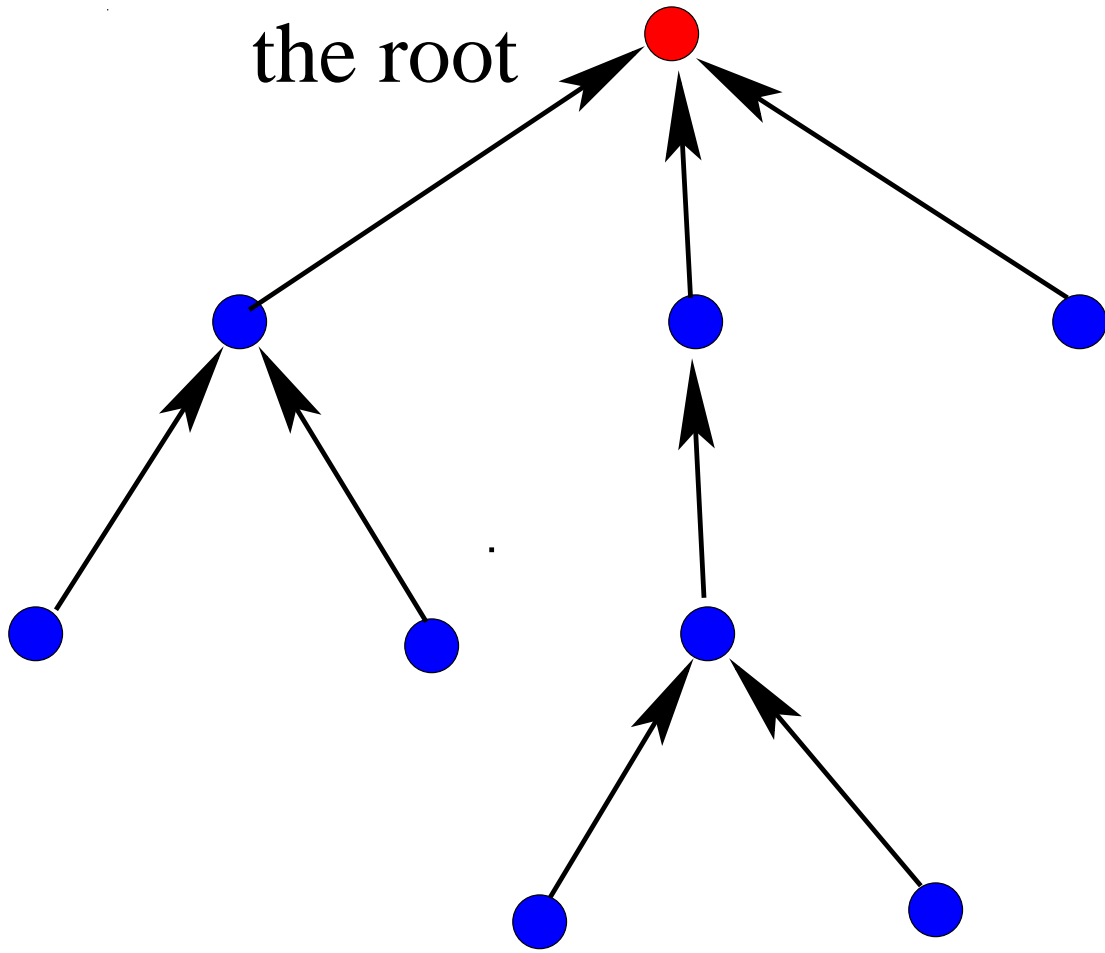


A tree  $t$  built on a set of nodes



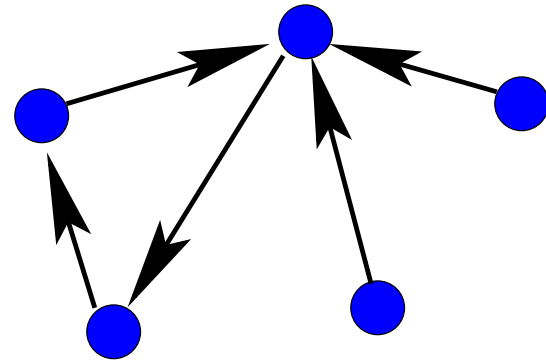


$t$  is a function defined on  $ND$  except at the root

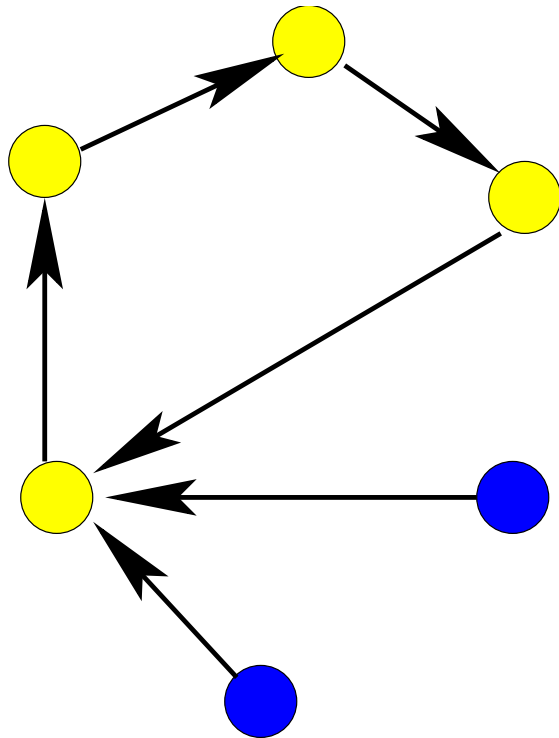


the root

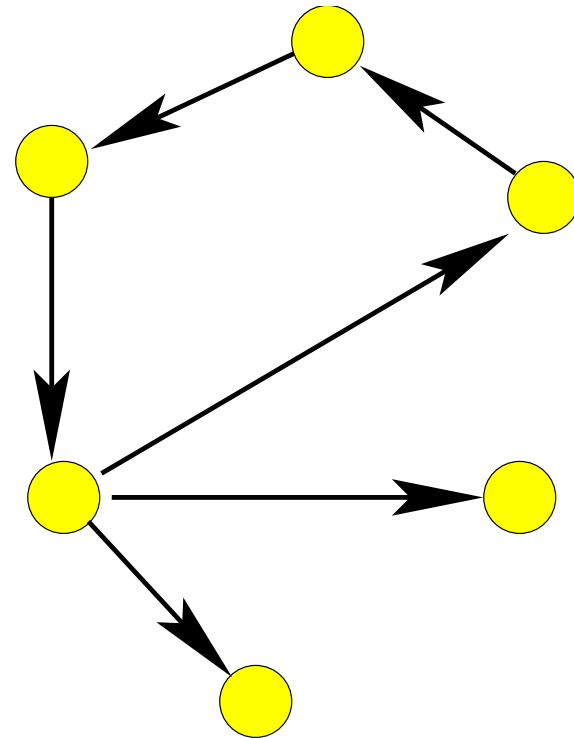
Avoiding cycles



BAD



A cycle



Its inverse image

The nodes of a cycle are included  
in their inverse image

- Given
  - a **set**  $ND$
  - a **subset**  $p$  of  $ND$
  - a **binary relation**  $t$  built on  $ND$
- The **inverse image** of  $p$  under  $t$  is denoted by  $t^{-1}[p]$

$$t^{-1}[p] \equiv \{ x \mid x \in ND \wedge \exists y \cdot (y \in p \wedge (x, y) \in t) \}$$

- When  $t$  is a **partial function**, this reduces to

$$\{ x \mid x \in \text{dom}(t) \wedge t(x) \in p \}$$

- If  $p$  is **included in its inverse image**, we have then:

$$\forall x \cdot (x \in p \Rightarrow x \in \text{dom}(t) \wedge t(x) \in p)$$

- Notice that the **empty set** enjoys this property

$$\emptyset \subseteq t^{-1}[\emptyset]$$

- The property of having **no cycle** is thus equivalent to:

The only subset  $p$  of  $ND$  s.t.  $p \subseteq t^{-1}[p]$  is **EMPTY**

$$\forall p \cdot \left( \begin{array}{l} p \subseteq ND \wedge \\ p \subseteq t^{-1}[p] \\ \Rightarrow \\ p = \emptyset \end{array} \right)$$

The predicate **tree** ( $r, t$ )

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$r$  is a member of  $ND$     $r \in ND$



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$t$  is a function     $t \in ND - \{r\} \rightarrow ND$

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$t$  is a function     $t \in ND - \{r\} \rightarrow ND$

$t$  is acyclic     $\forall p \cdot \left( \begin{array}{l} p \subseteq ND \wedge \\ p \subseteq t^{-1}[p] \\ \Rightarrow \\ p = \emptyset \end{array} \right)$

$t$  is acyclic: **equivalent formulations**

$$\forall p \cdot \left( \begin{array}{l} p \subseteq ND \wedge \\ p \subseteq t^{-1}[p] \\ \Rightarrow \\ p = \emptyset \end{array} \right) \Leftrightarrow \forall q \cdot \left( \begin{array}{l} q \subseteq ND \wedge \\ r \in q \wedge \\ t^{-1}[q] \subseteq q \\ \Rightarrow \\ ND \subseteq q \end{array} \right)$$

This gives an **Induction Rule**

$$\forall q \cdot \left( \begin{array}{l} q \subseteq ND \wedge \\ r \in q \wedge \\ \forall x \cdot (x \in ND - \{r\} \wedge t(x) \in q \Rightarrow x \in q) \\ \Rightarrow \\ ND \subseteq q \end{array} \right)$$

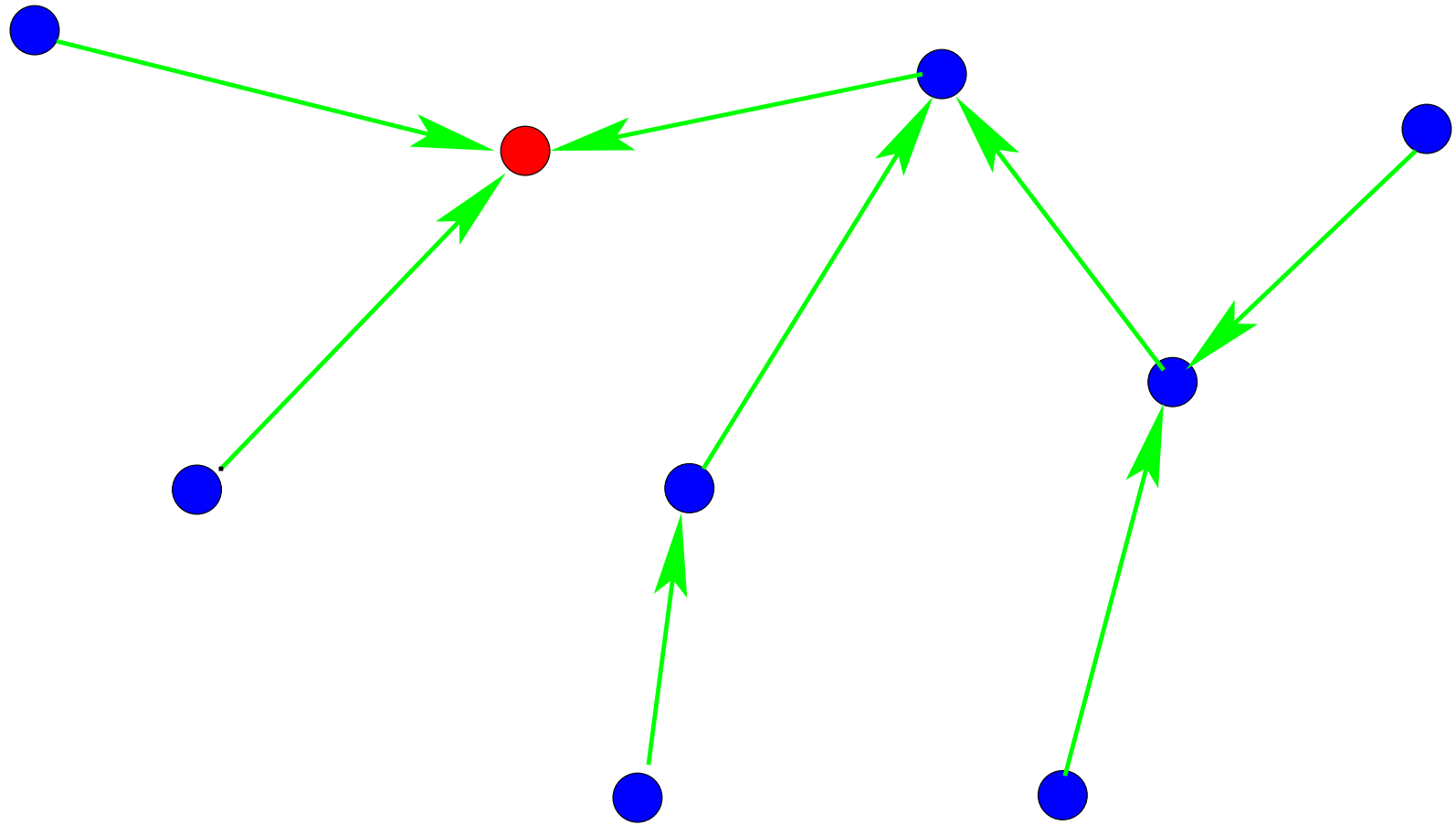
The predicate  $\text{tree}(r, t)$

$r$  is a member of  $ND$   $r \in ND$

$t$  is a function  $t \in ND - \{r\} \rightarrow ND$

$t$  is acyclic

$$\forall q \cdot \left( \begin{array}{l} q \subseteq ND \wedge \\ r \in q \wedge \\ t^{-1}[q] \subseteq q \\ \Rightarrow \\ ND \subseteq q \end{array} \right)$$



A spanning tree  $t$  of the graph  $gr$

The predicate **spanning** ( $r, t, gr$ )

$r, t$  is a tree

$\text{tree}(r, t)$

$t$  is included in  $gr$

$t \subseteq gr$

# The graph $gr$ is connected and acyclic (1)

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- Defining a relation  $f_n$  linking a node to the possible spanning trees of  $gr$  having that node as a root:

$$f_n \subseteq ND \times (ND \rightarrow ND)$$

$$\forall (r, t) \cdot \left( \begin{array}{l} r \in ND \wedge \\ t \in ND \rightarrow ND \\ \Rightarrow \\ (r, t) \in f_n \Leftrightarrow \text{spanning}(r, t, gr) \end{array} \right)$$

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## The graph $gr$ is connected and acyclic (2)

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Totality of relation  $fn \Rightarrow$  Connectivity of  $gr$

Functionality of relation  $fn \Rightarrow$  Acyclicity of  $gr$

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# Summary of constants $gr$ and $fn$

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$$gr \subseteq ND \times ND$$

$$\text{dom}(gr) = ND$$

$$gr = gr^{-1}$$

$$\text{id}(ND) \cap gr = \emptyset$$

$$fn \in ND \rightarrow (ND \leftrightarrow ND)$$

$$\forall(r, t) \cdot \left( \begin{array}{l} r \in ND \wedge \\ t \in ND \leftrightarrow ND \\ \Rightarrow \\ t = fn(r) \Leftrightarrow \text{spanning}(r, t, gr) \end{array} \right)$$

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# Election in One Shot: Building a Spanning Tree

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- Variables  $rt$  and  $ts$

$$rt \in ND$$

$$ts \in ND \leftrightarrow ND$$

elect  $\hat{=}$

**BEGIN**

$rt, ts : \text{spanning}(rt, ts, gr)$

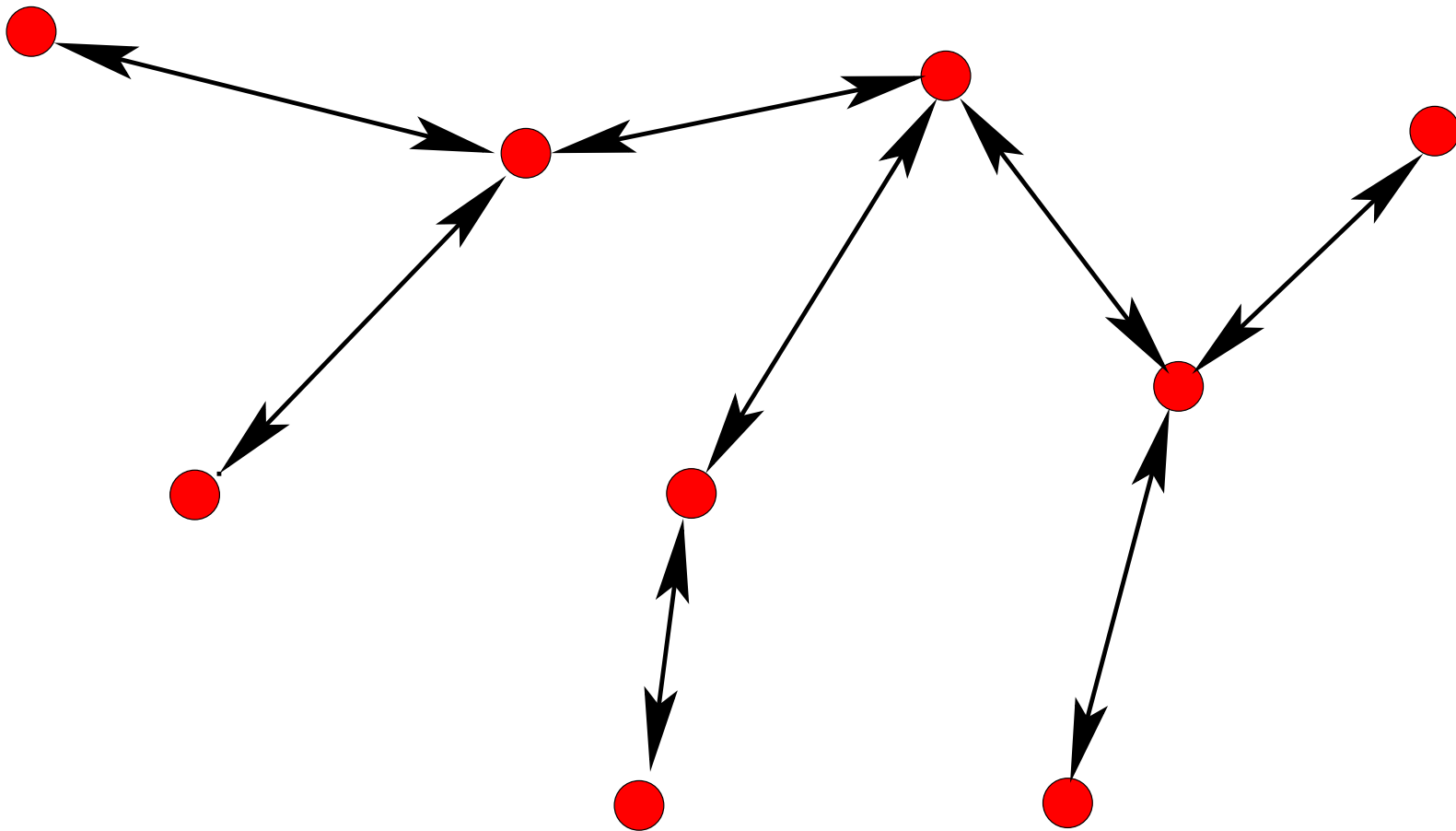
**END**

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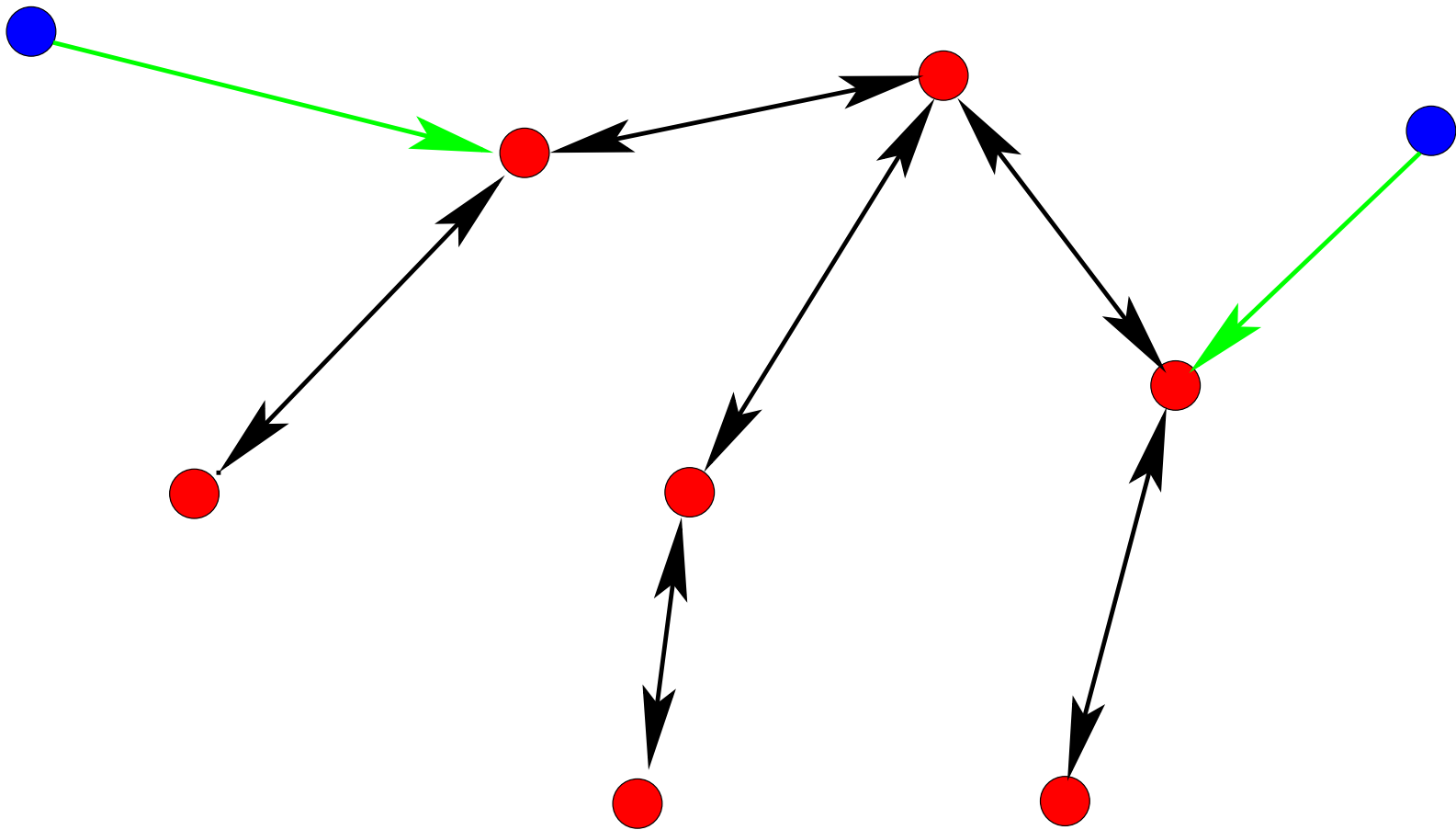
# First Refinement (1)

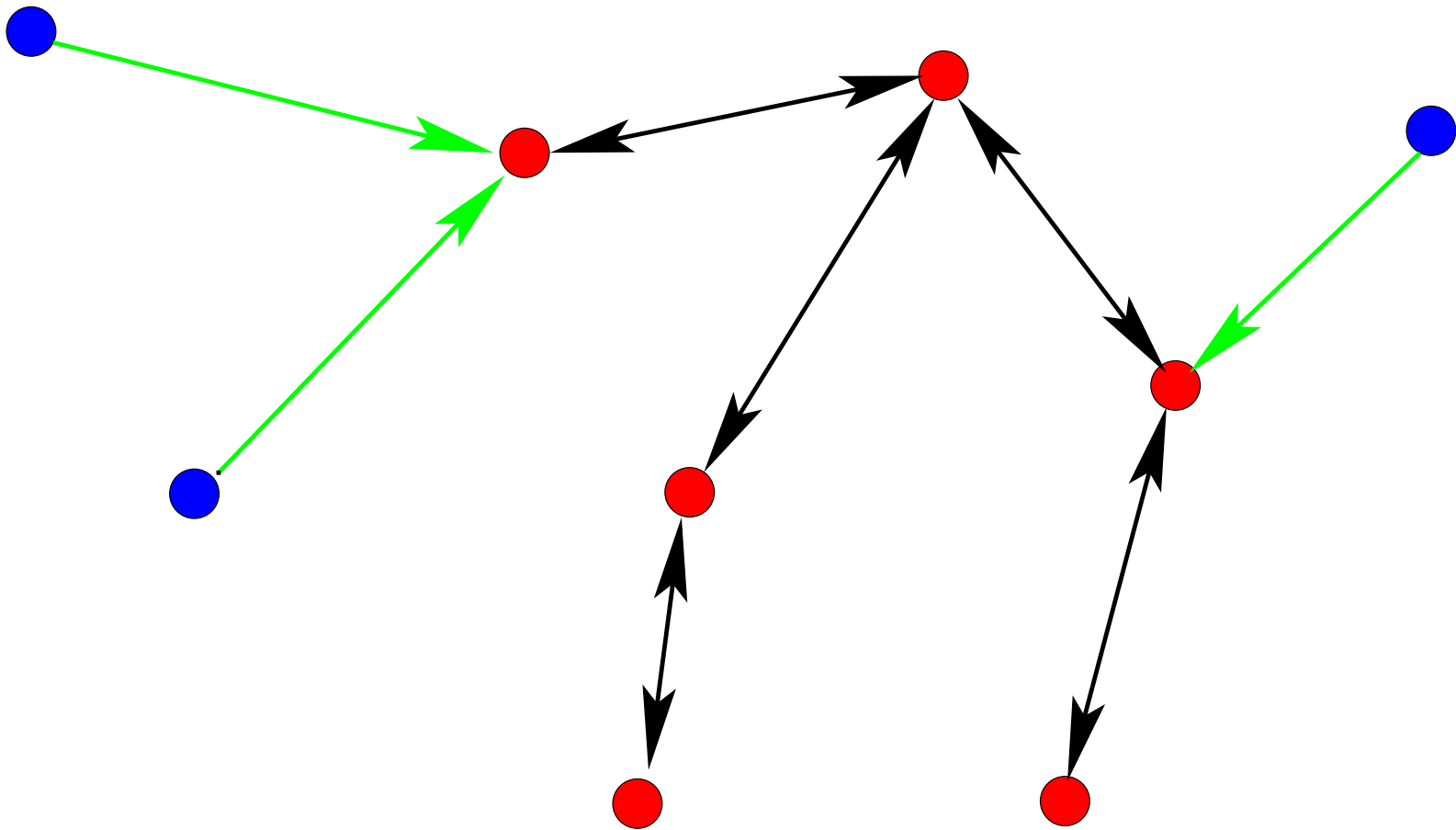
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- Introducing a **new variable**,  $tr$ , corresponding to the **"tree" in construction**
  - Introducing a **new event**: the **progression event**
  - Defining the **invariant**
  - Back to the **animation** : Observe the construction of the tree
-

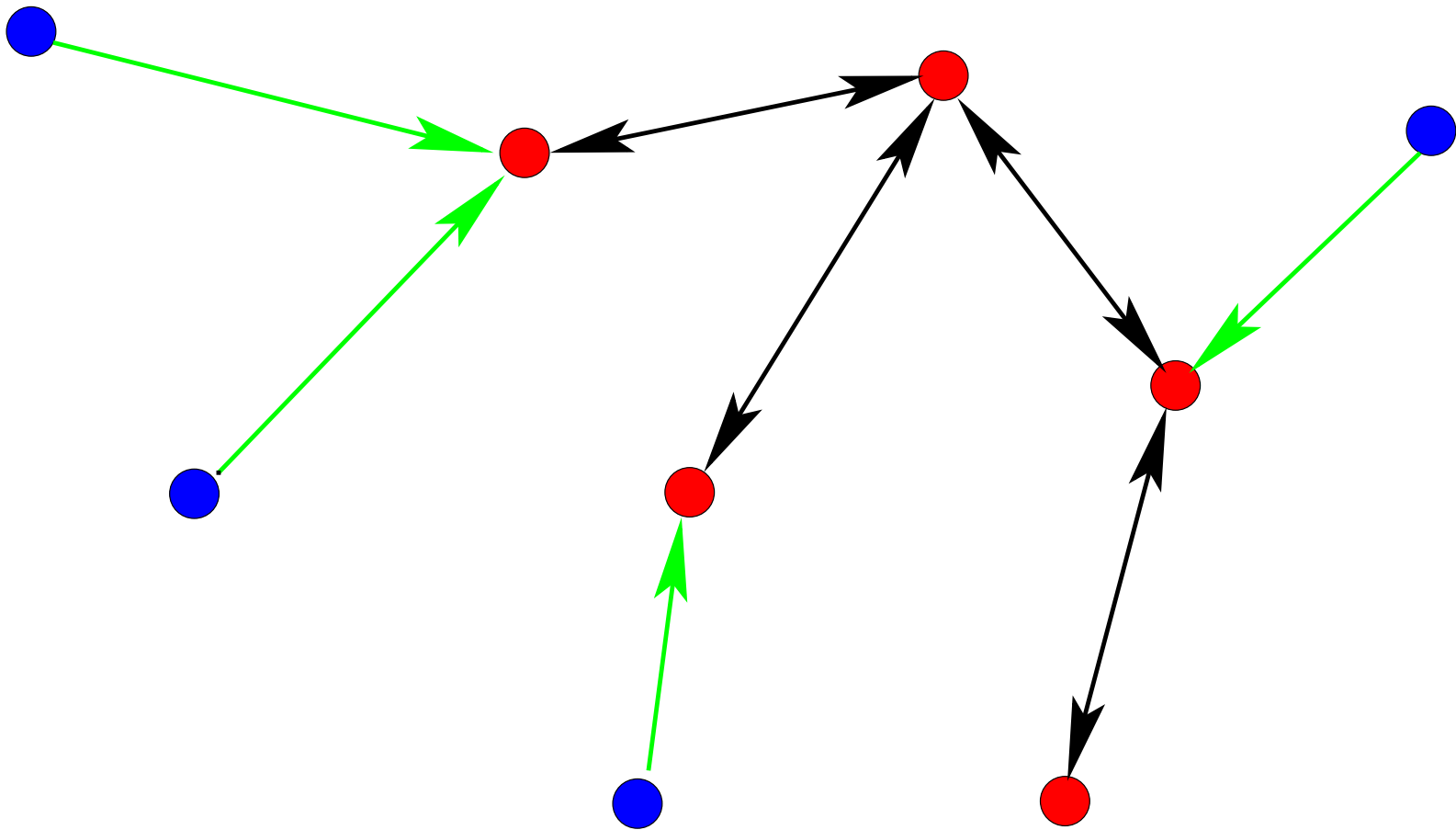


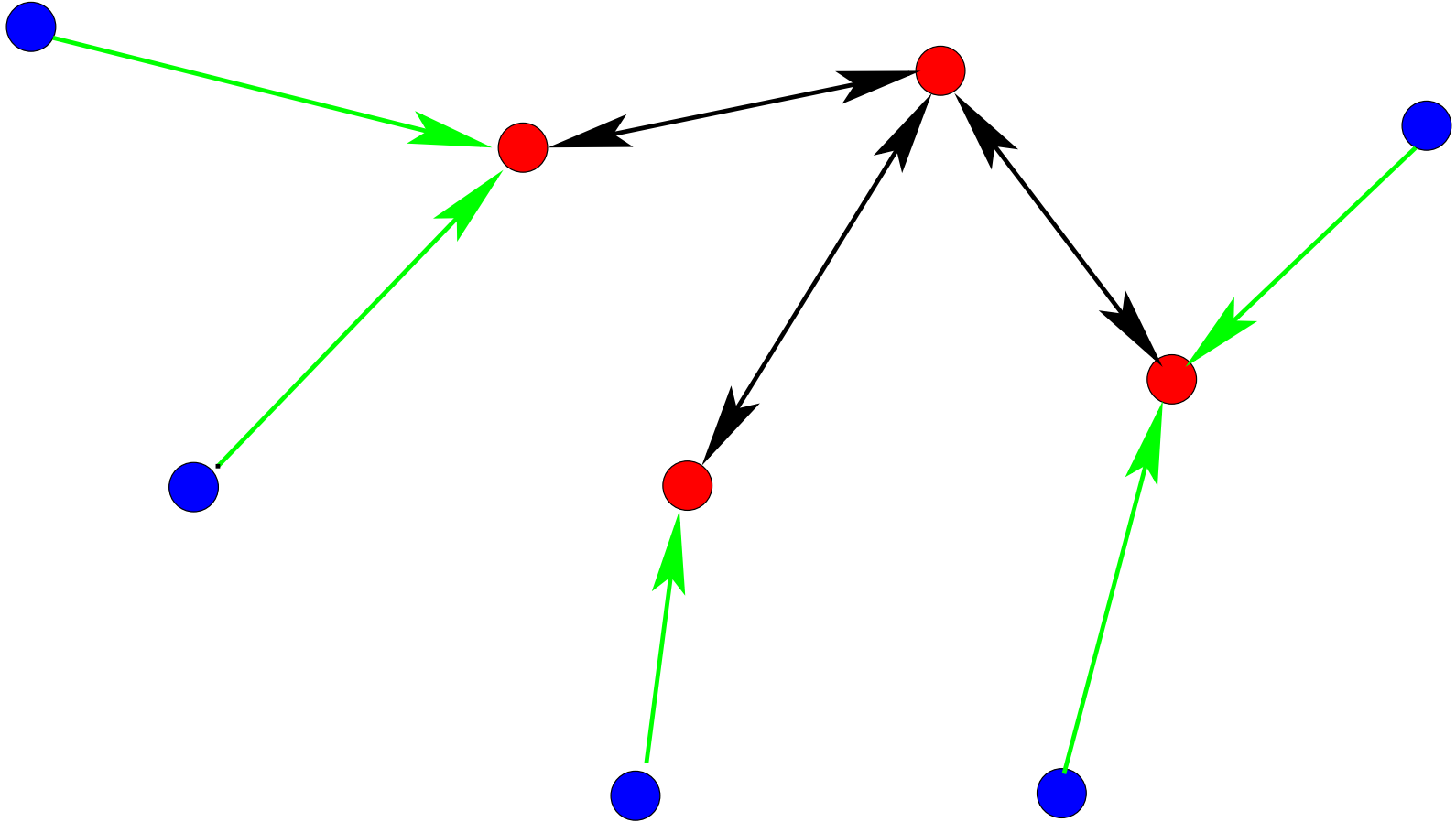


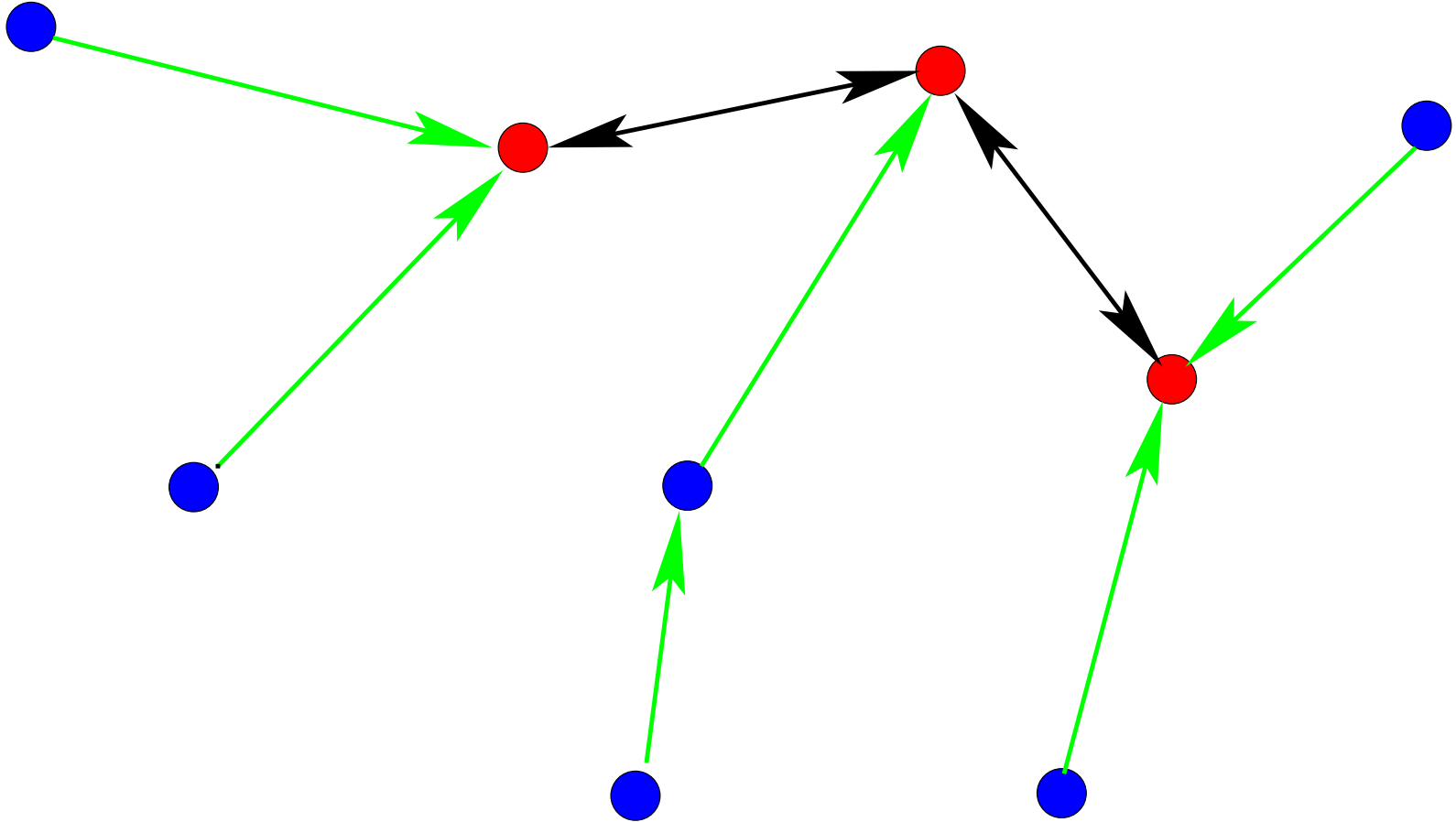


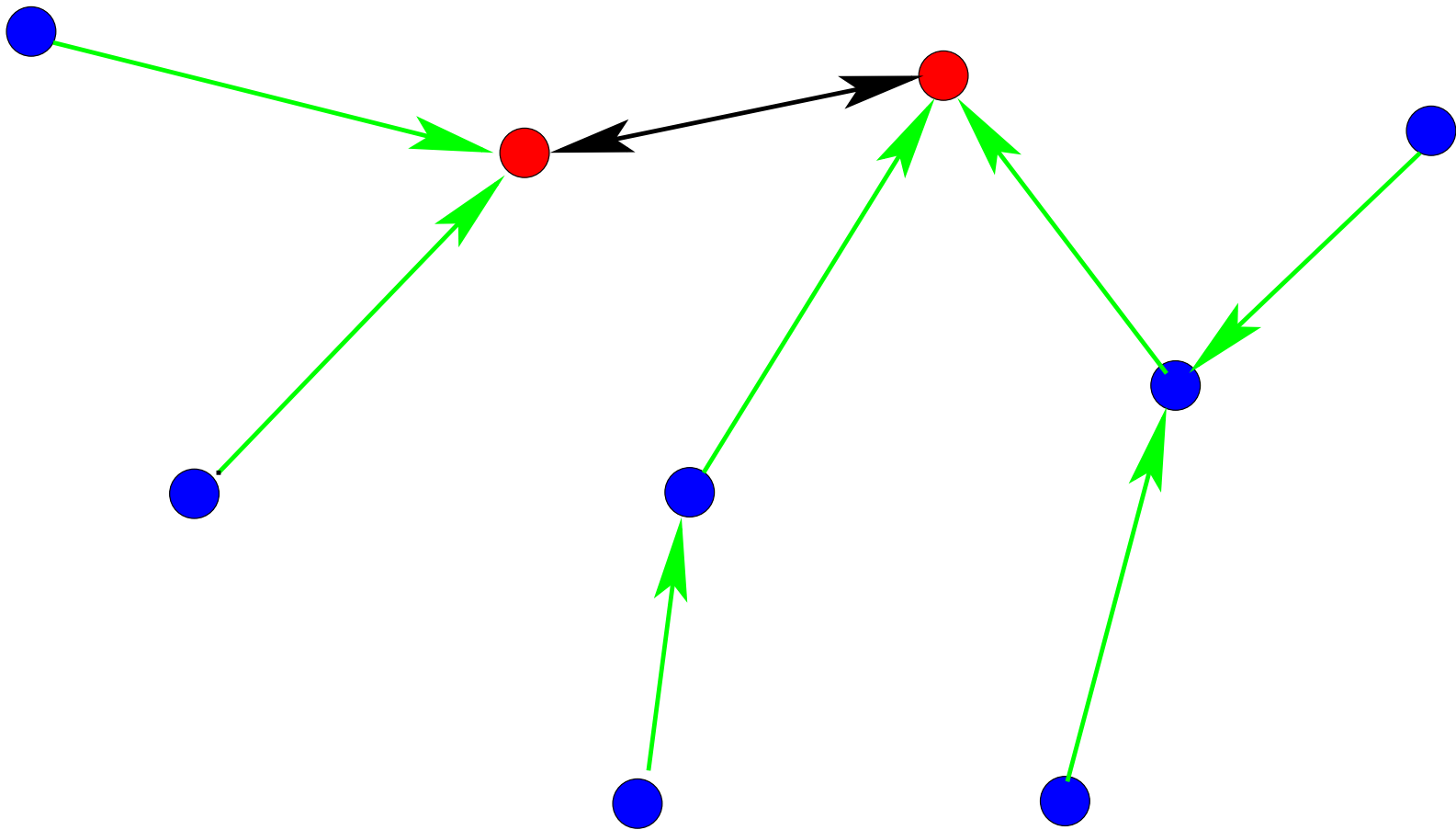


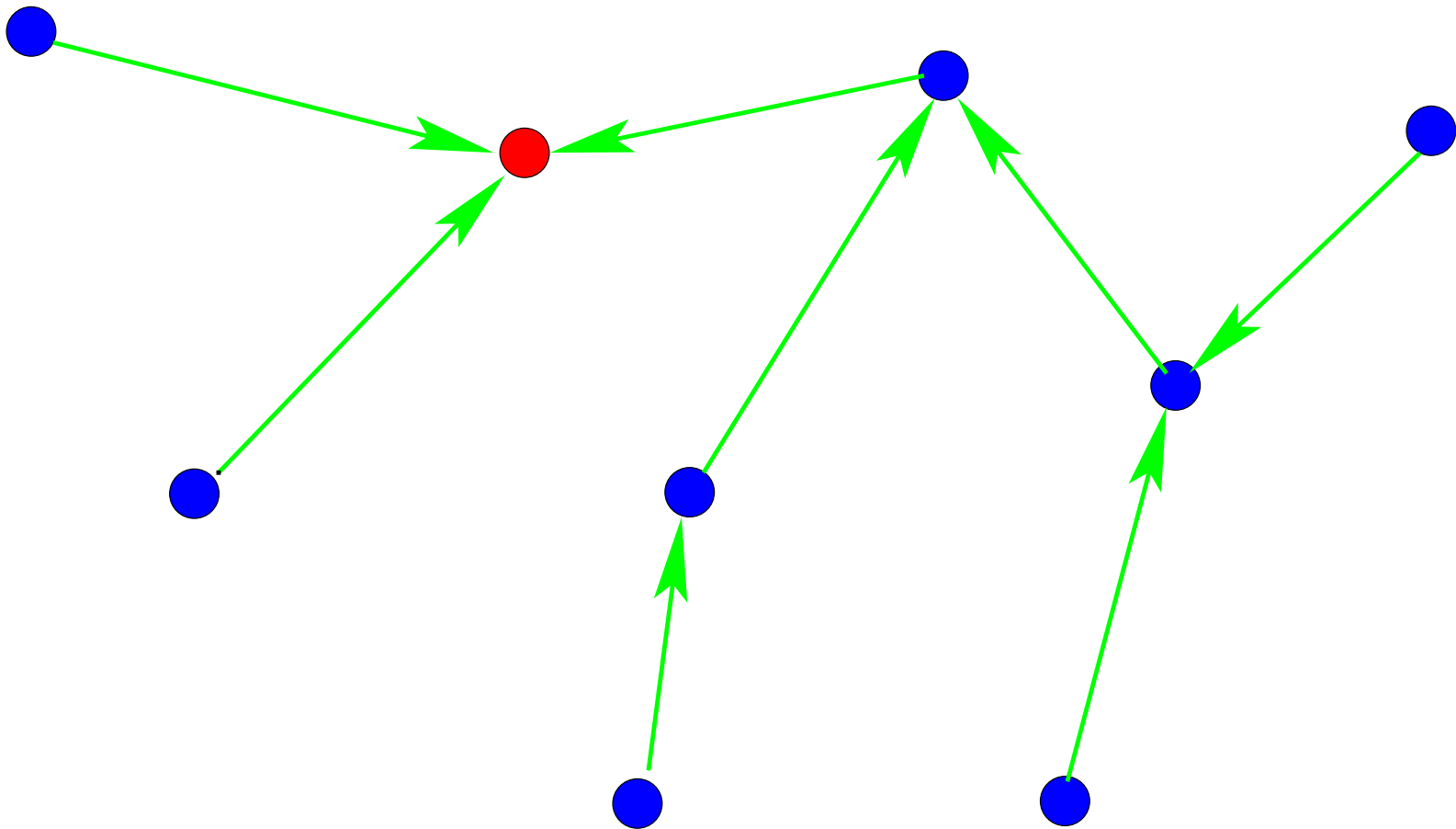












- The **green arrows** correspond to the  $tr$  function
- The **blue nodes** are the **domain** of  $tr$
- The function  $tr$  is a **forest (multi-tree)** on nodes
- The **red nodes** are the **roots** of these trees

The predicate **invariant** ( $tr$ )

$$tr \in ND \leftrightarrow ND$$

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$$\forall p \cdot \left( \begin{array}{l} p \subseteq ND \quad \wedge \\ ND - \text{dom}(tr) \subseteq p \quad \wedge \\ tr^{-1}[p] \subseteq p \\ \Rightarrow \\ ND \subseteq p \end{array} \right)$$

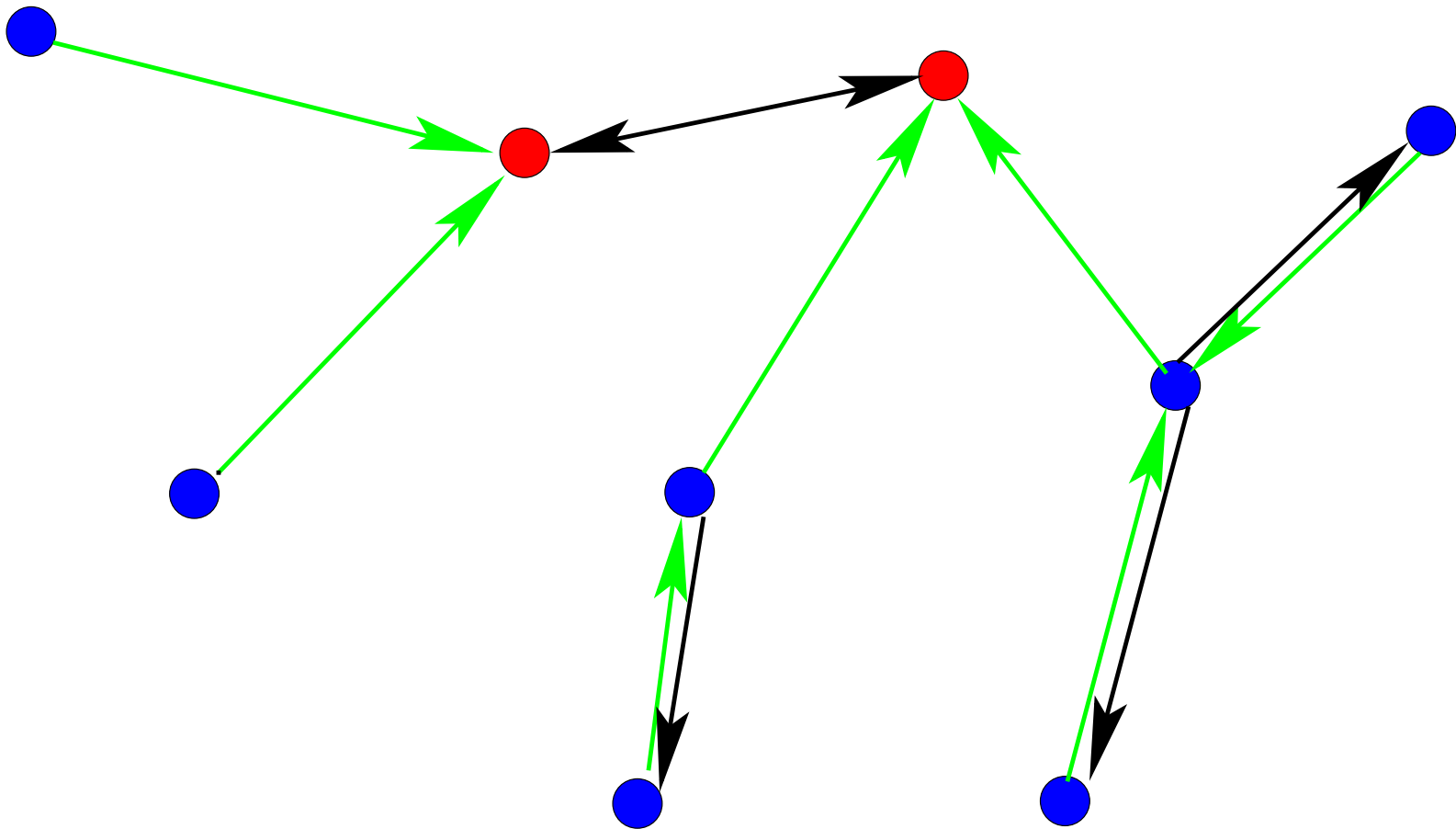


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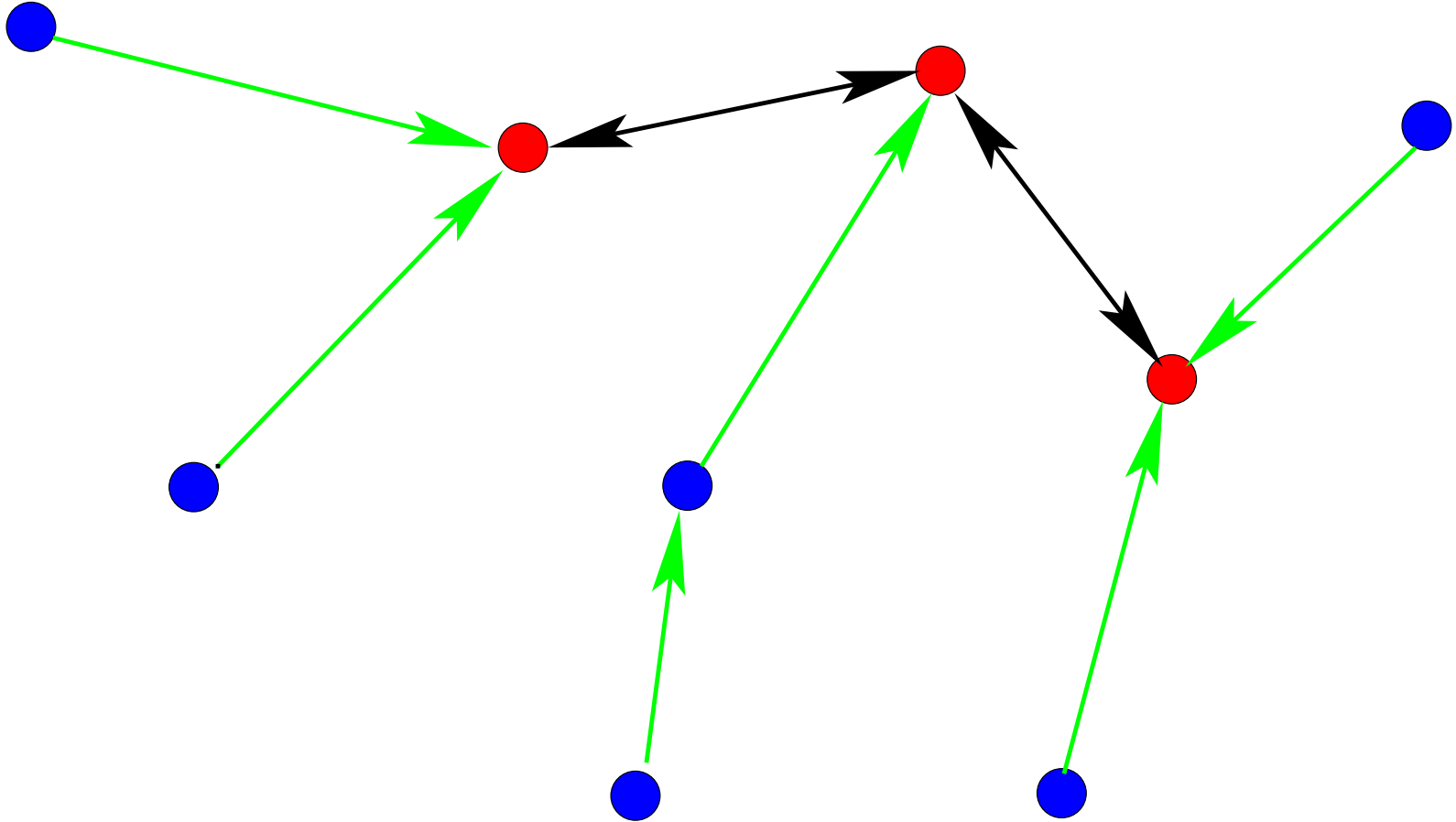
$$\text{dom}(tr) \triangleleft (tr \cup tr^{-1}) = \text{dom}(tr) \triangleleft gr$$

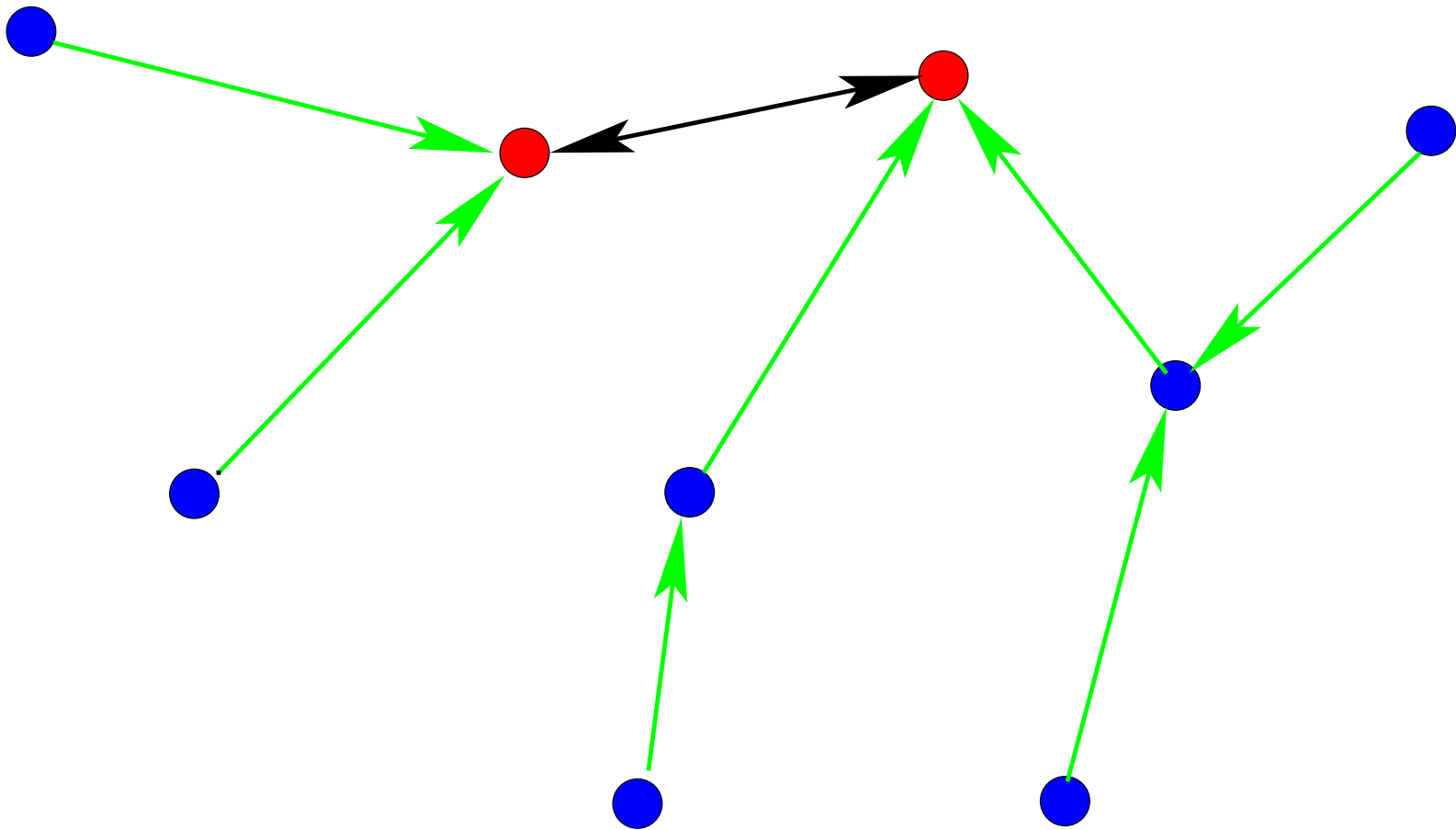


## First Refinement (2)

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- Introducing the **new event** "progress"
  - Refining the abstract event "elect"
  - Back to the **animation** : Observe the "guard" of progress
-





When a red node  $x$  is connected to **AT MOST** one other red node  $y$  then event "progress" can take place

progress  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$$x, y \in gr \wedge$$

$$x \notin \text{dom}(tr) \wedge$$

$$y \notin \text{dom}(tr) \wedge$$

$$gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\}$$

**THEN**

$$tr := tr \cup \{x \mapsto y\}$$

**END**

## To be proved

$$\text{invariant}(tr) \quad \wedge$$

$$x, y \in gr \quad \wedge$$

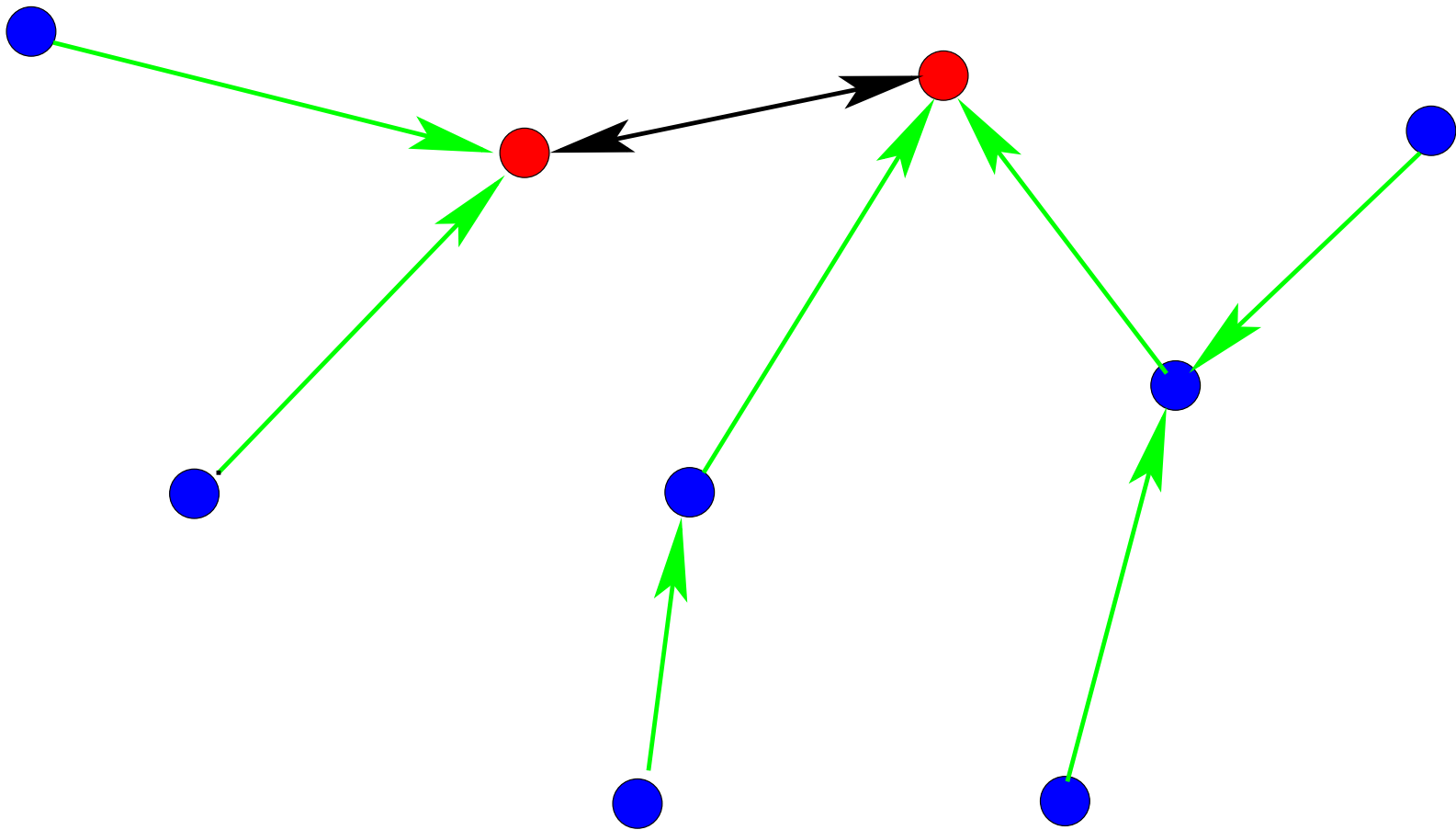
$$x \notin tr \quad \wedge$$

$$y \notin tr \quad \wedge$$

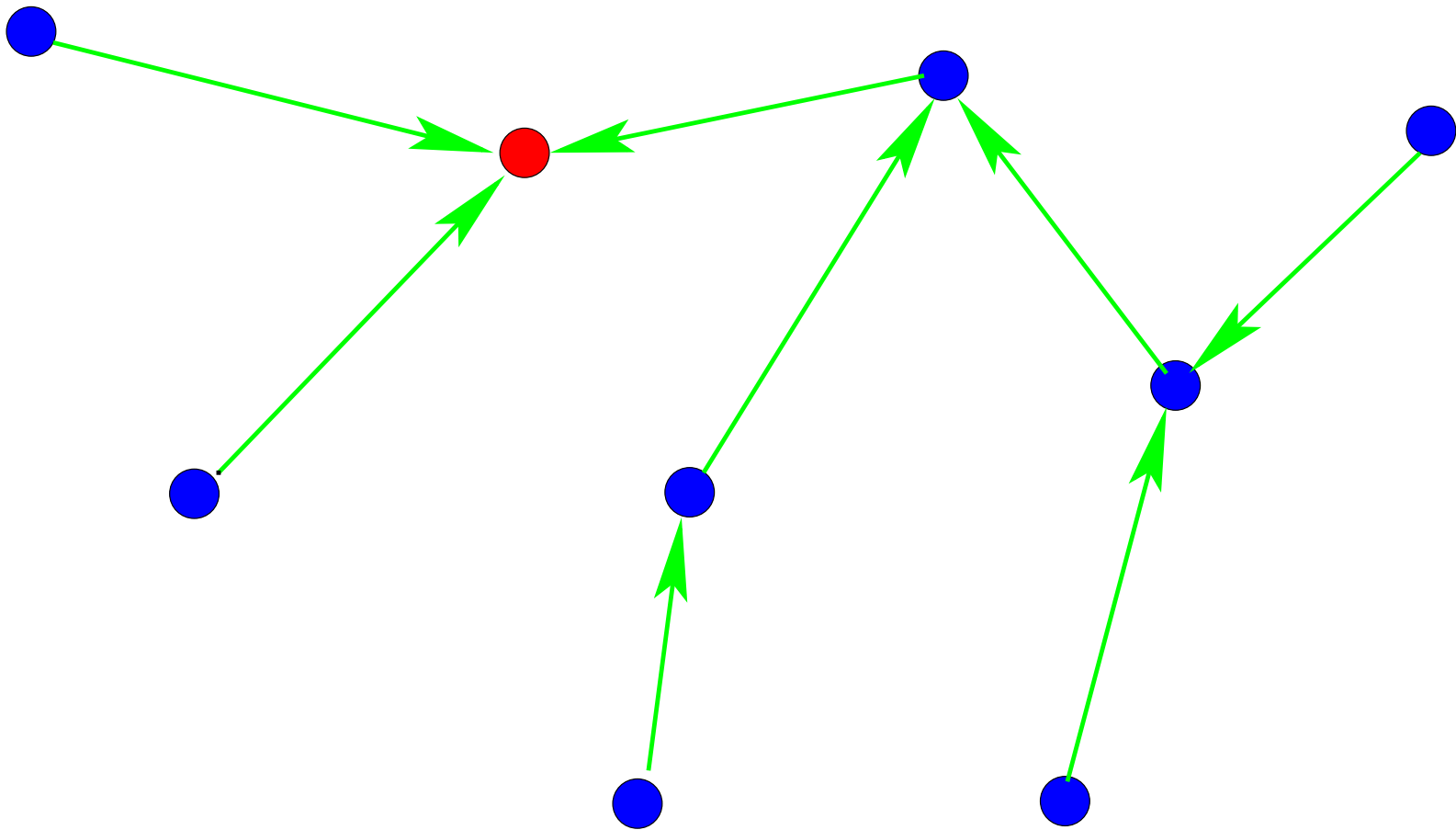
$$gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\}$$

$\Rightarrow$

$$\text{invariant}(tr \cup \{x \mapsto y\})$$







When a **red node**  $x$  is **ONLY** connected to **blue nodes** then event "elect" can take place

elect  $\hat{=}$

**ANY**  $x$  **WHERE**

$x \in ND \wedge$

$gr[\{x\}] = tr^{-1}[\{x\}]$

**THEN**

$rt, ts := x, tr$

**END**

elect  $\hat{=}$

**BEGIN**

$rt, ts : \text{spanning}(rt, ts, gr)$

**END**

elect  $\hat{=}$

**ANY**  $x$  **WHERE**

$x \in ND \wedge$

$gr[\{x\}] = tr^{-1}[\{x\}]$

**THEN**

$rt, ts := x, tr$

**END**

## To be proved

$\text{invariant}(tr) \quad \wedge$

$x \in ND \quad \wedge$

$gr[\{x\}] = tr^{-1}[\{x\}]$

$ts = tr$

$\Rightarrow$

$\text{spanning}(x, ts, gr)$

# Summary of First Refinement

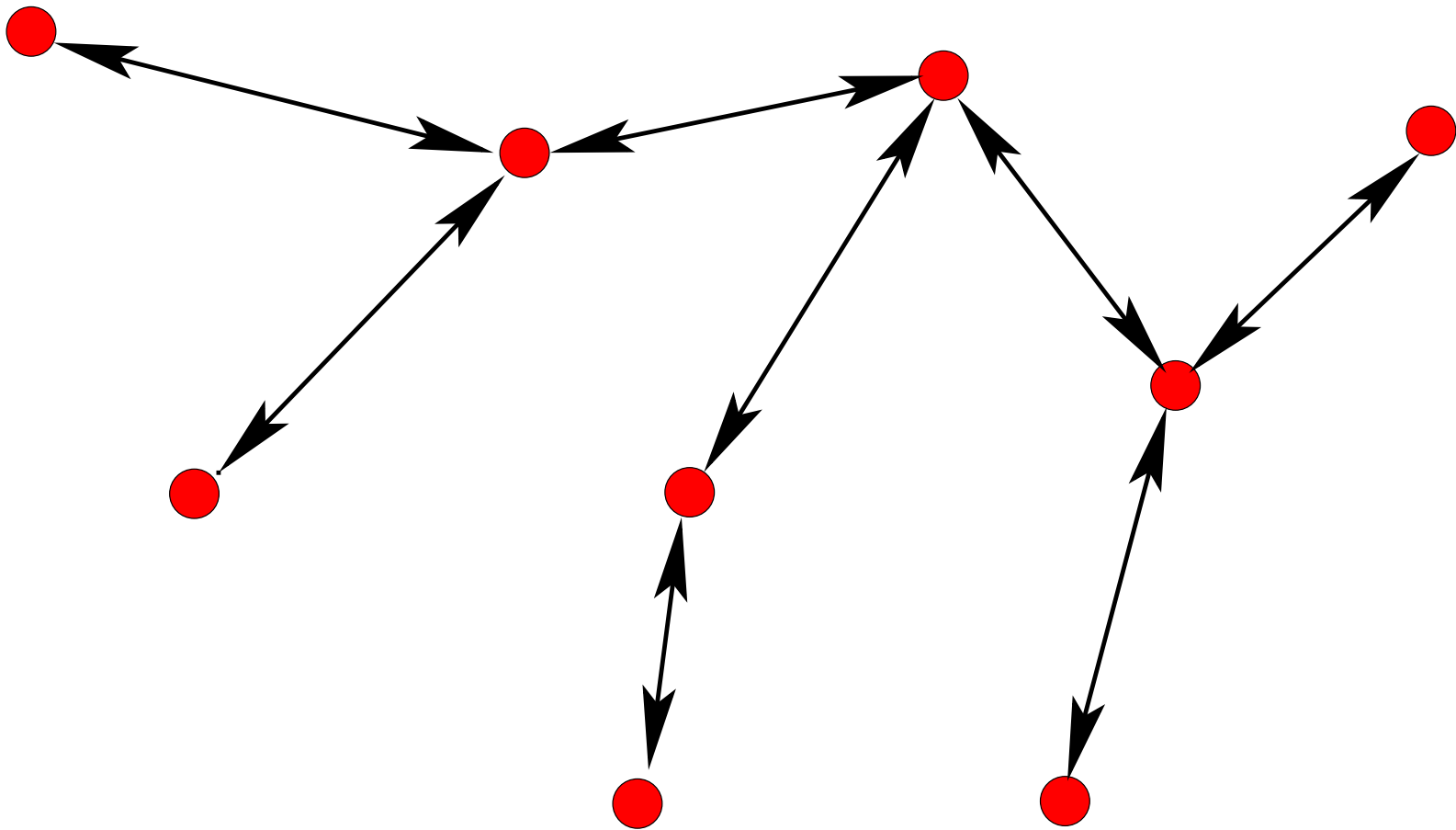
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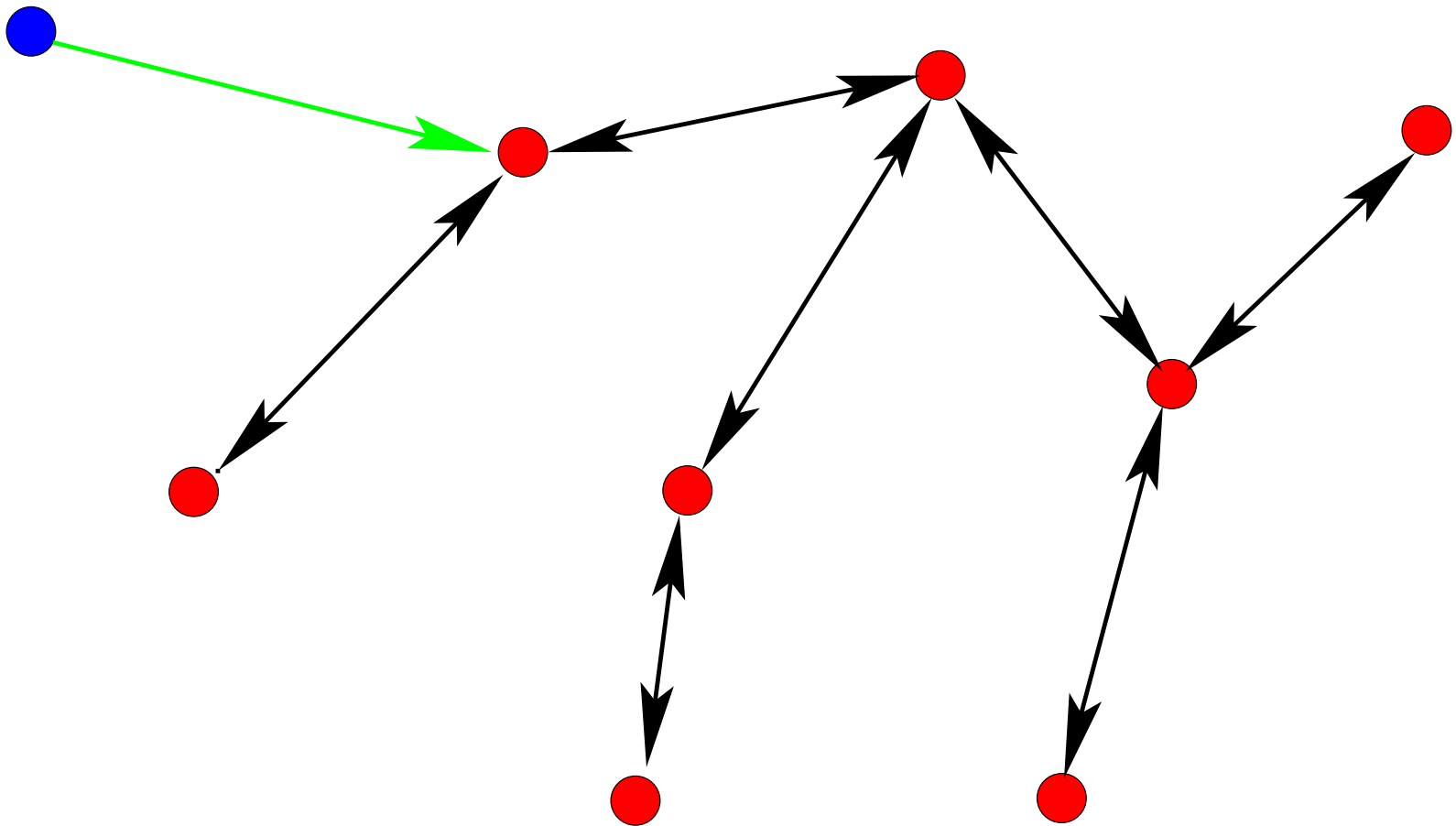
- 15 proofs
- Among which 9 were interactive (one is a bit difficult !)

# Current state of the model

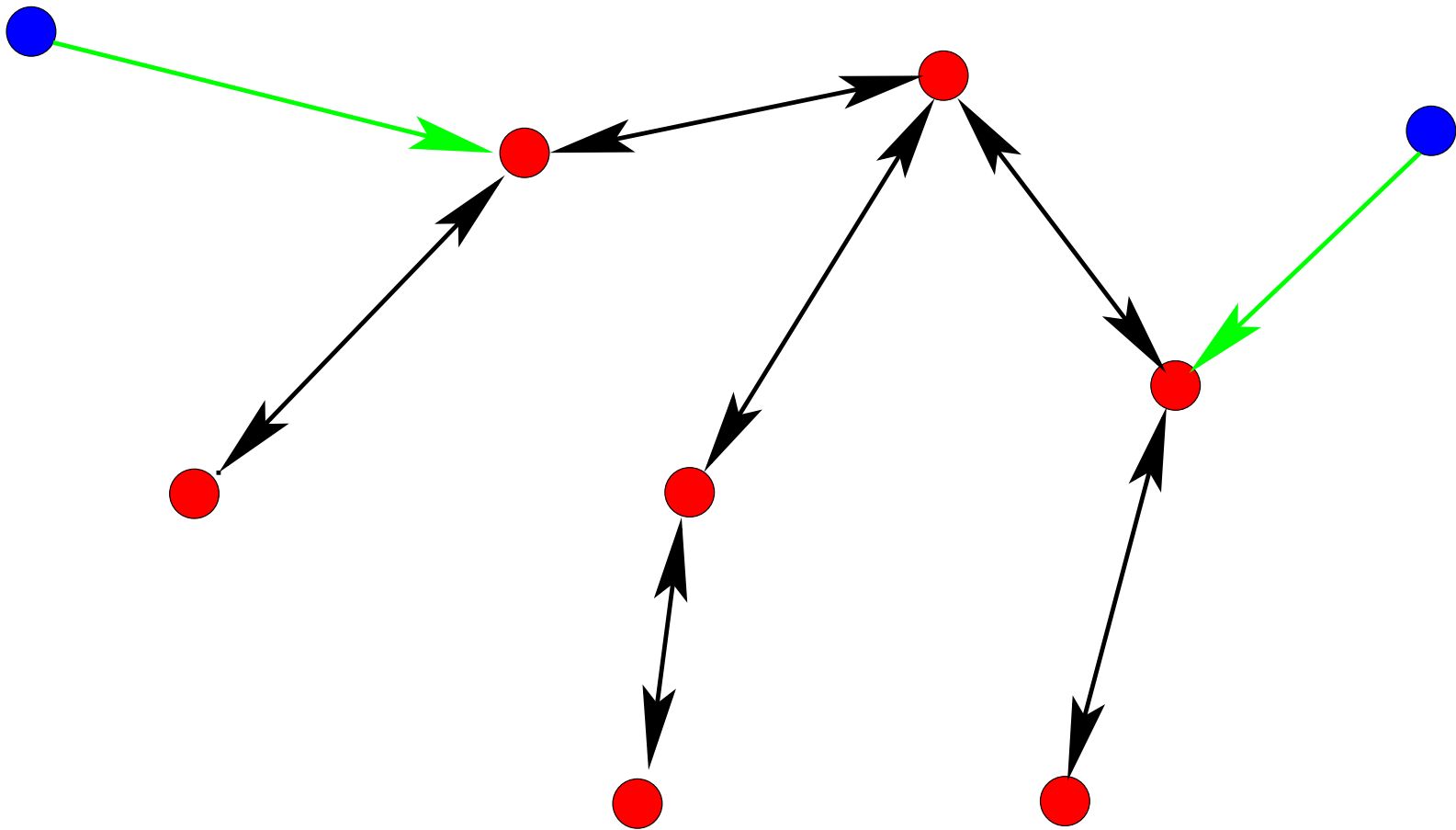
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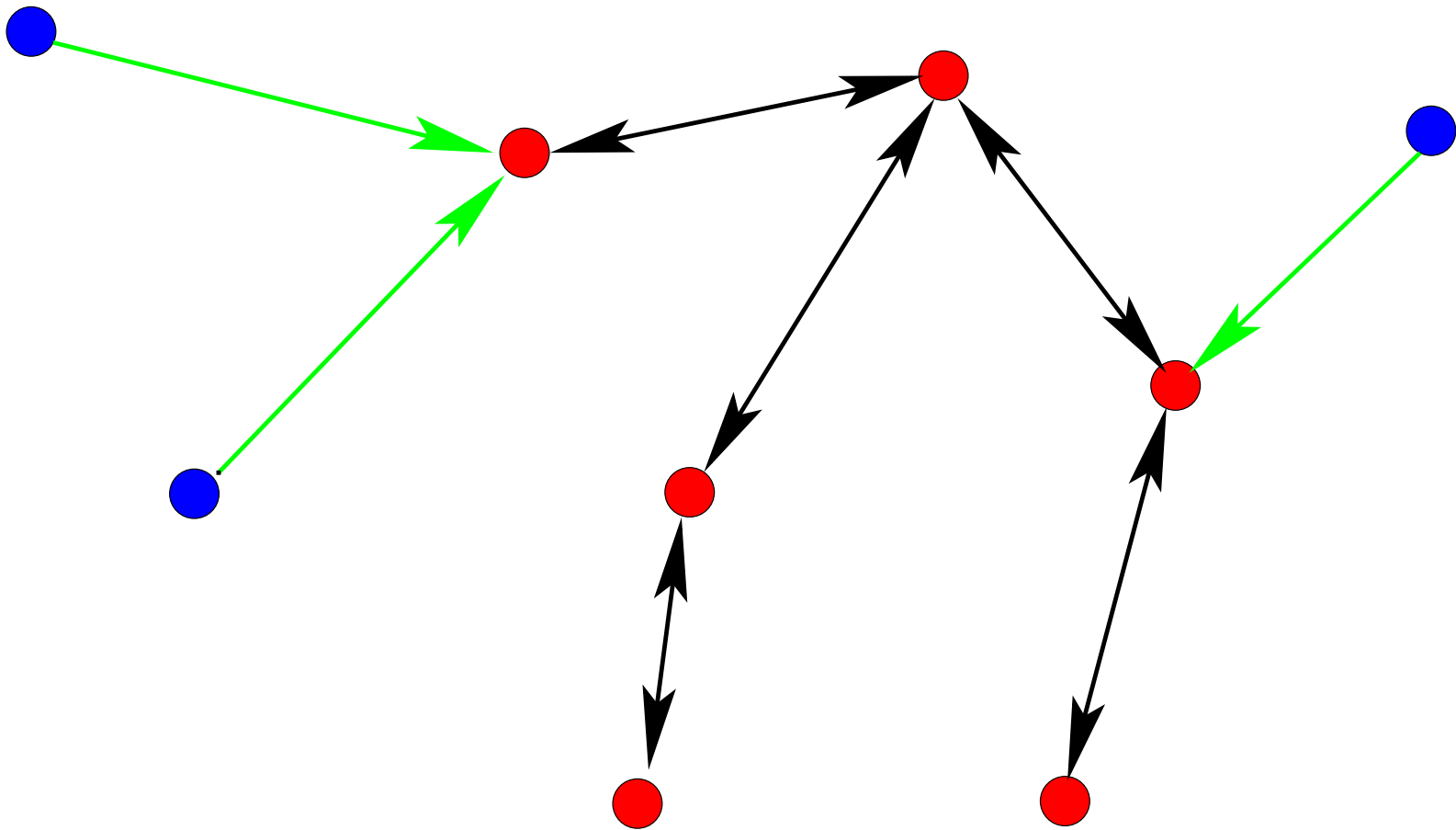
- 12 proofs
  - Among which 5 were interactive (one is a bit difficult !)
  - Animation of the current model
-

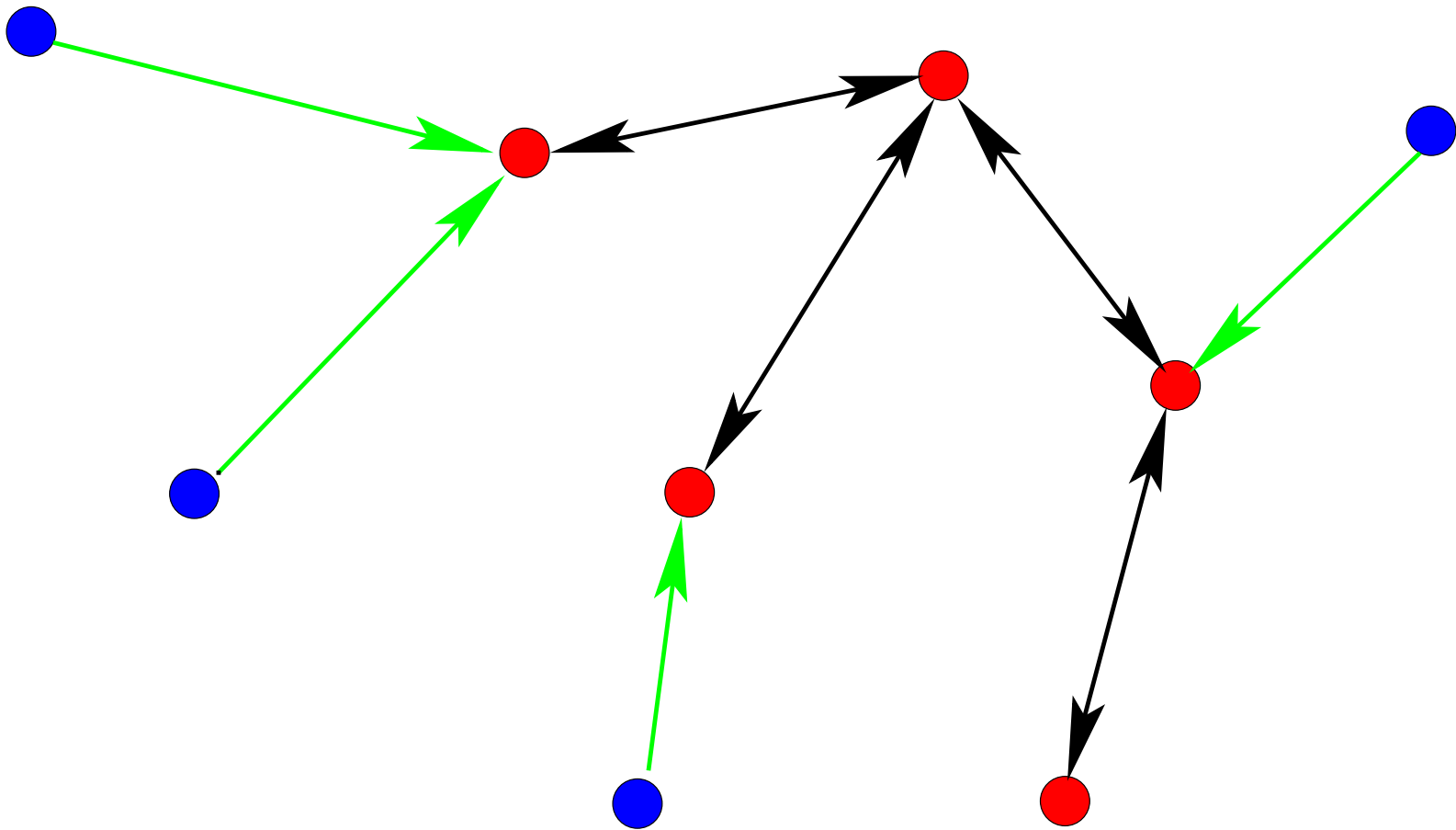


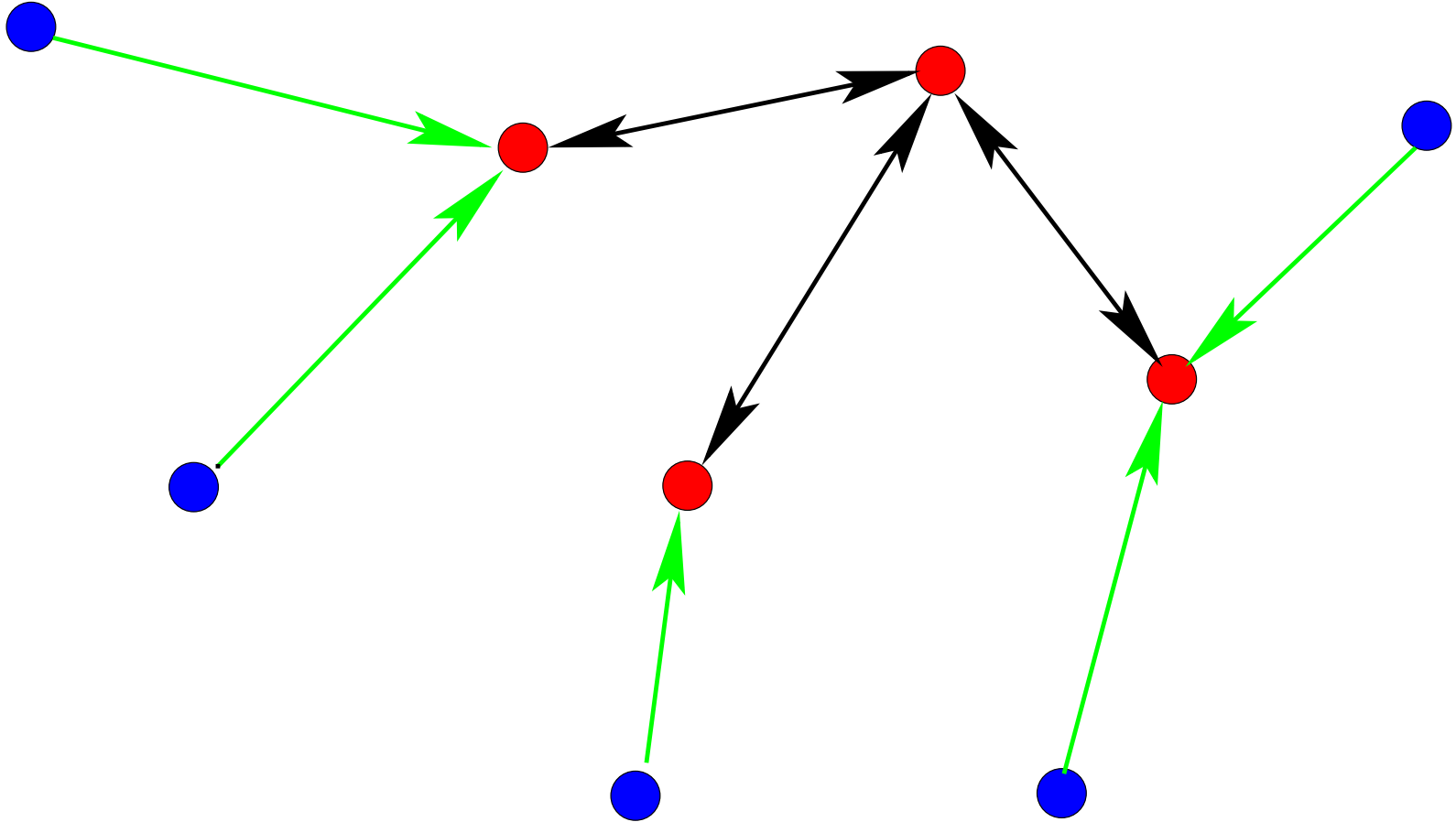


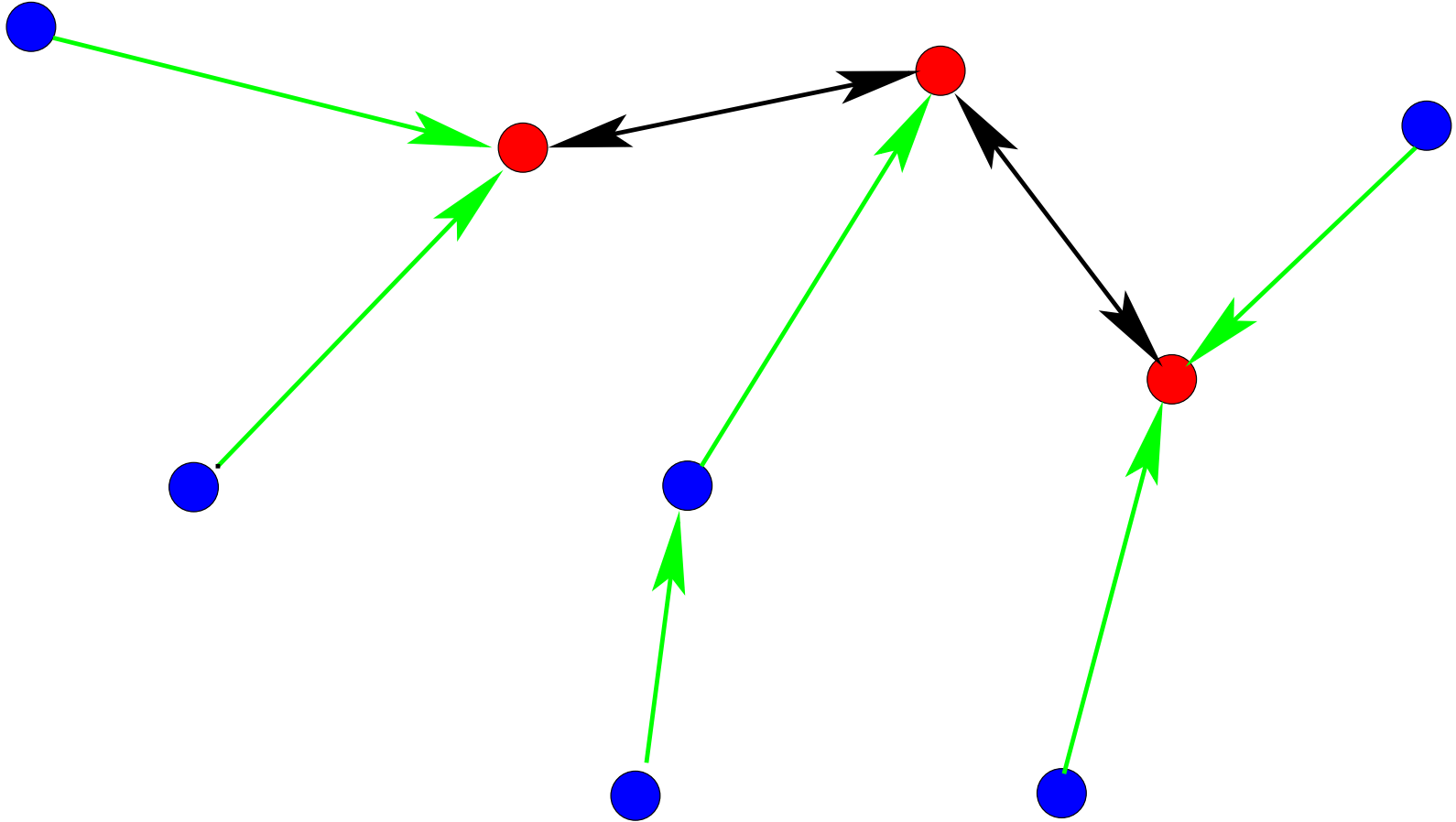


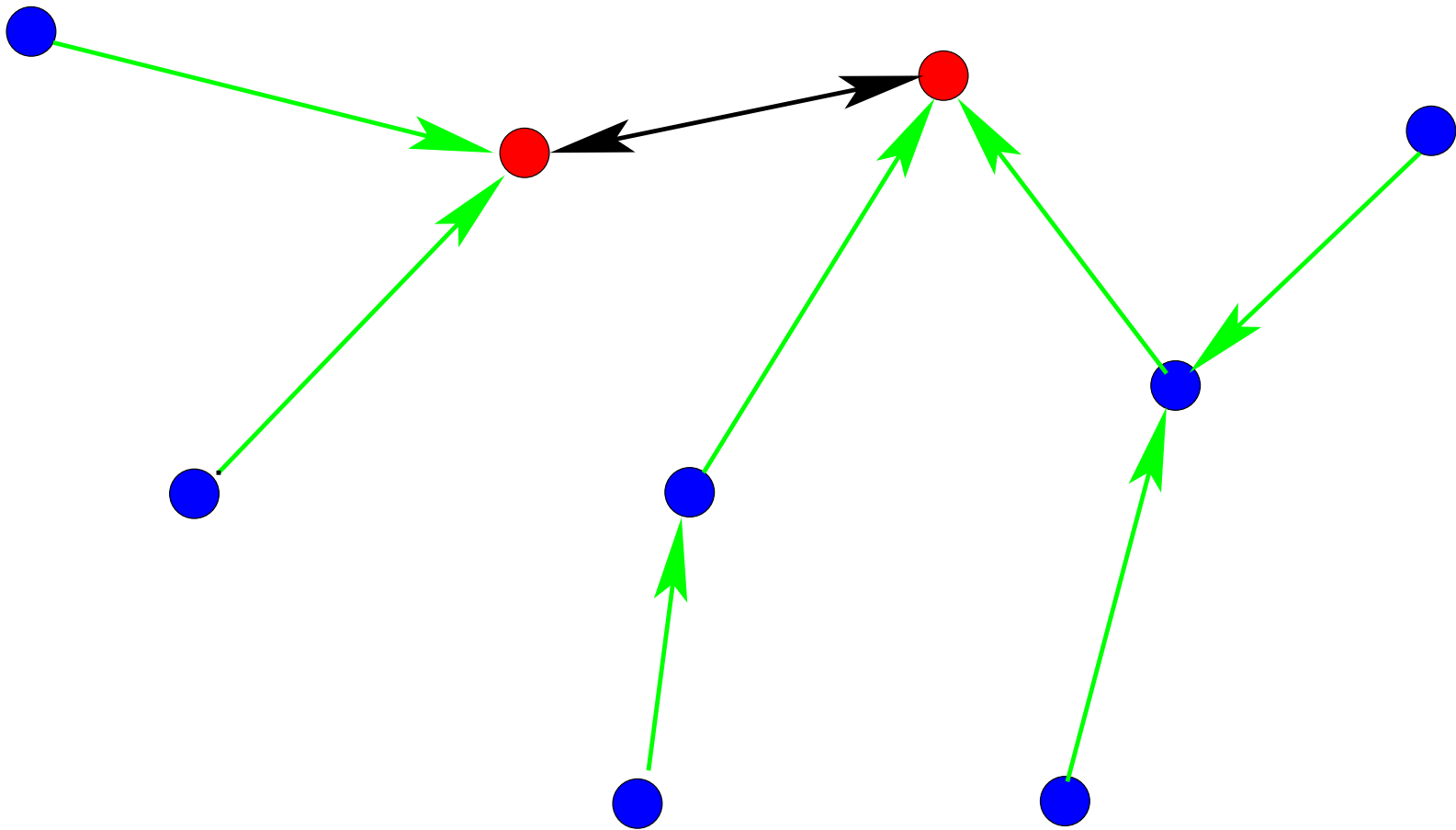


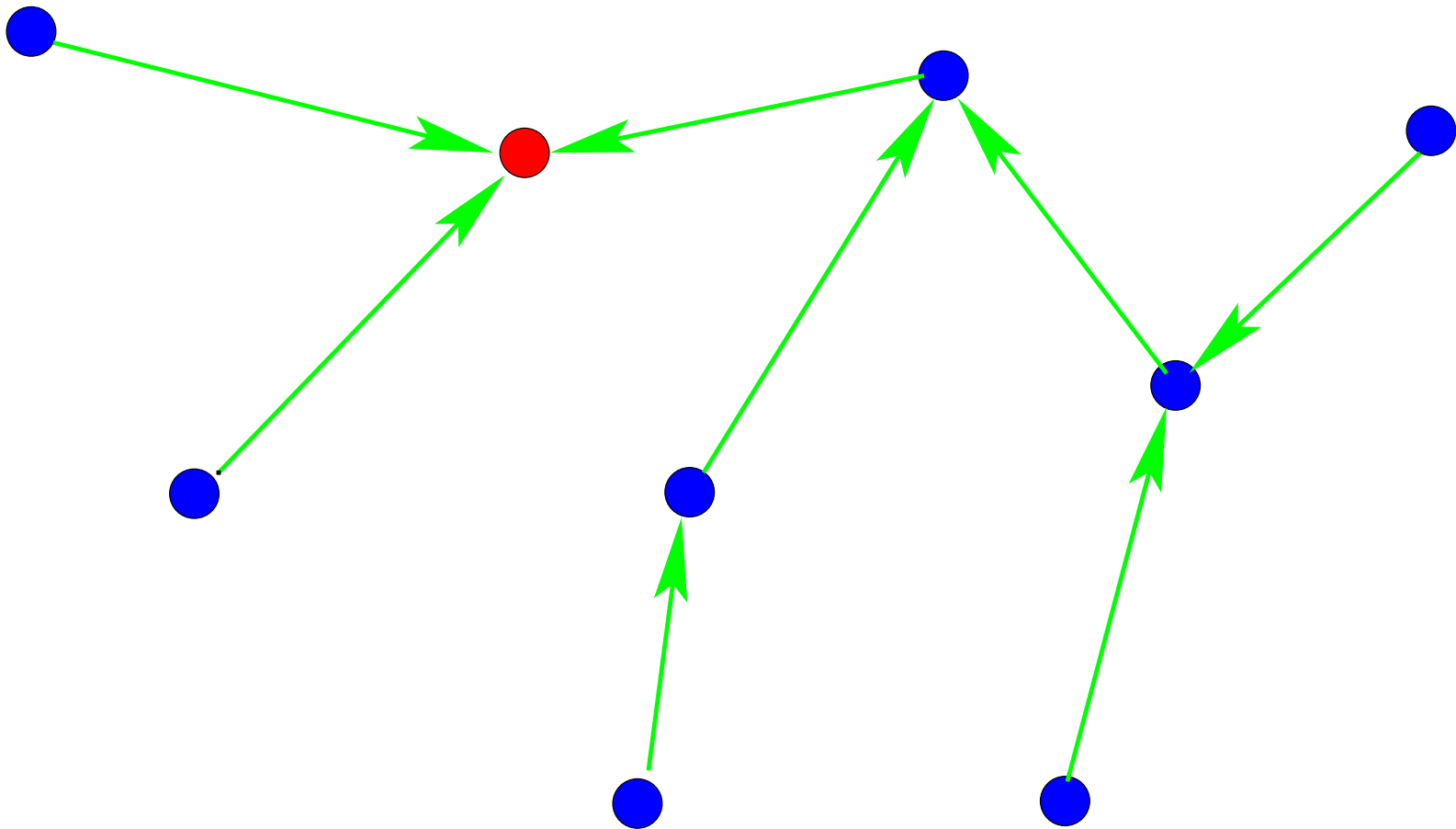


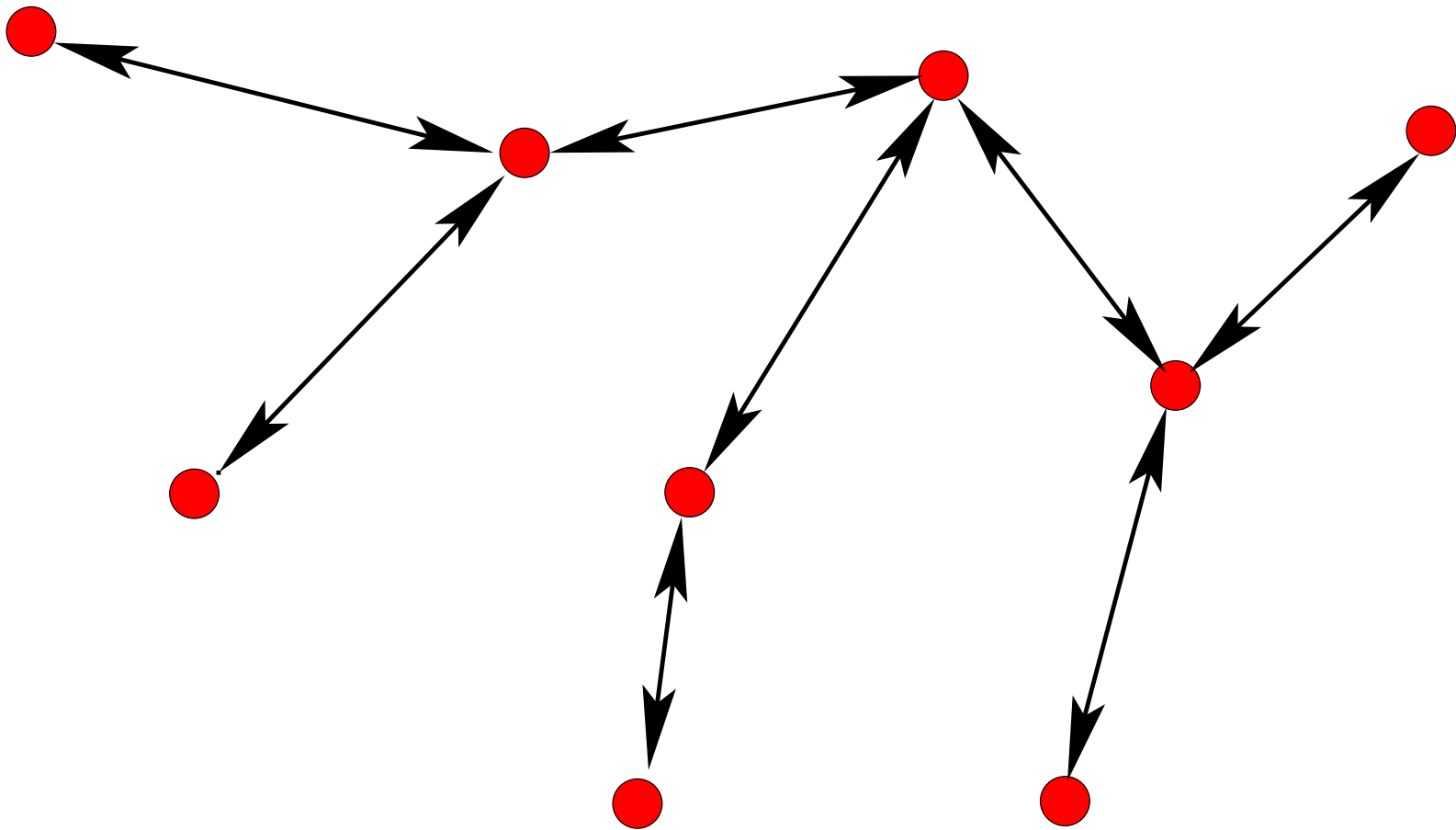




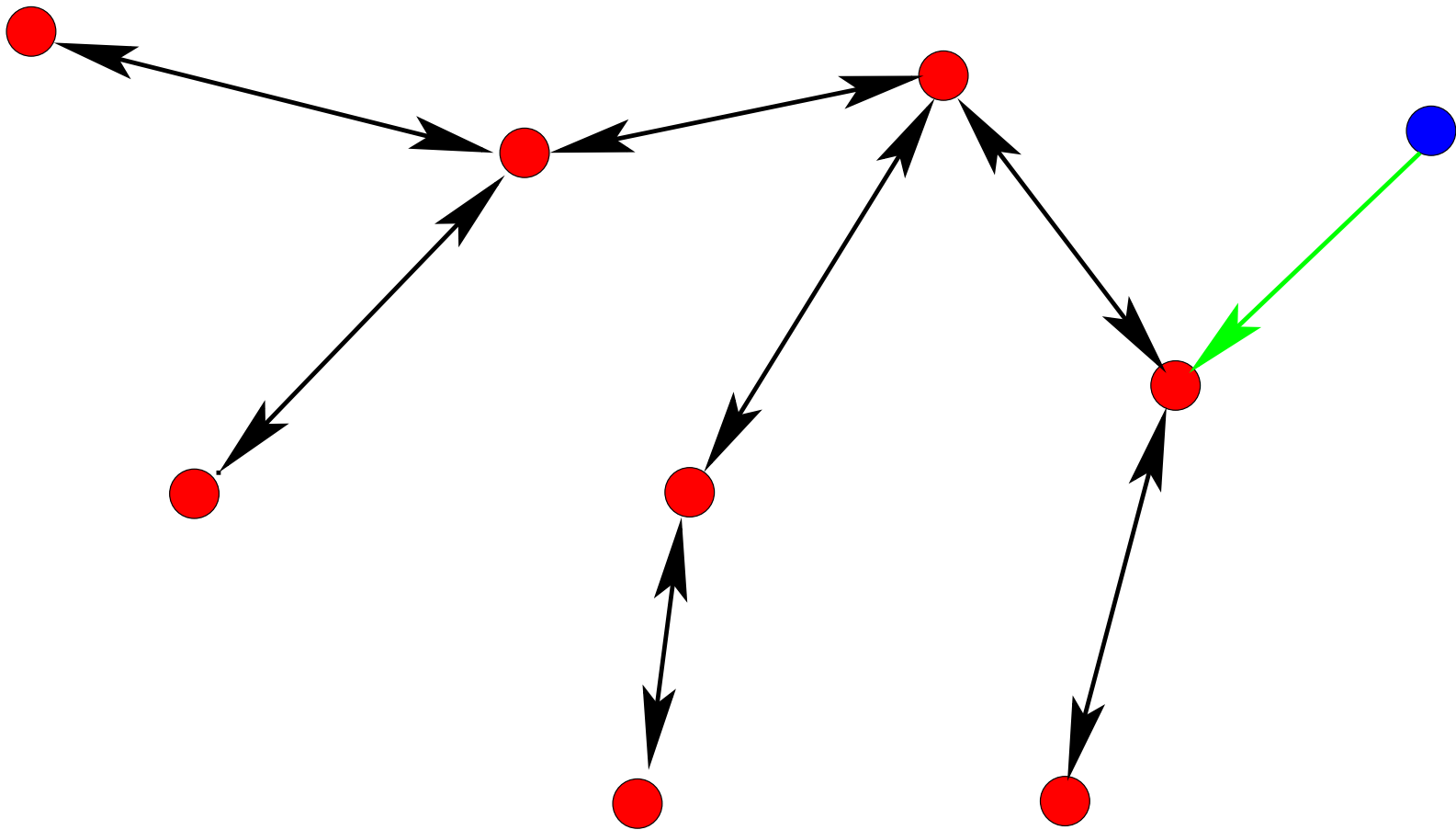


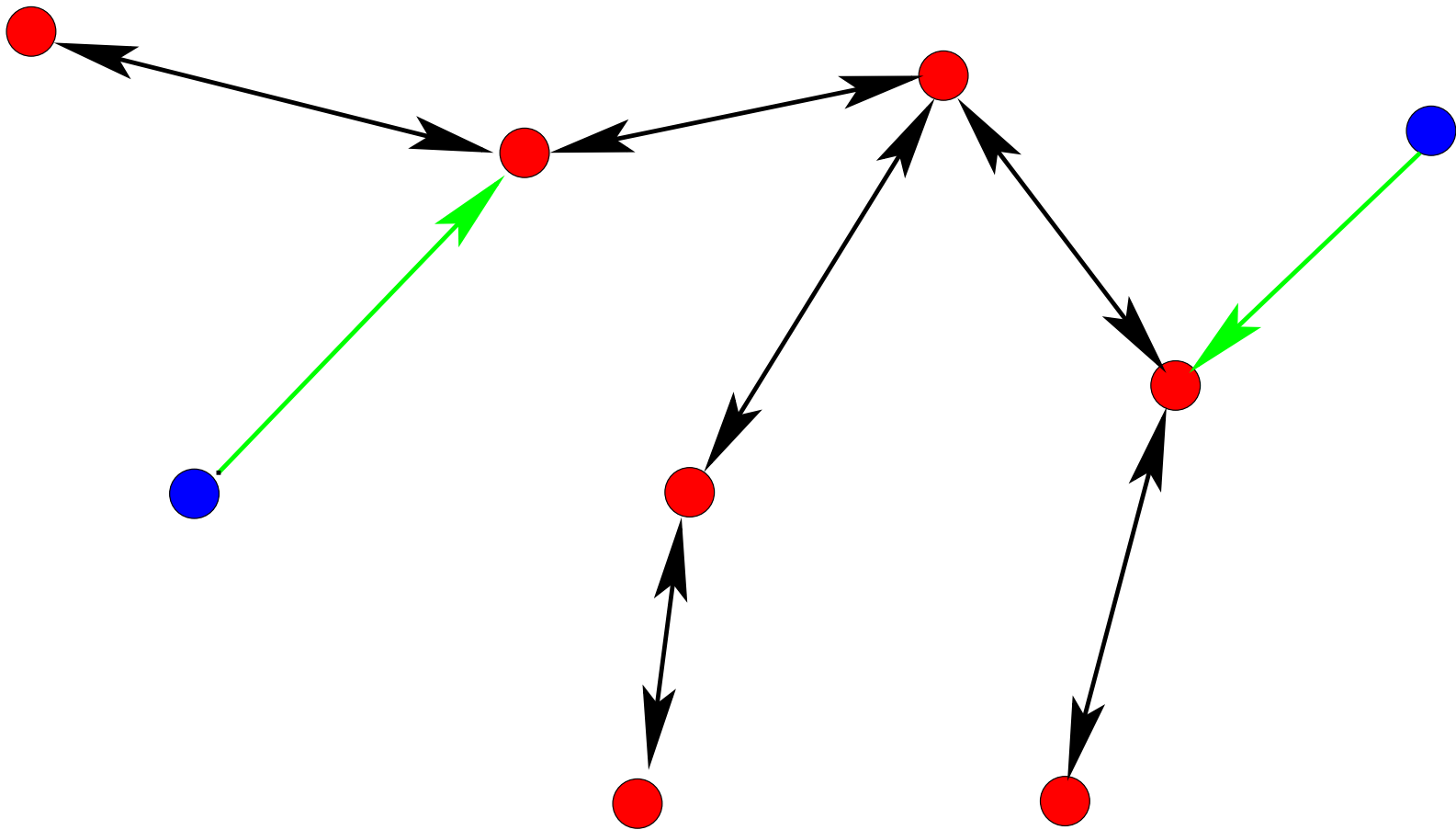


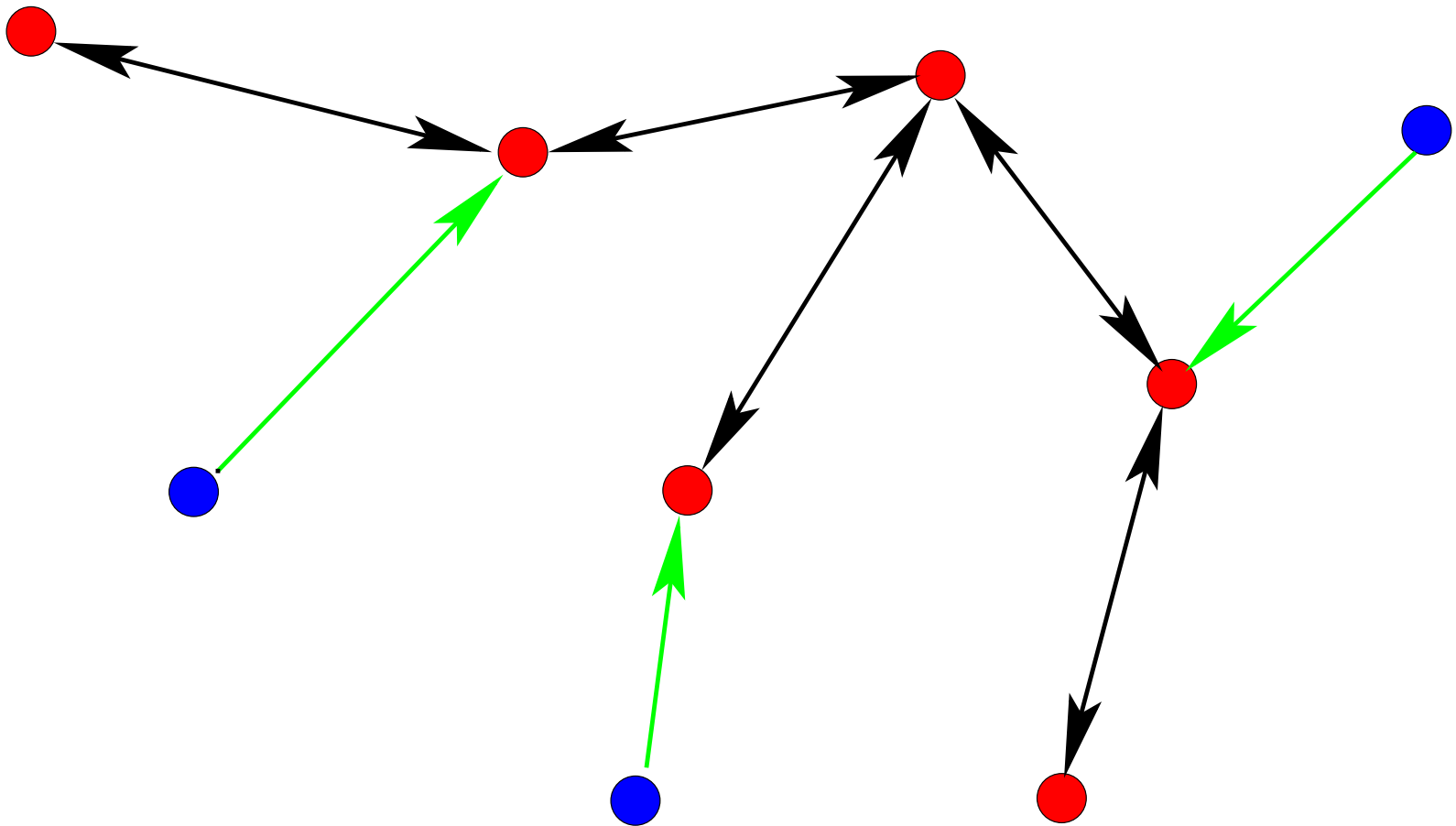


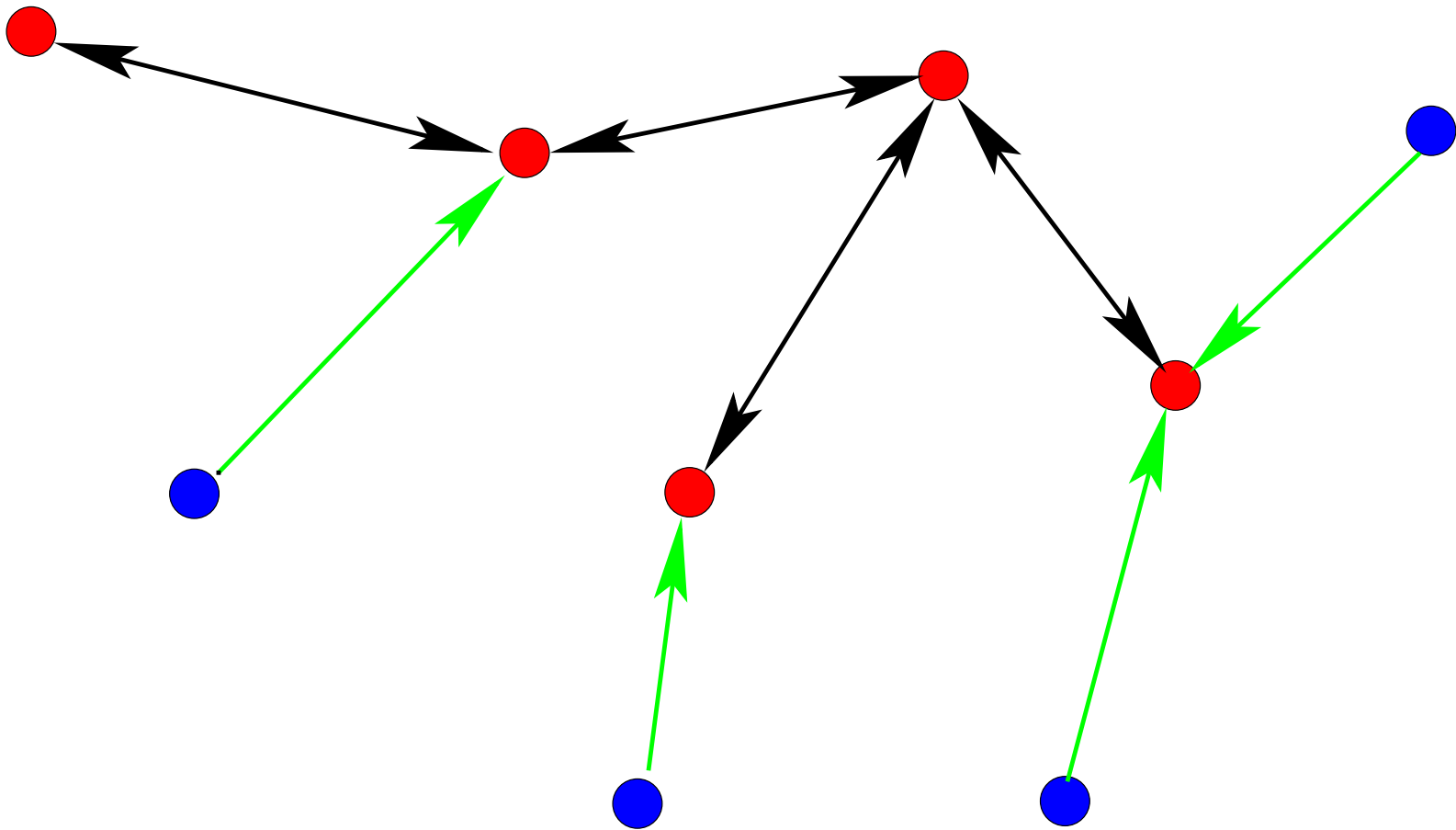


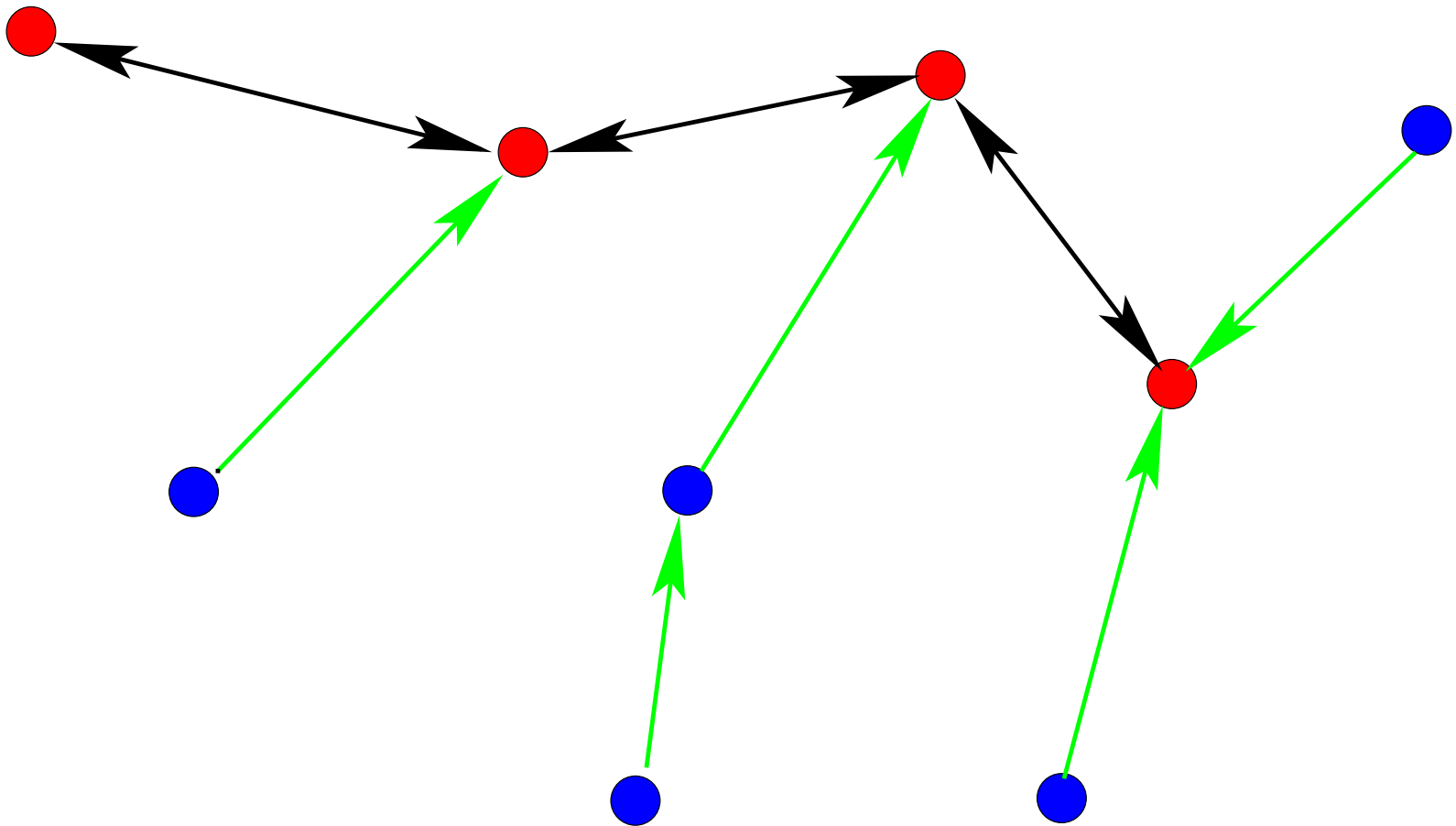


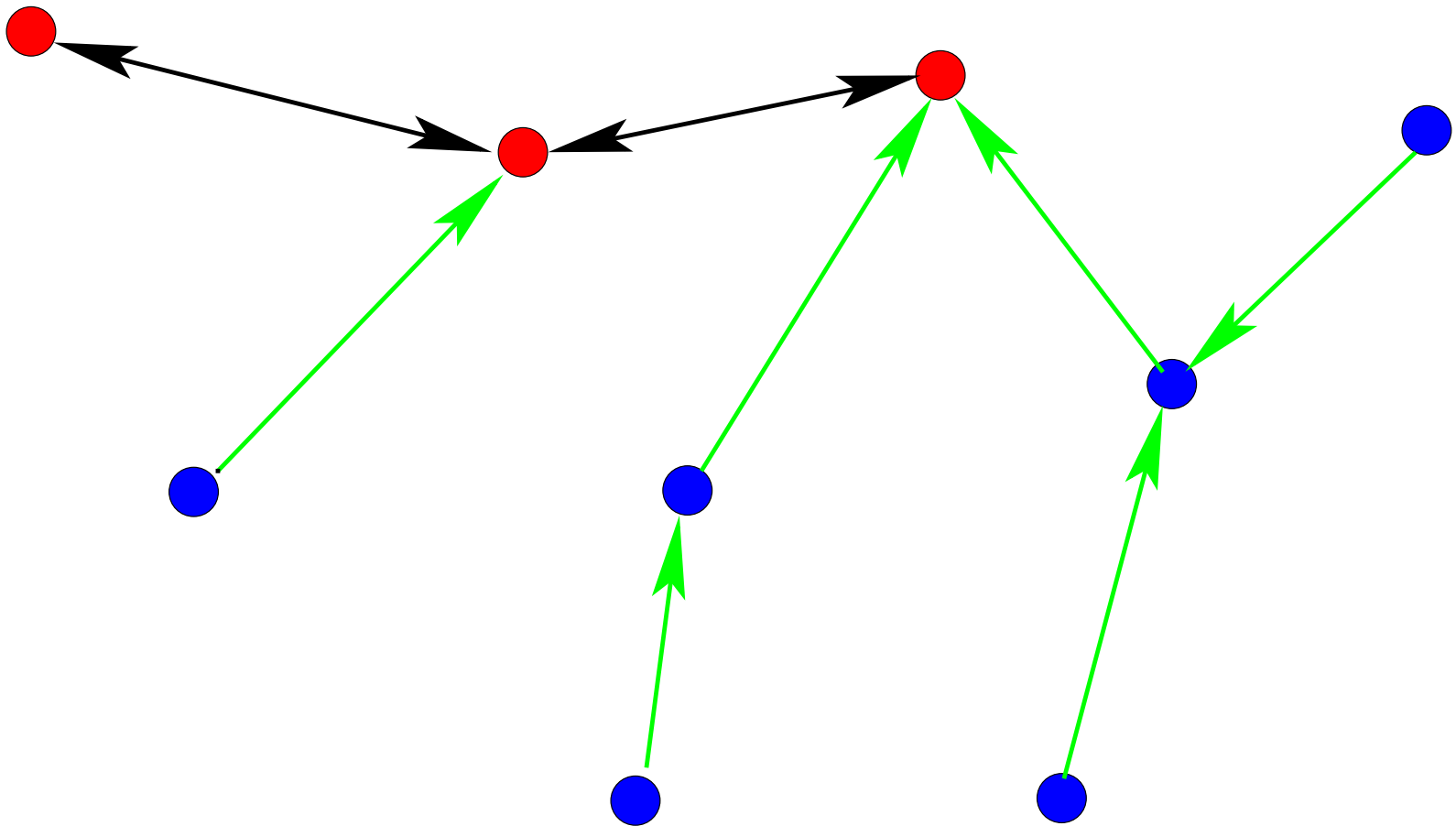


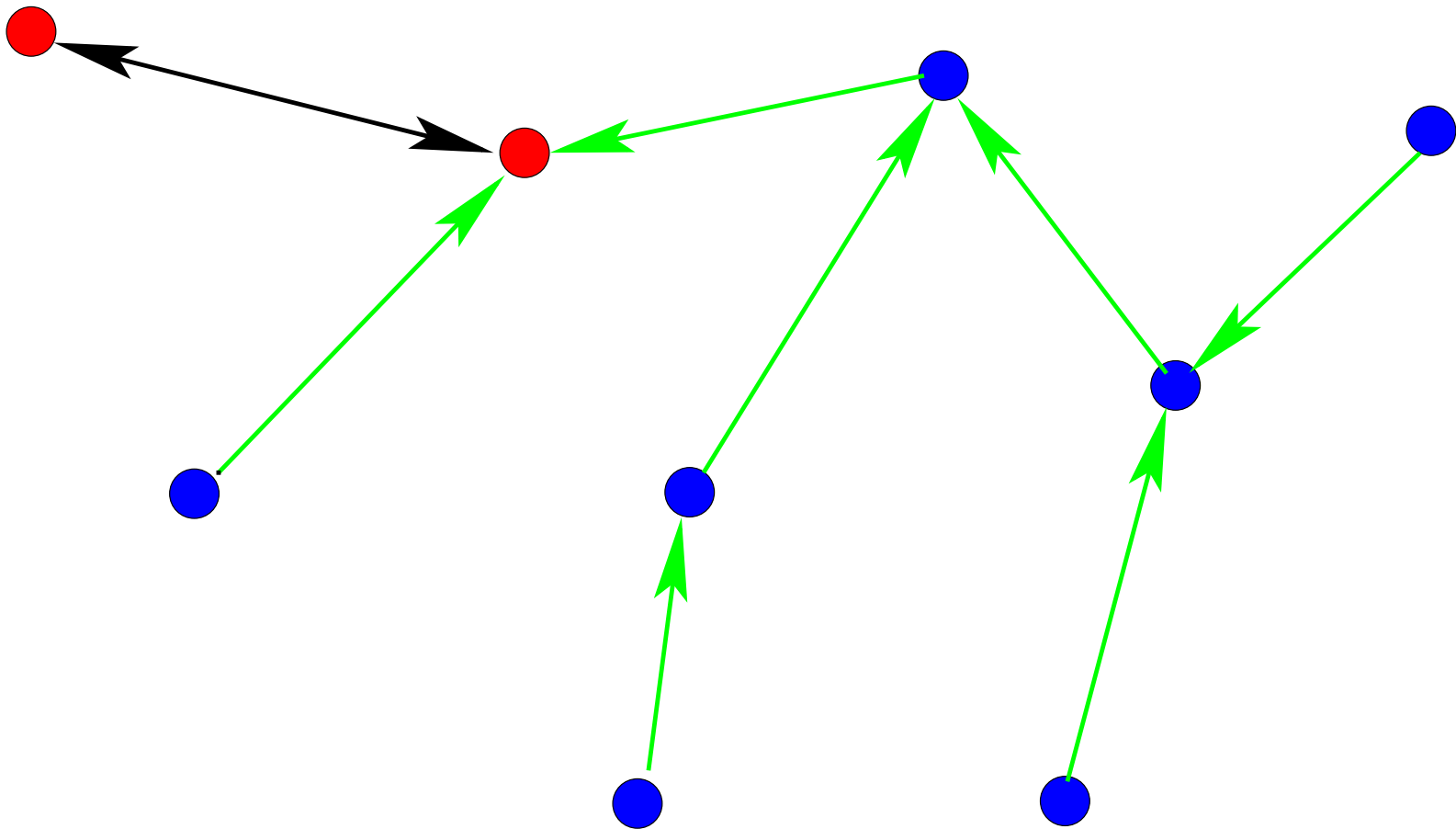


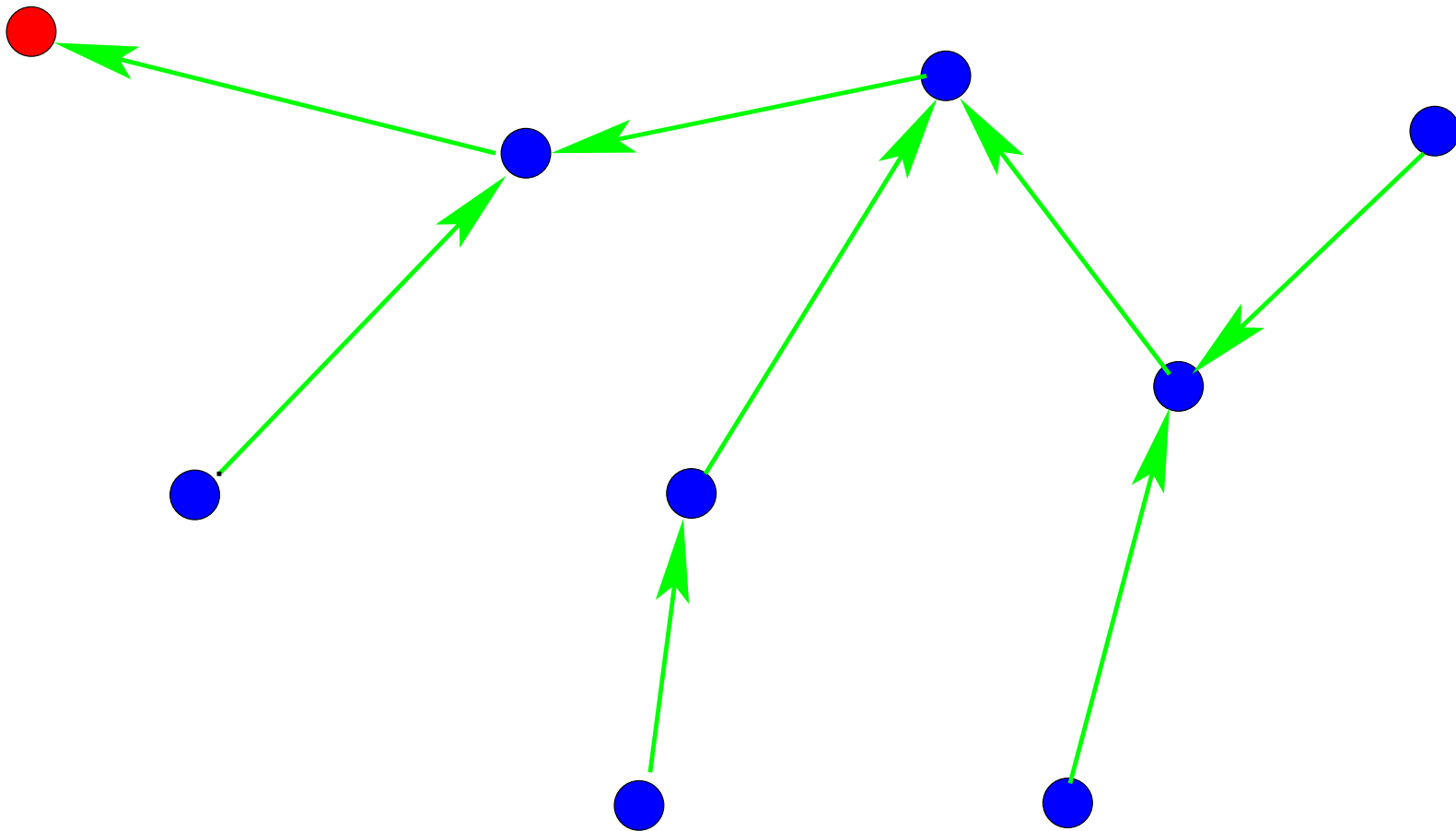










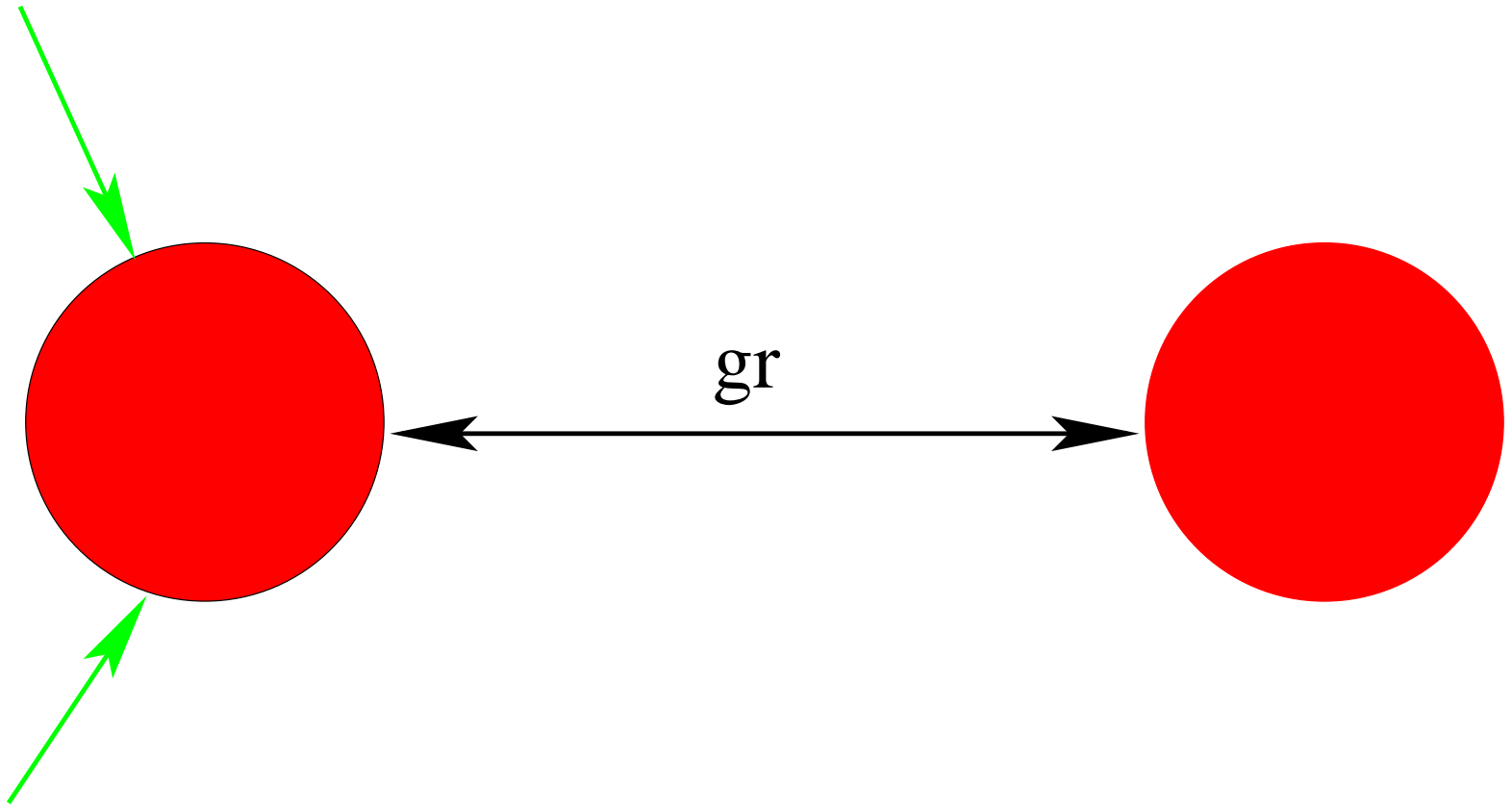


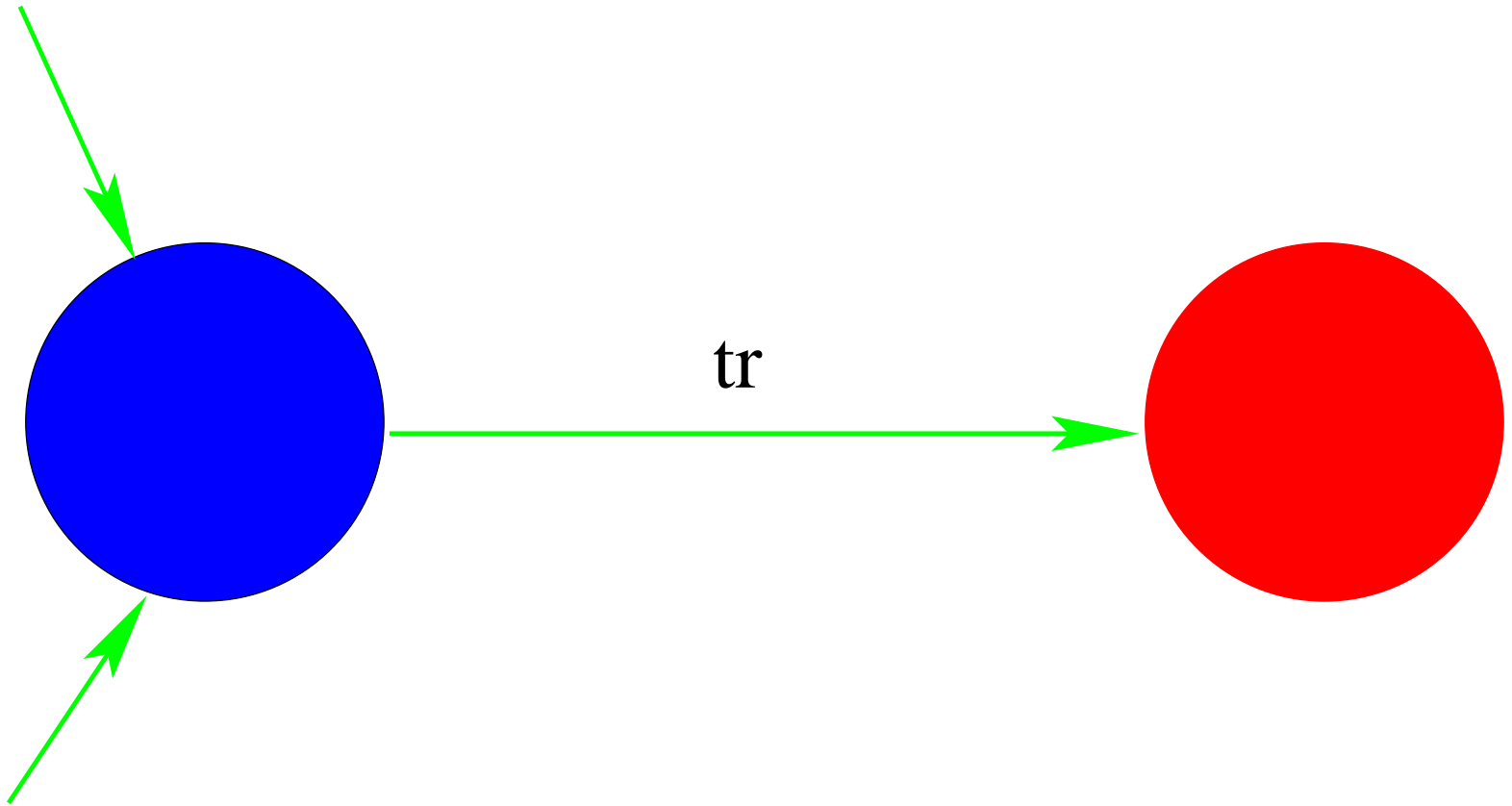


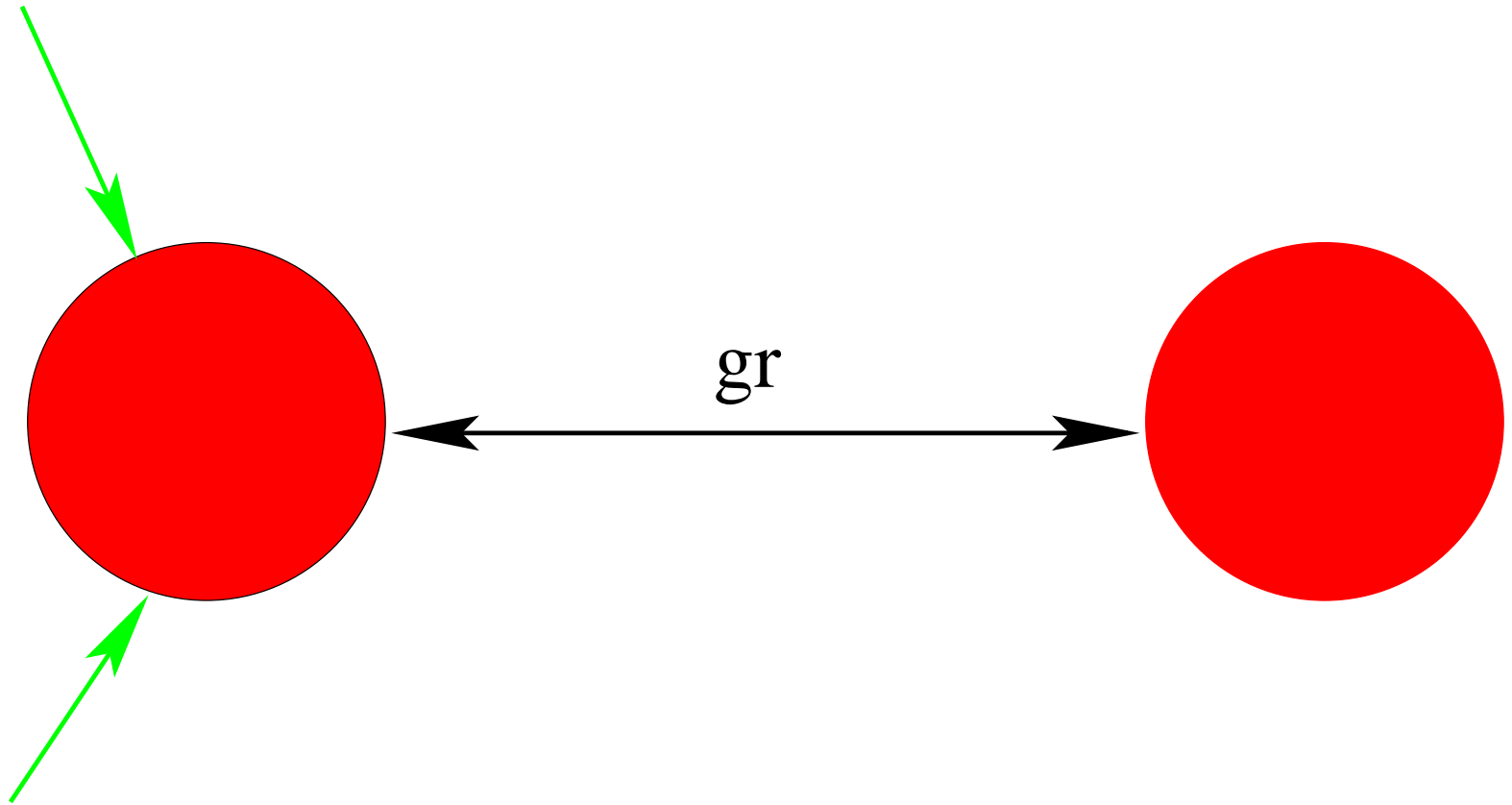
## Second Refinement

---

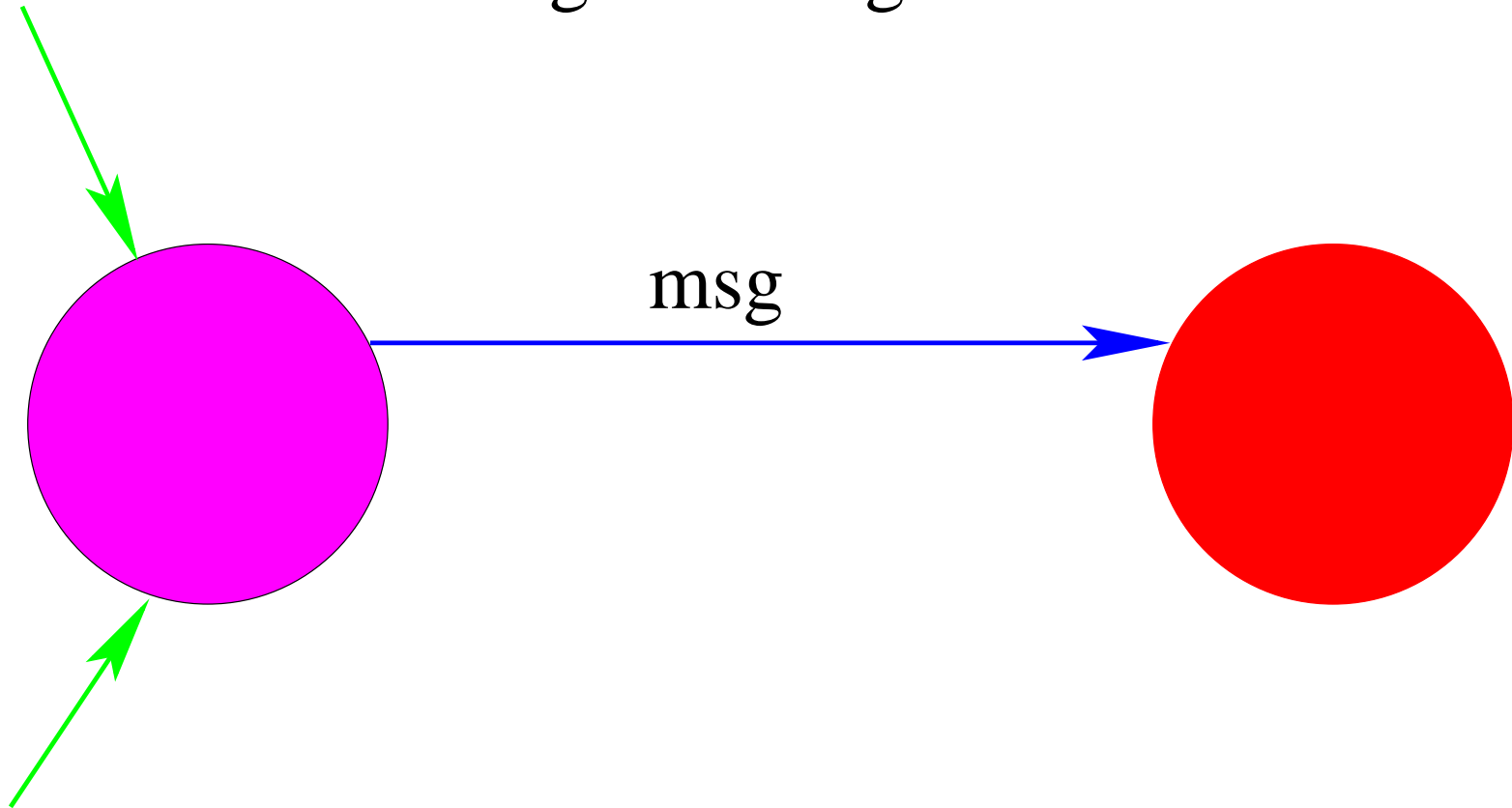
- Nodes are **communicating** with their neighbors
  - This is done by means of **messages**
  - Messages are **acknowledged**
  - Acknowledgements are **confirmed**
  - Next is a **local** animation
-





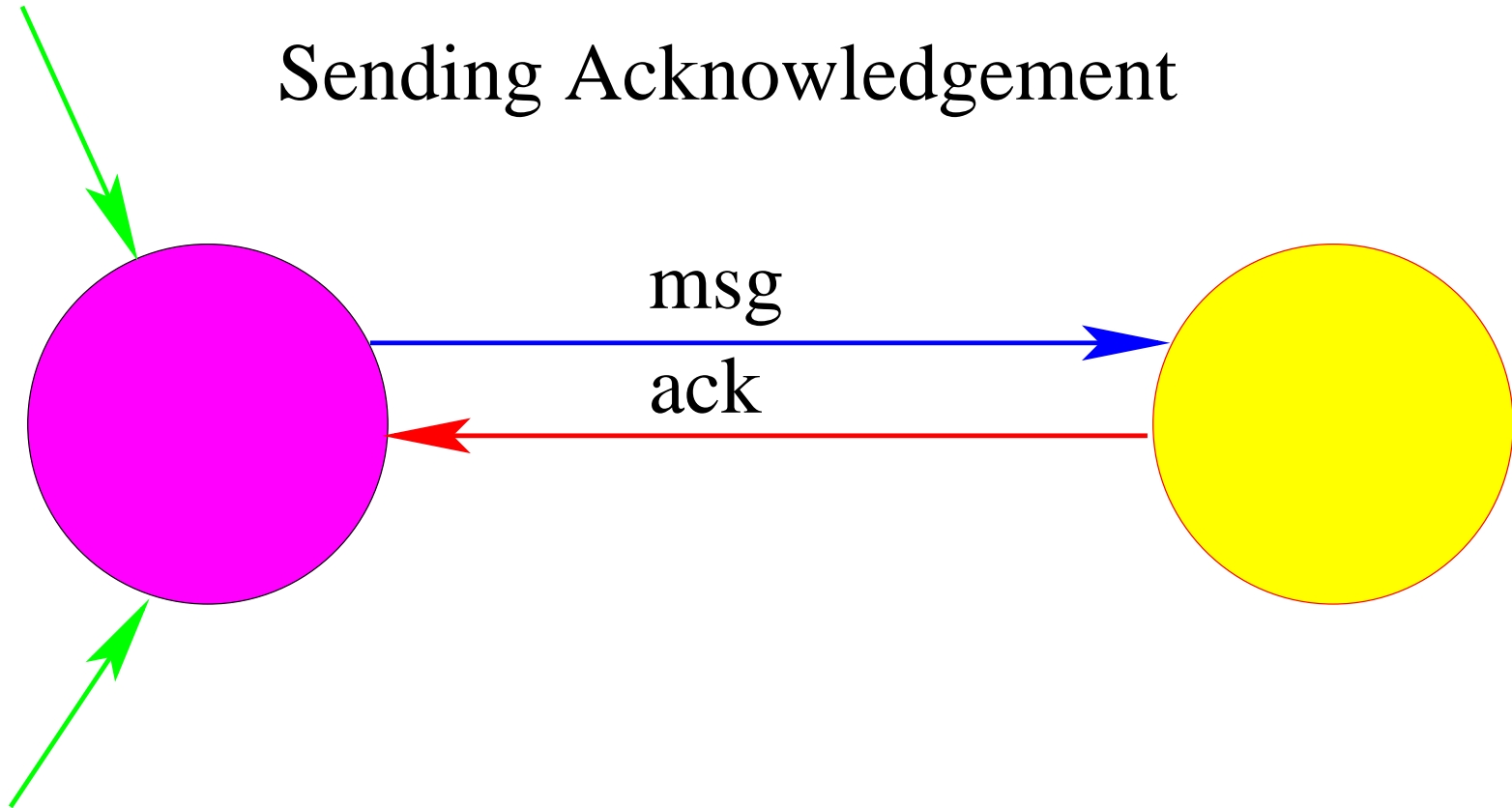


Sending a message



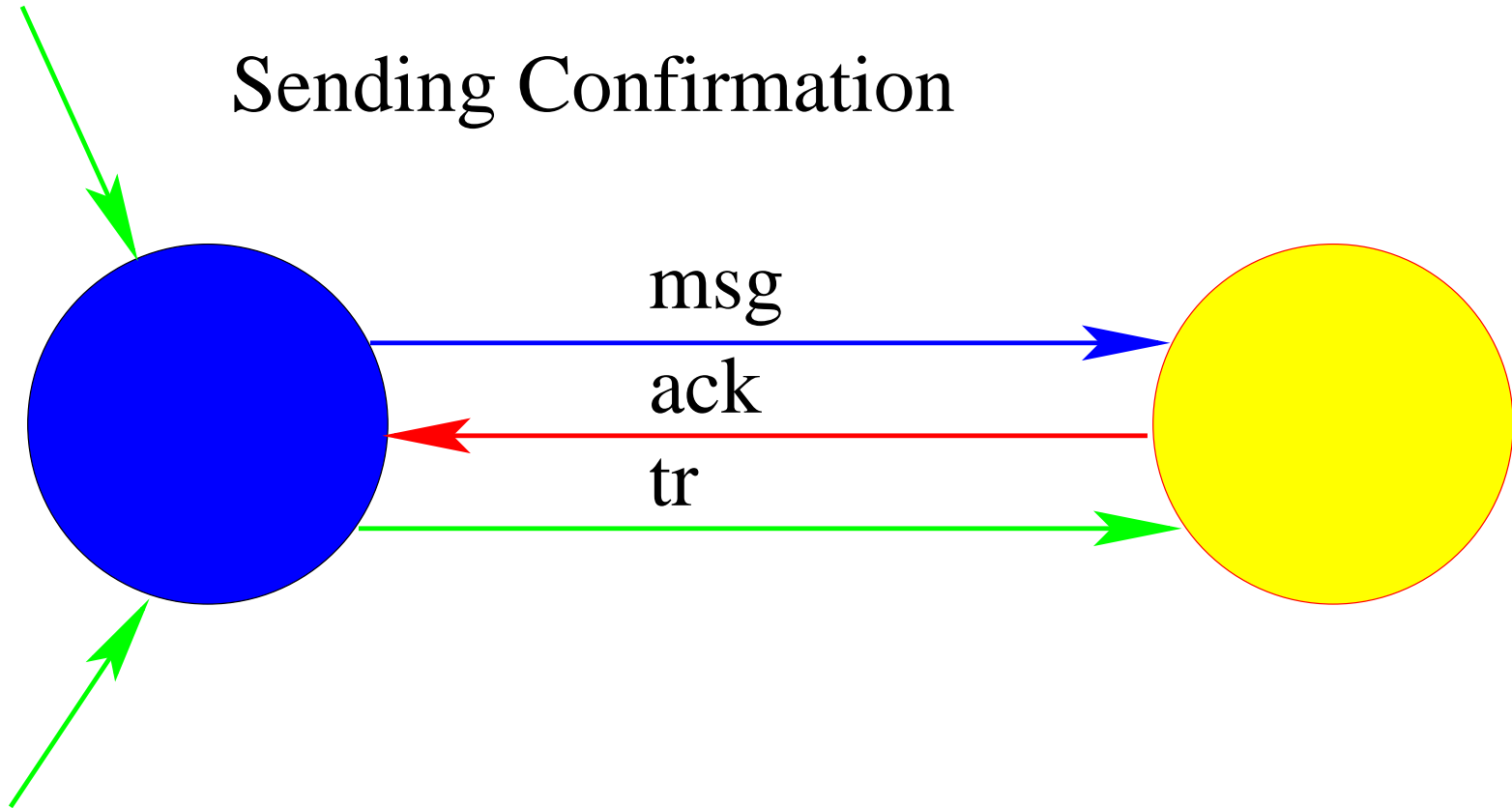
Receiving a message

Sending Acknowledgement

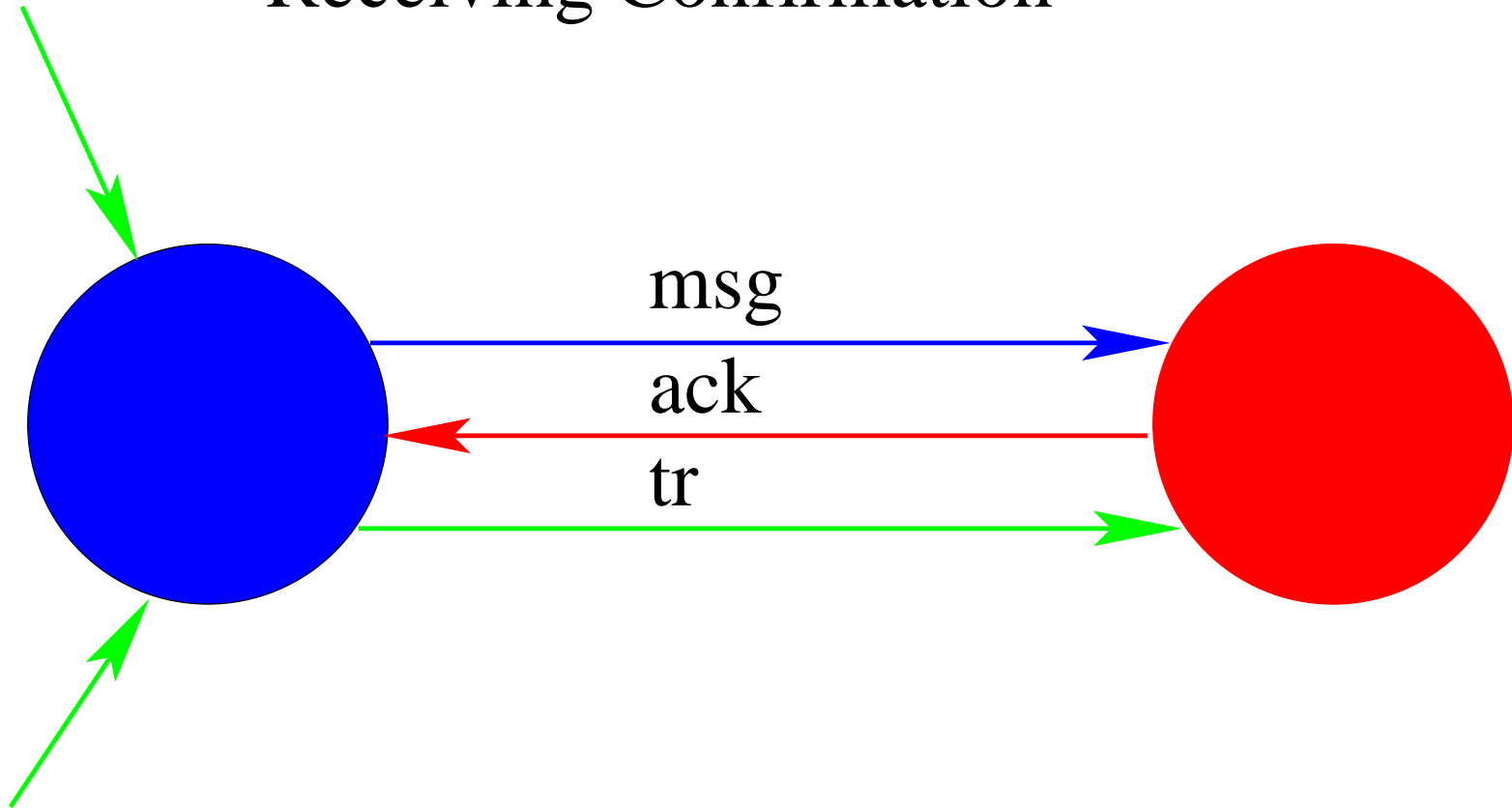


Receiving Acknowledgement

Sending Confirmation



# Receiving Confirmation





## Invariant (1)

$$msg \in ND \leftrightarrow ND$$

$$ack \in ND \leftrightarrow ND$$

$$tr \subseteq ack \subseteq msg \subseteq gr$$

Node  $x$  sends a **message** to node  $y$

$\text{send\_msg} \equiv$

**ANY**  $x, y$  **WHERE**

$x, y \in gr \wedge$

$x \notin \text{dom}(tr) \wedge$

$y, x \notin tr \wedge$

$gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\} \wedge$

$y, x \notin \text{ack} \wedge$

$x \notin \text{dom}(msg)$

**THEN**

$msg := msg \cup \{x \mapsto y\}$

**END**

Node  $y$  sends an **acknowledgement** to node  $x$

$\text{send\_ack} \equiv$

**ANY**  $x, y$  **WHERE**

$x, y \in \text{msg-ack} \wedge$

$y \notin \text{dom}(\text{msg})$

**THEN**

$\text{ack} := \text{ack} \cup \{x \mapsto y\}$

**END**

Node  $x$  sends a **confirmation** to node  $y$

progress  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in ack \wedge$

$x \notin \text{dom}(tr)$

**THEN**

$tr := tr \cup \{x \mapsto y\}$

**END**

## Invariant (2)

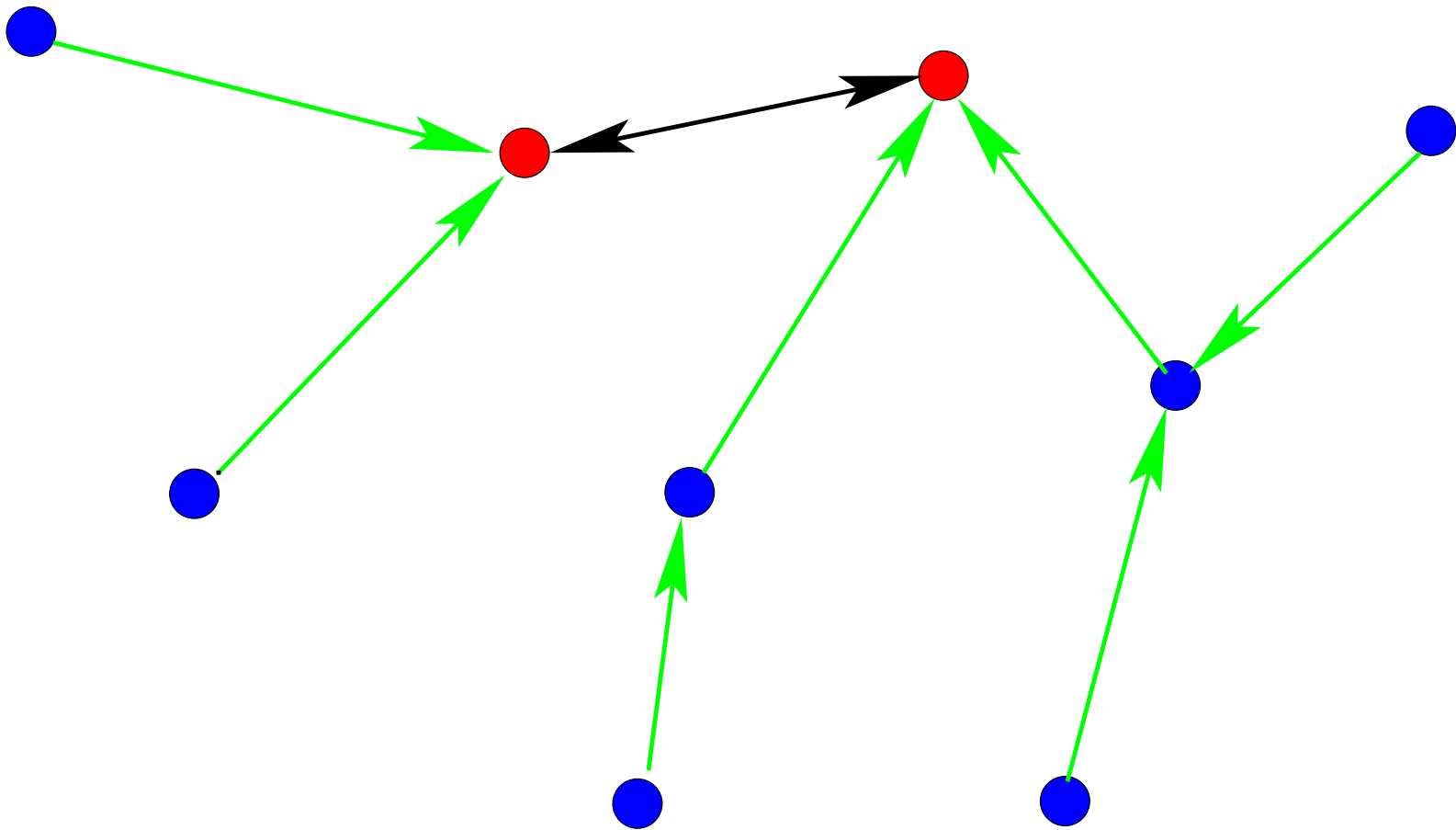
$$\forall (x, y) \cdot \left( \begin{array}{l} x, y \in msg-ack \\ \Rightarrow \\ x, y \in gr \quad \wedge \\ x \notin \text{dom}(tr) \quad \wedge \quad y \notin \text{dom}(tr) \quad \wedge \\ gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\} \end{array} \right)$$

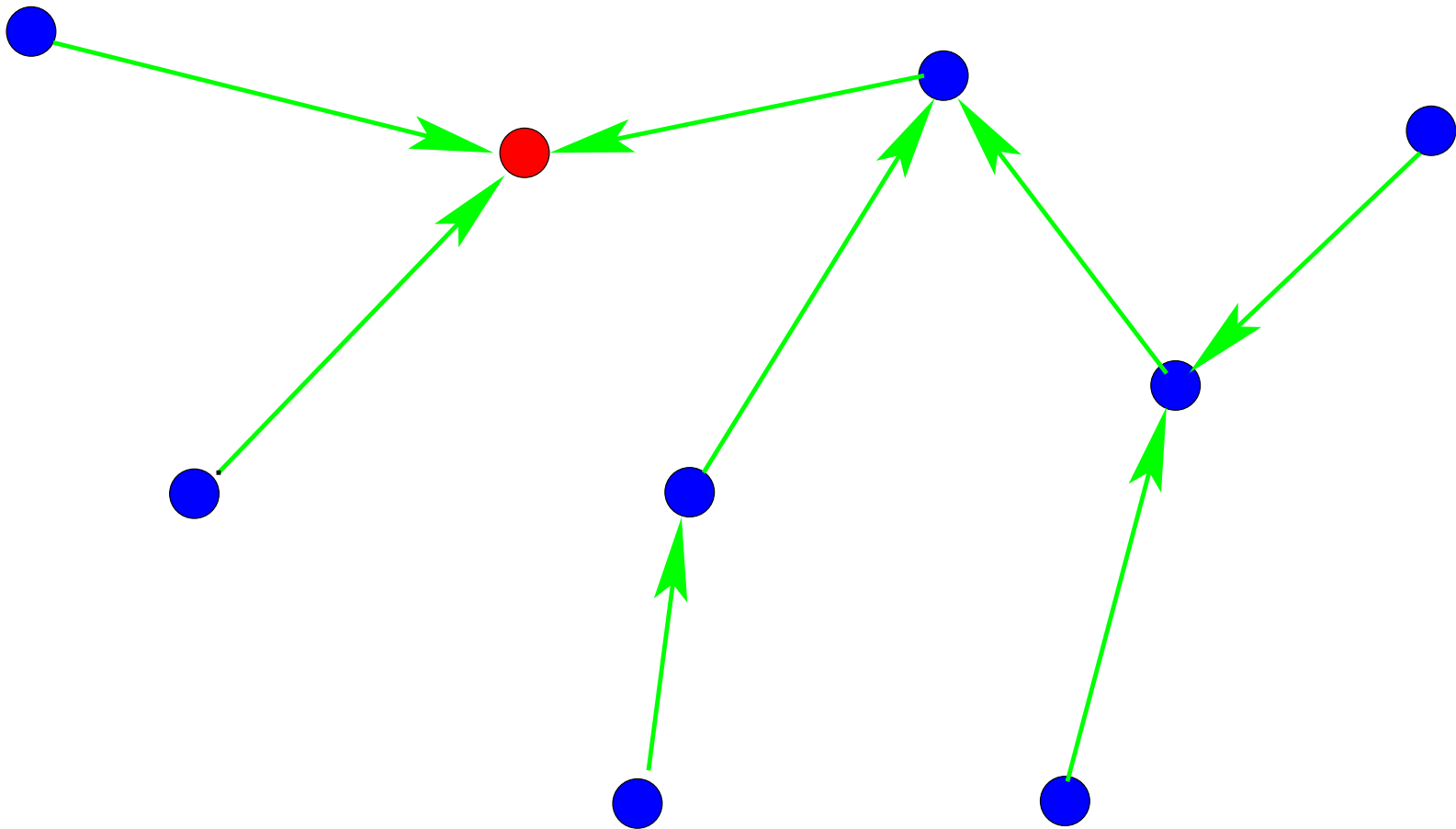
$$\forall (x, y) \cdot \left( \begin{array}{l} x, y \in ack \quad \wedge \\ x \notin \text{dom}(tr) \\ \Rightarrow \\ x, y \in gr \quad \wedge \\ y \notin \text{dom}(tr) \quad \wedge \\ gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\} \end{array} \right)$$

# Second Refinement: The problem of contention

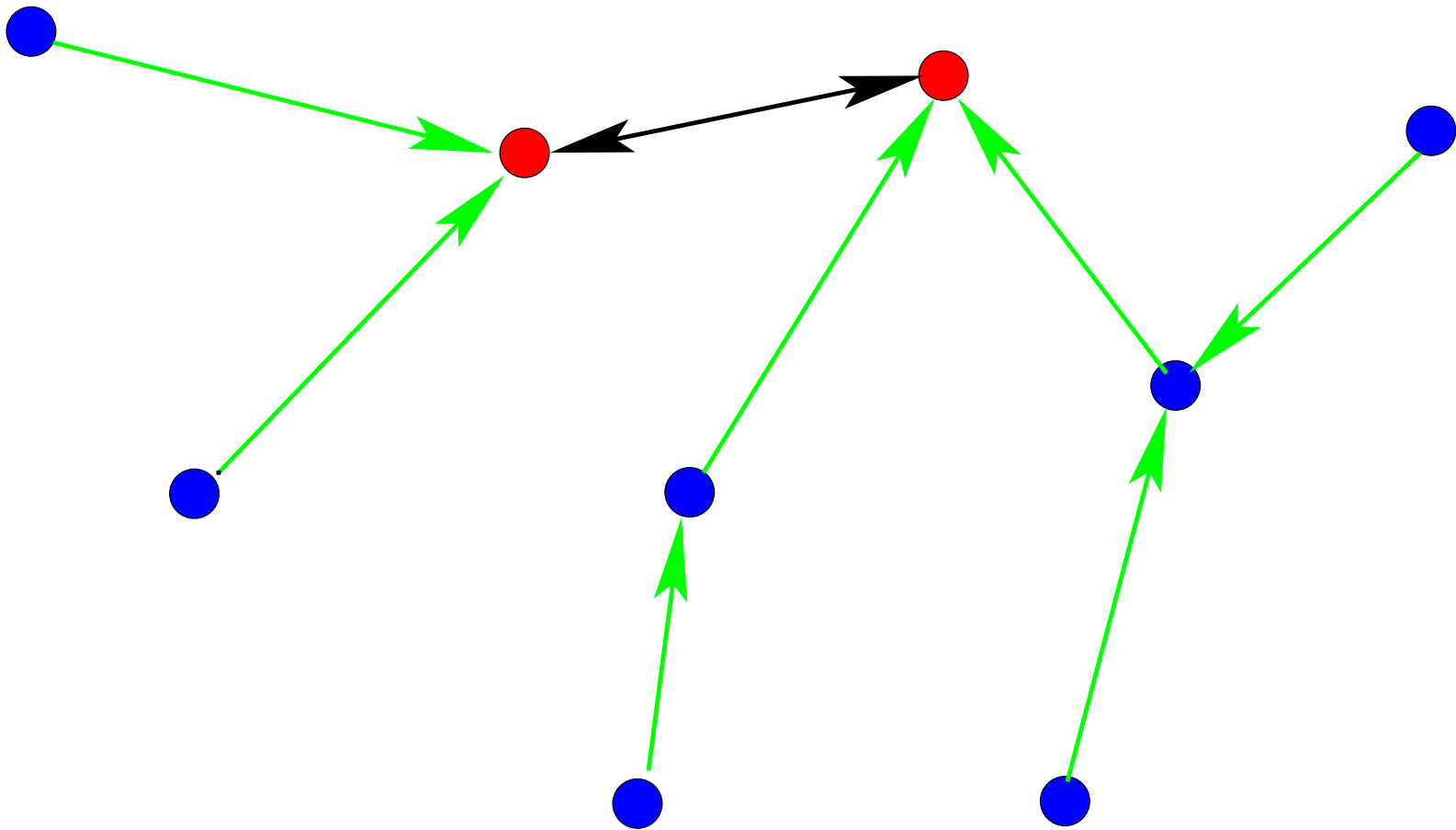
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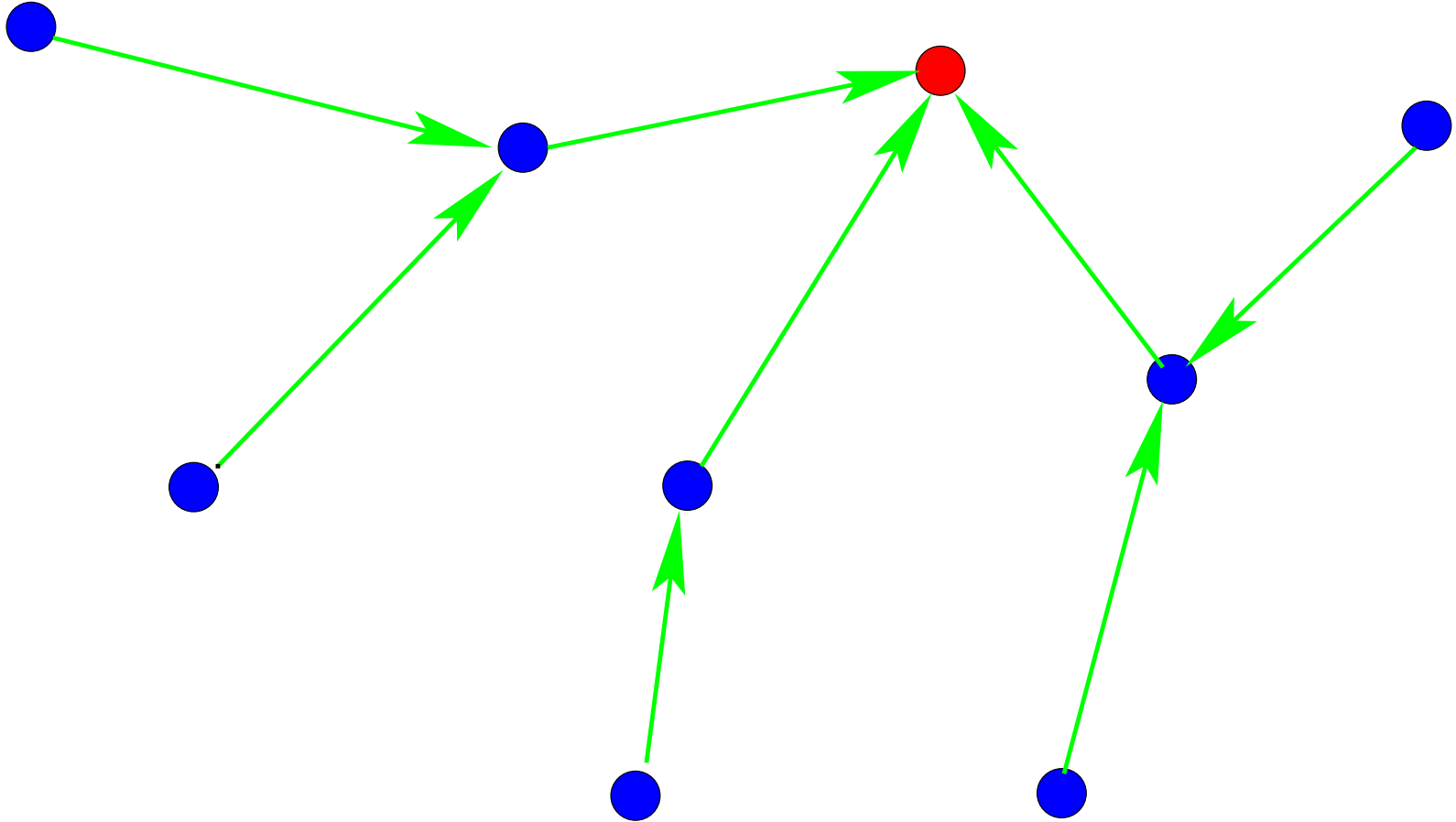
- Explaining the **problem**
  - Proposing a **partial** solution
  - Towards a **better** treatment
  - Back to the **local** animation
-

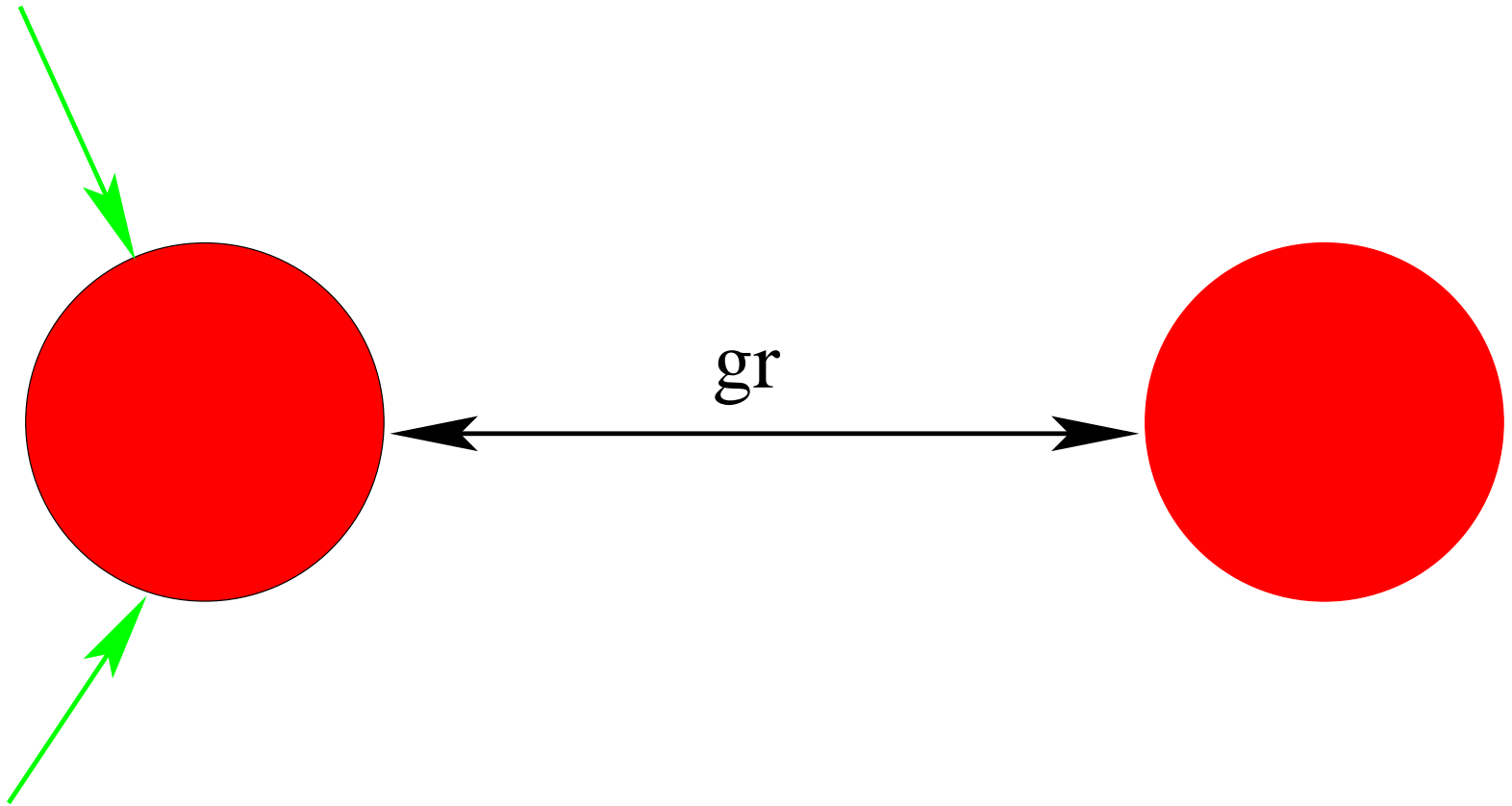




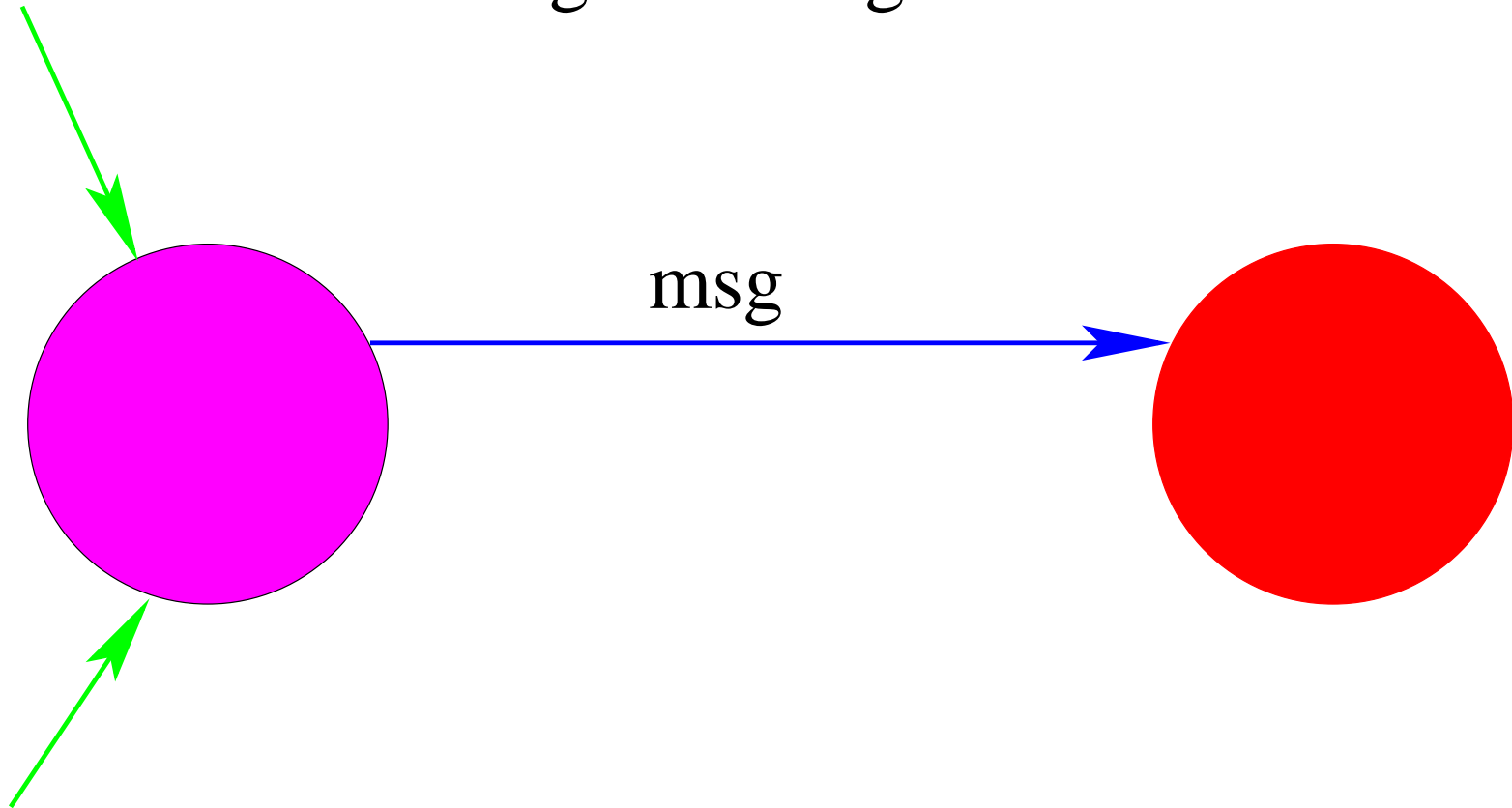




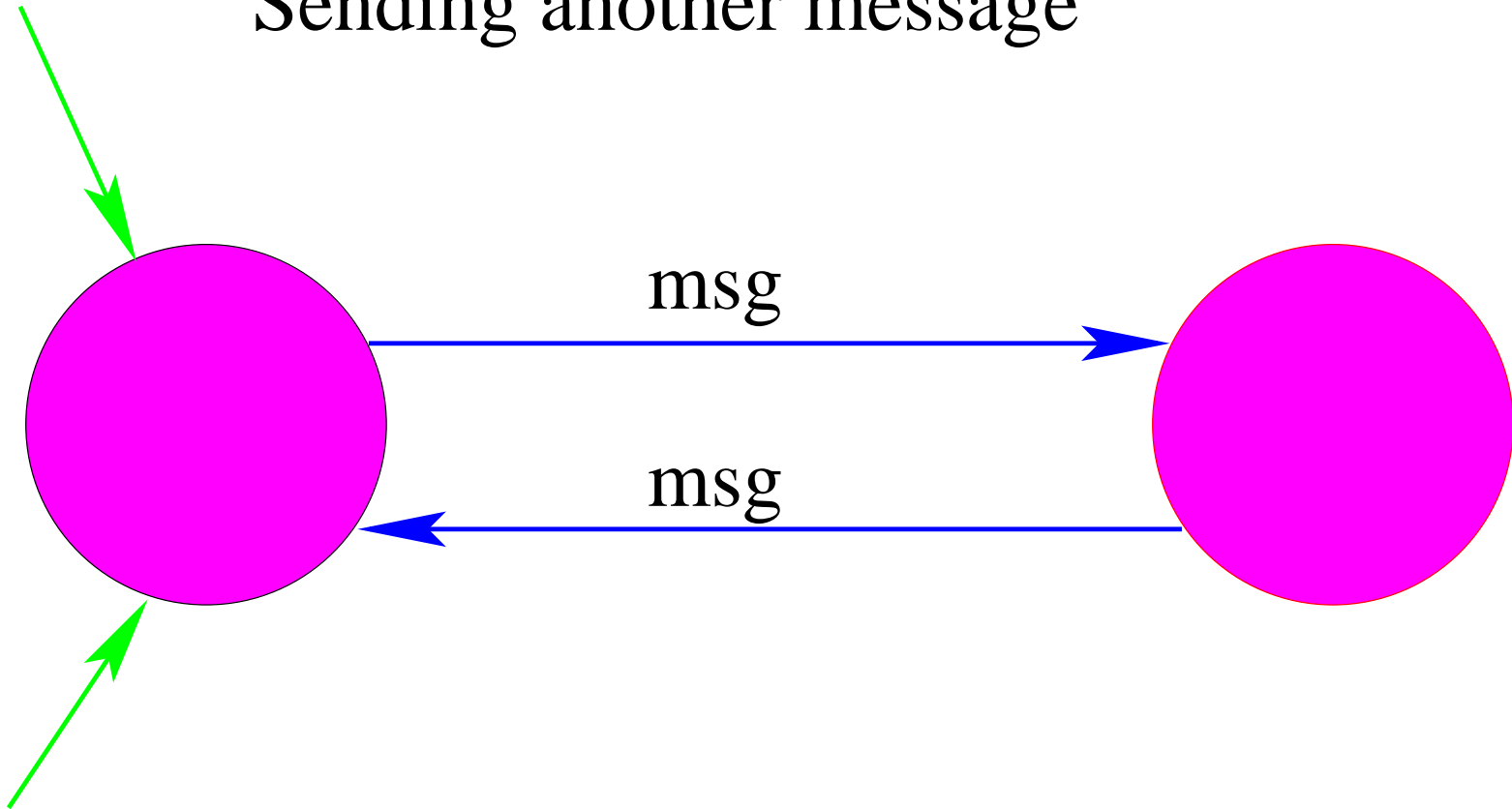




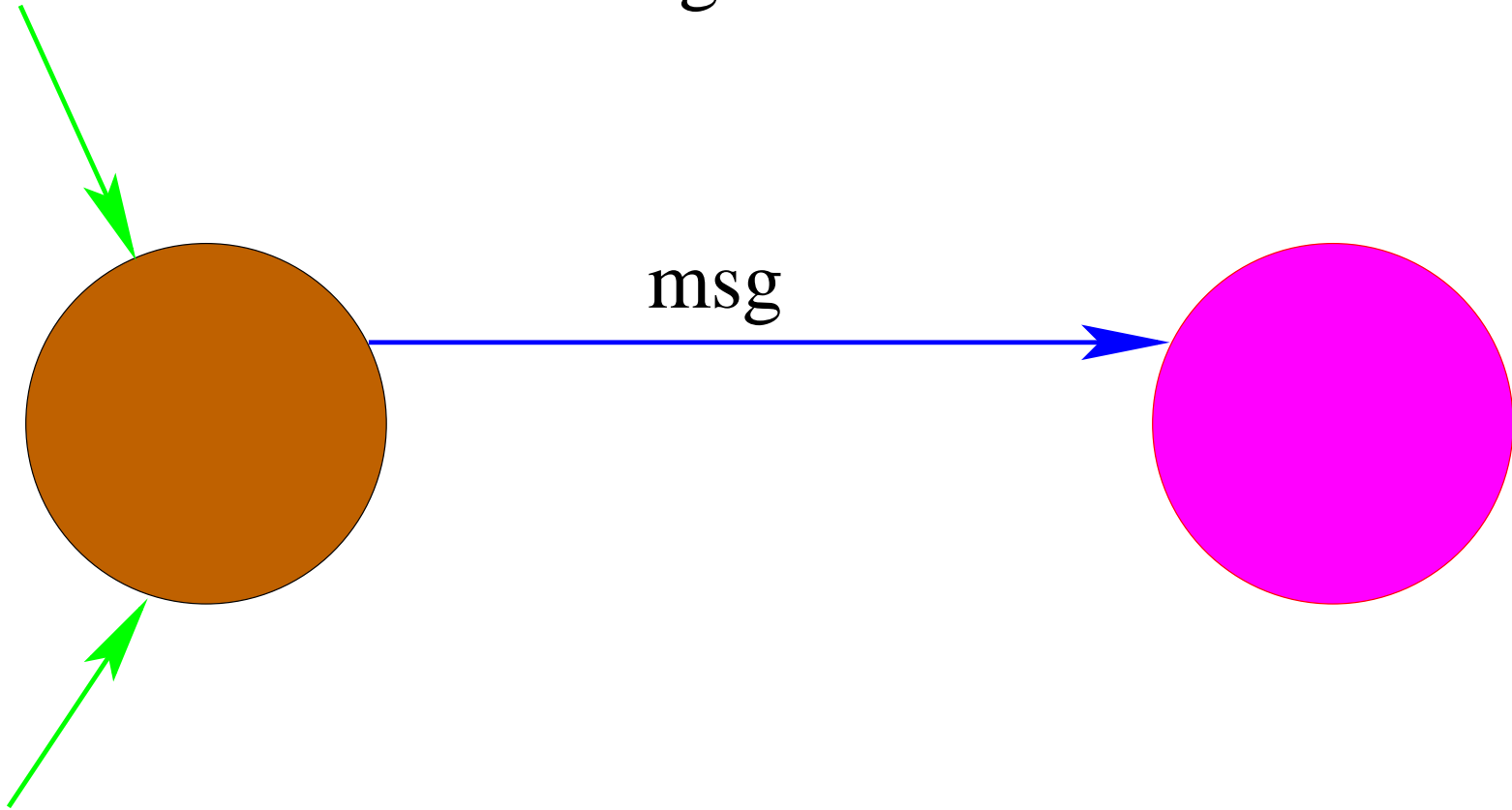
Sending a message



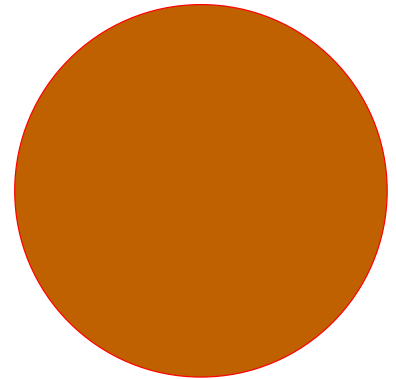
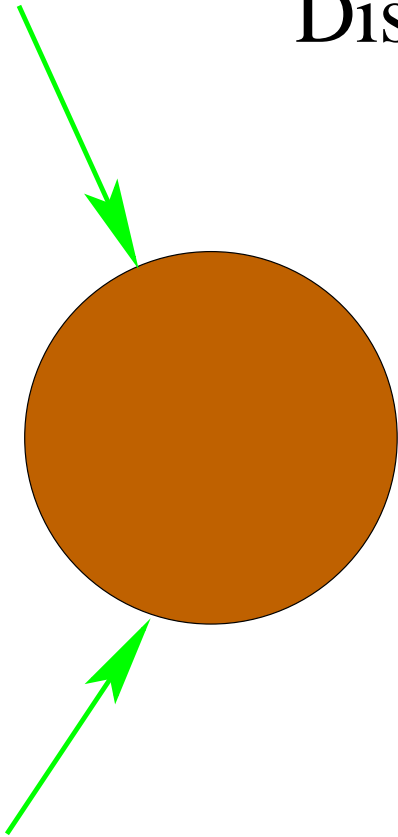
Sending another message



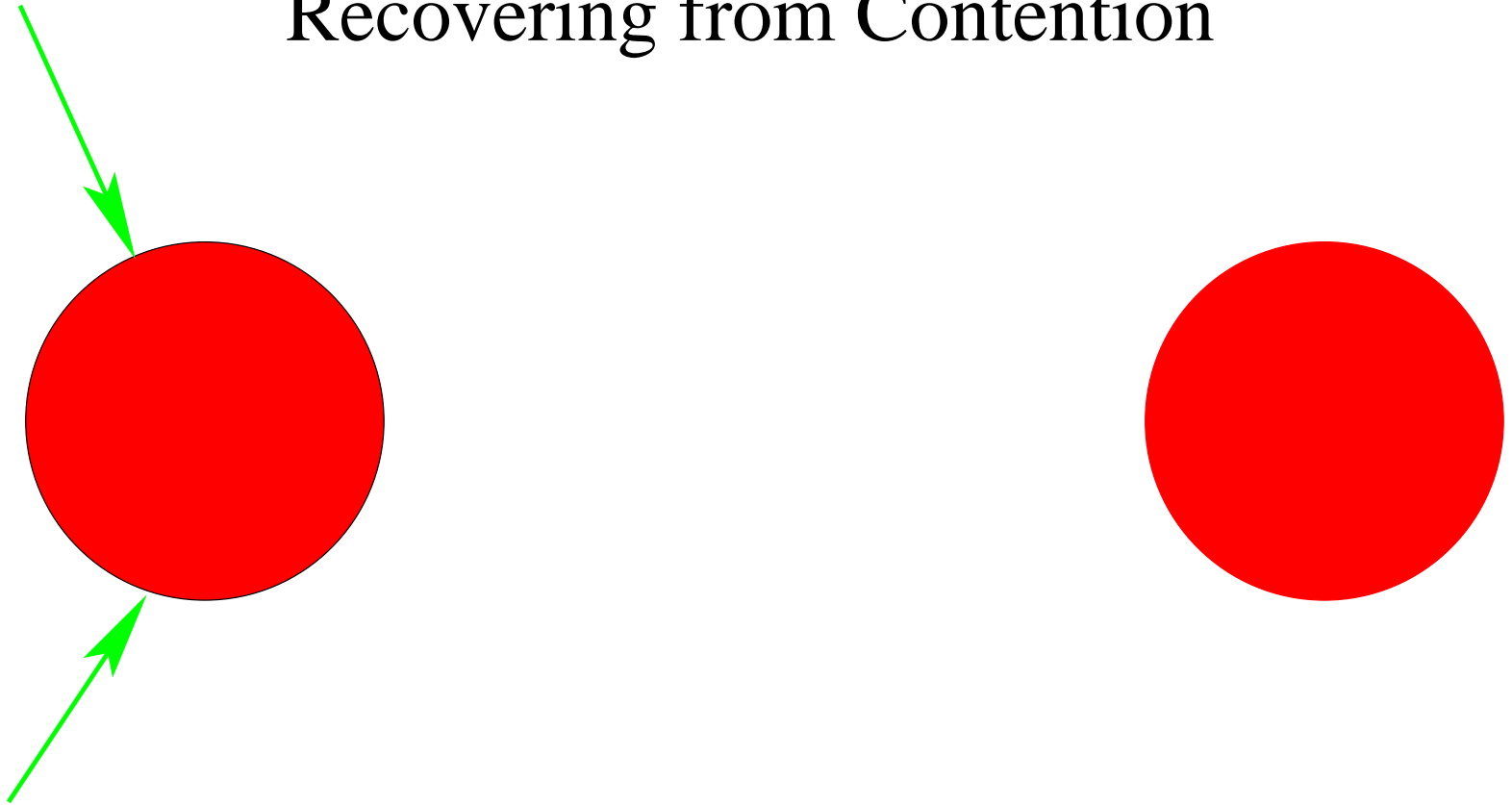
# Discovering Contention



# Discovering Contention

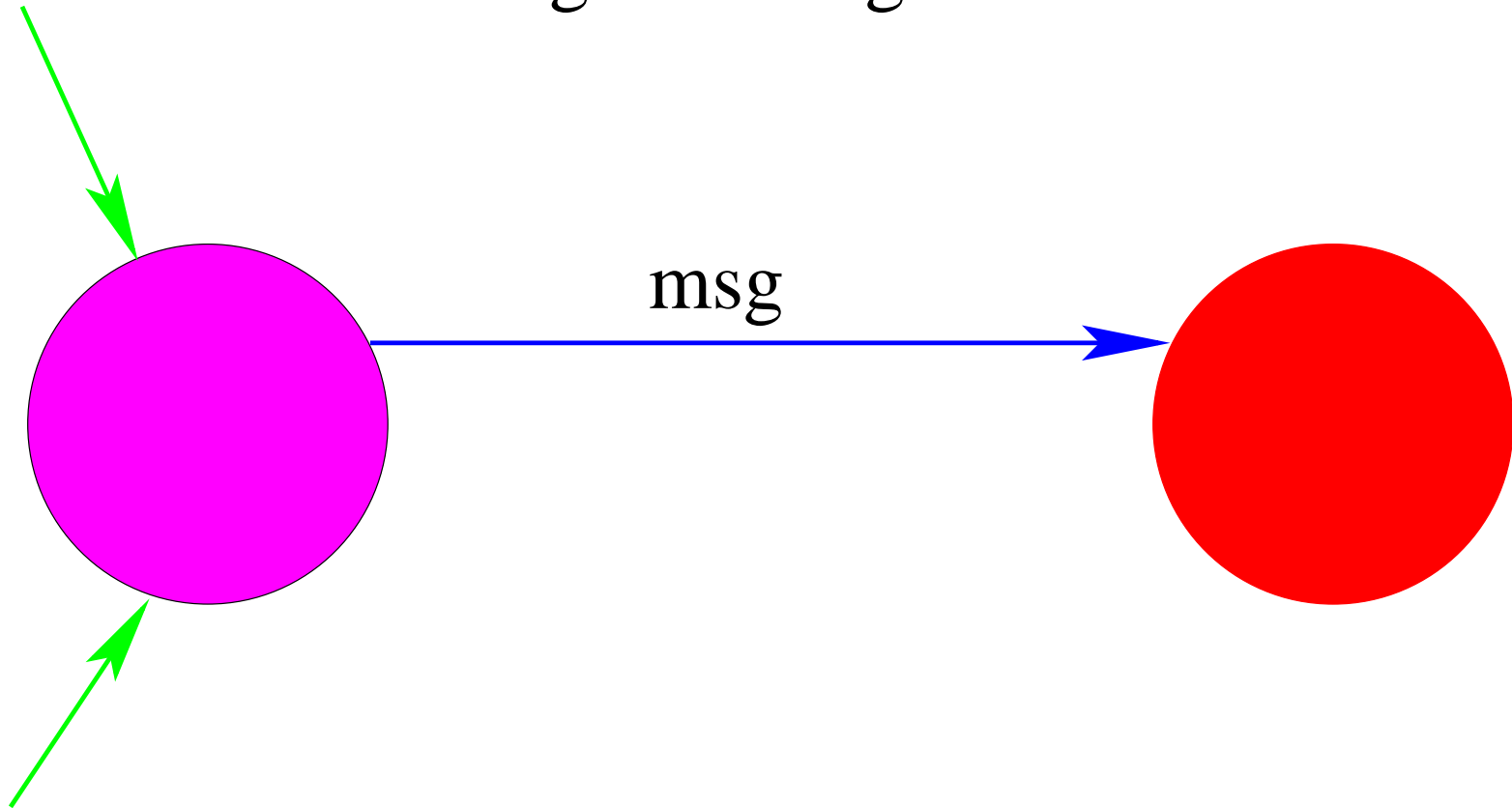


## Recovering from Contention

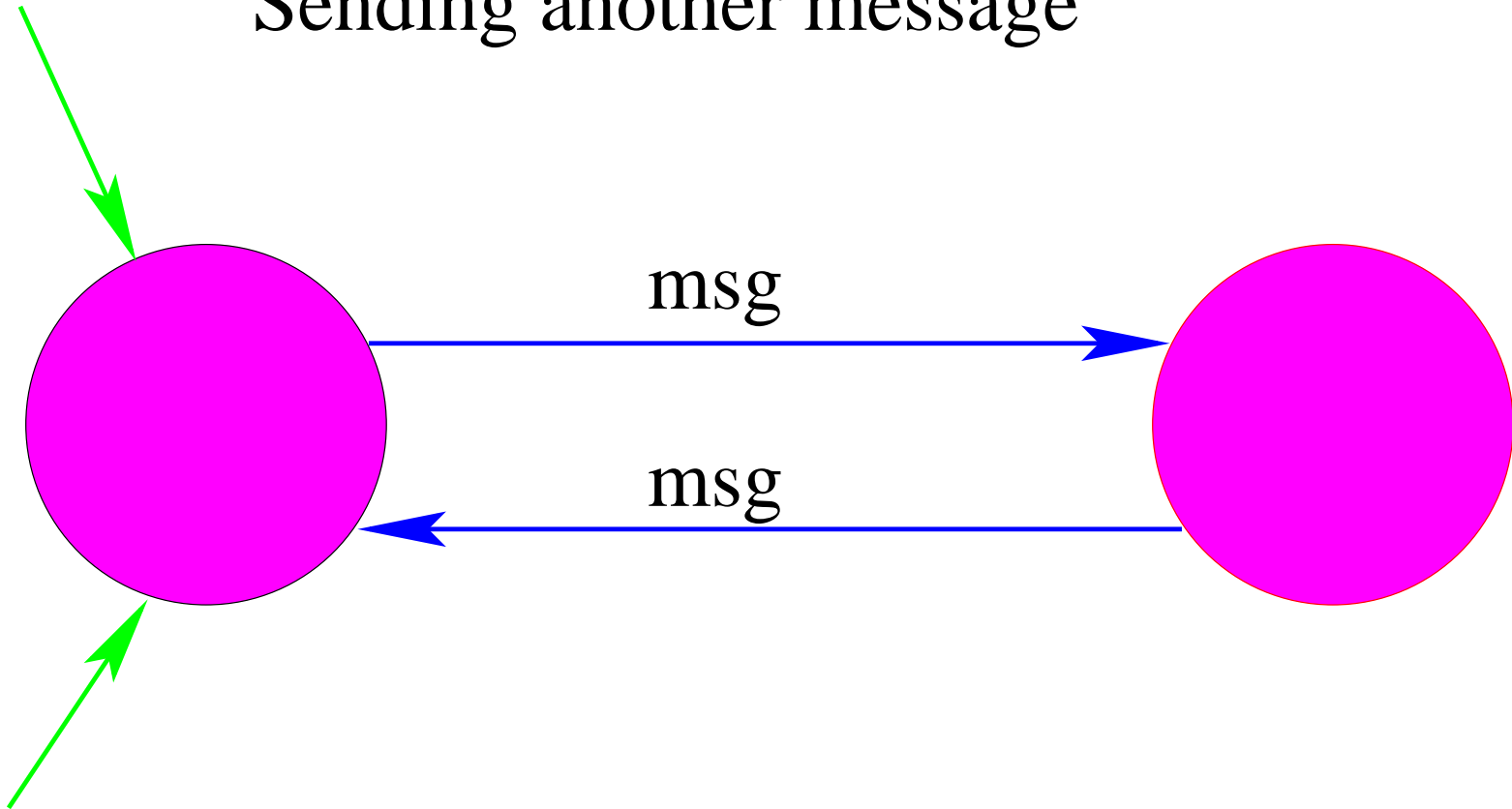




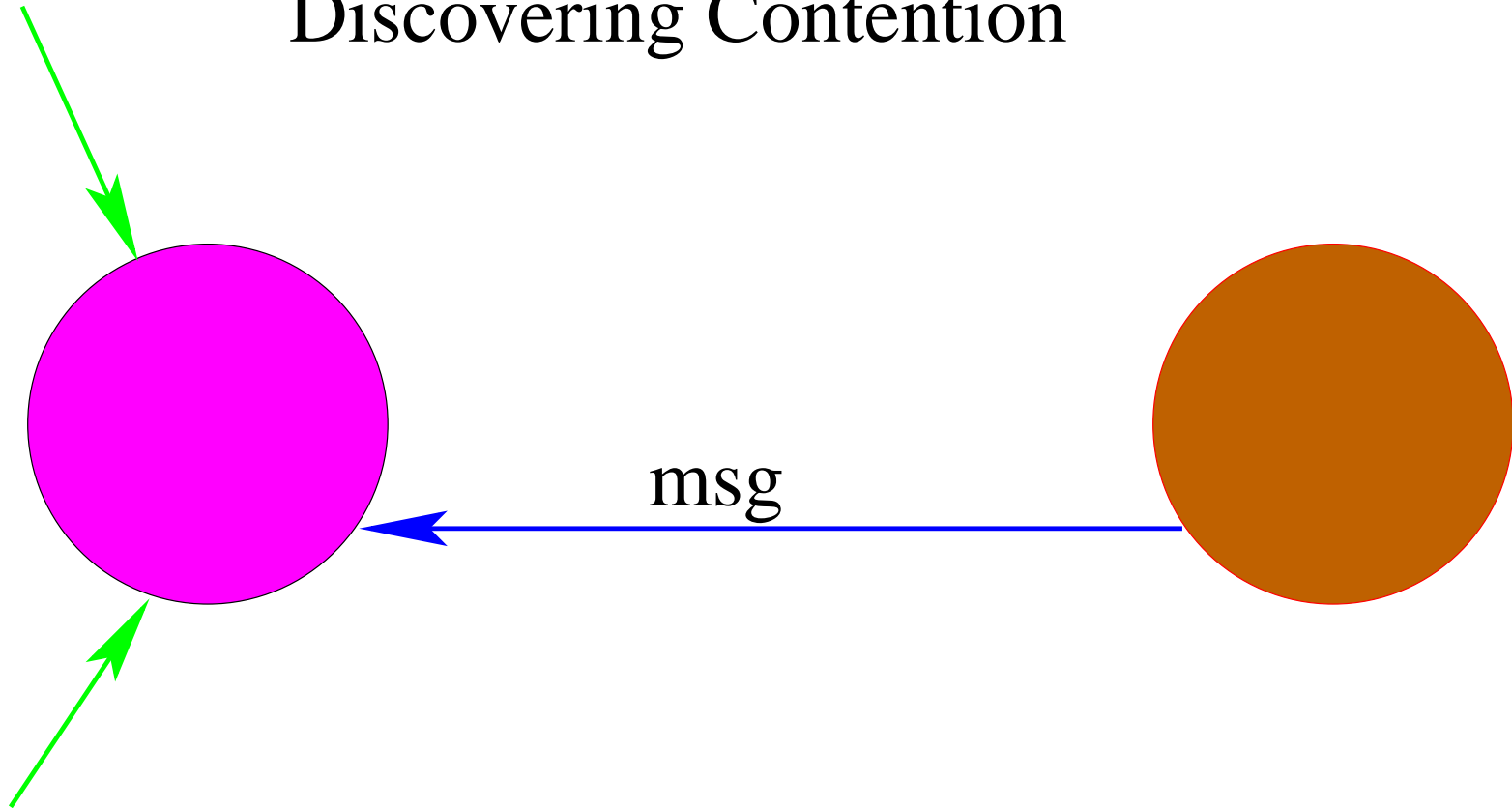
Sending a message



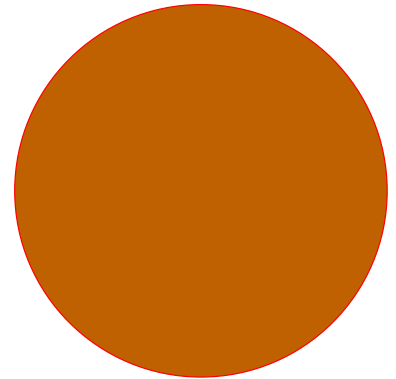
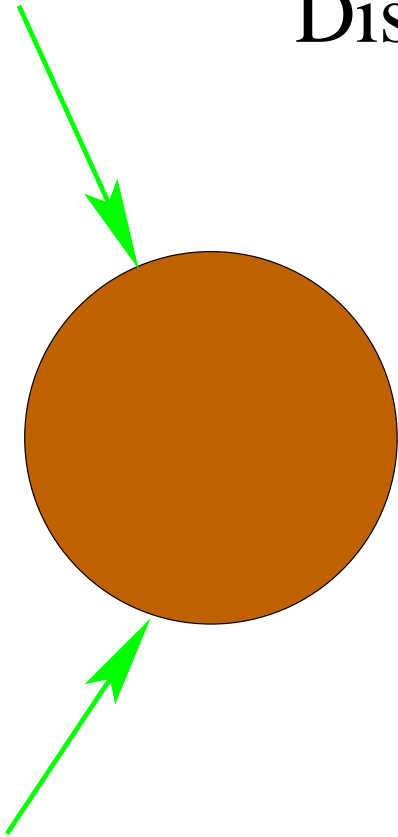
Sending another message



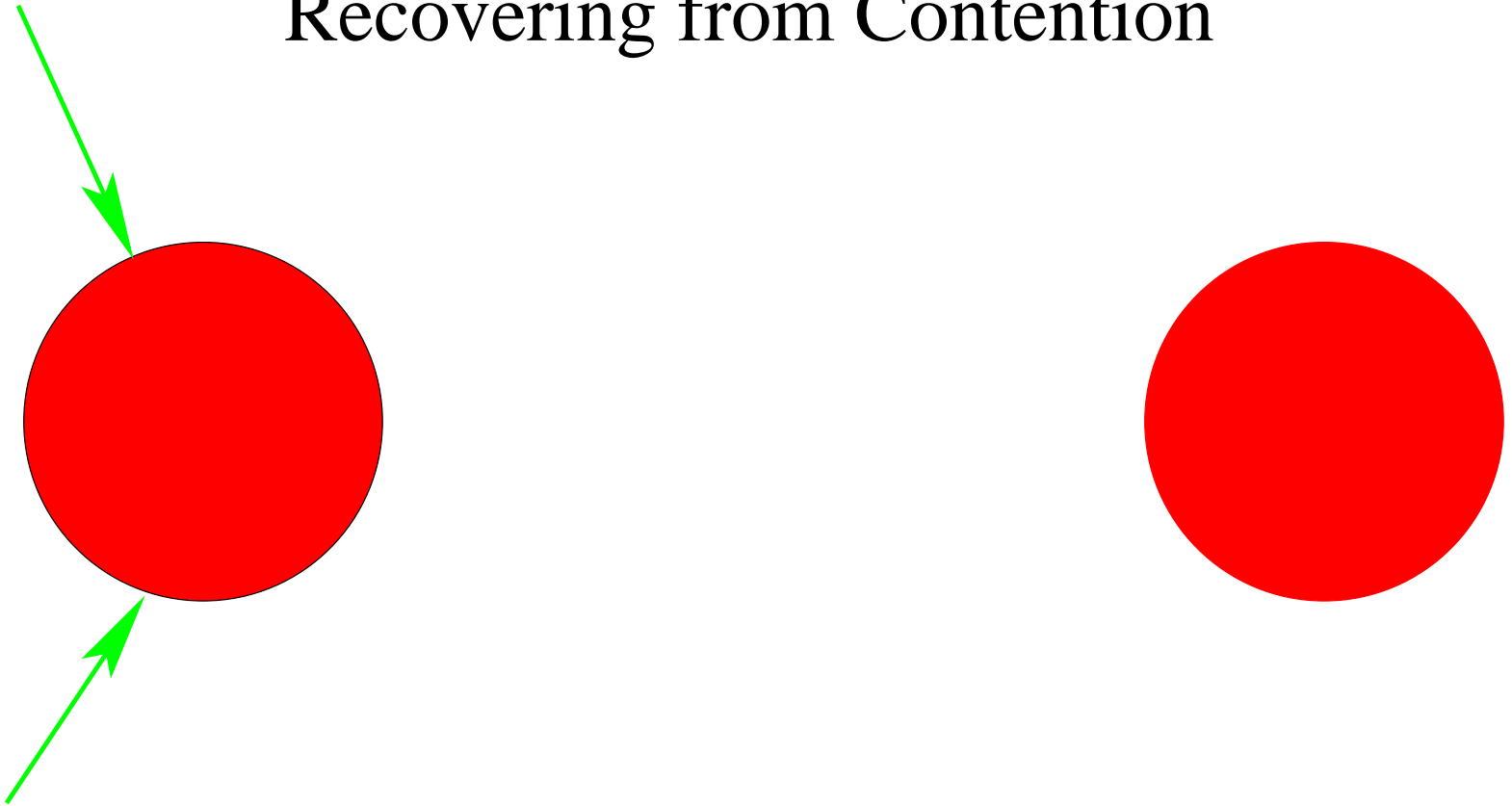
# Discovering Contention



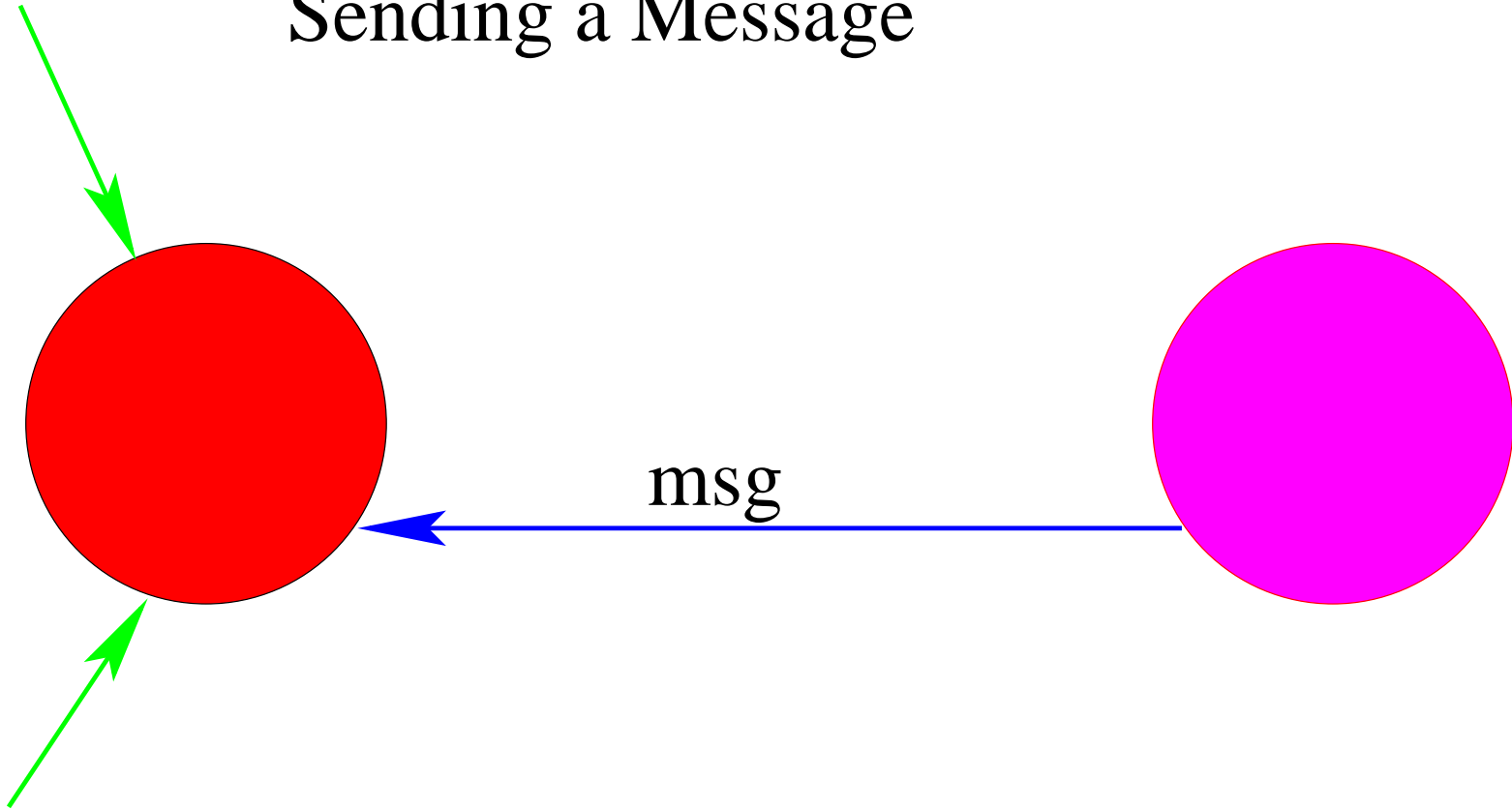
# Discovering Contention



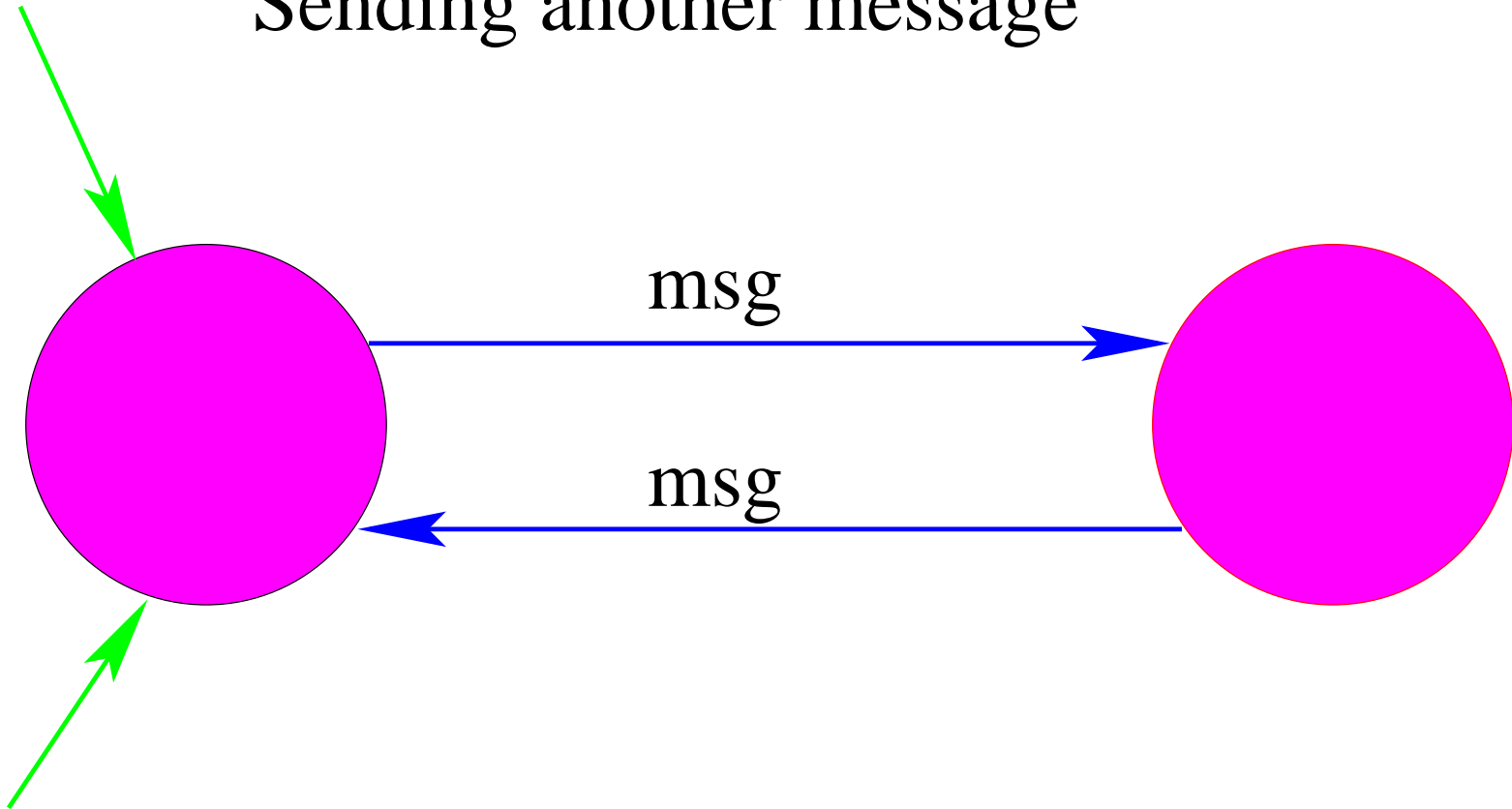
# Recovering from Contention



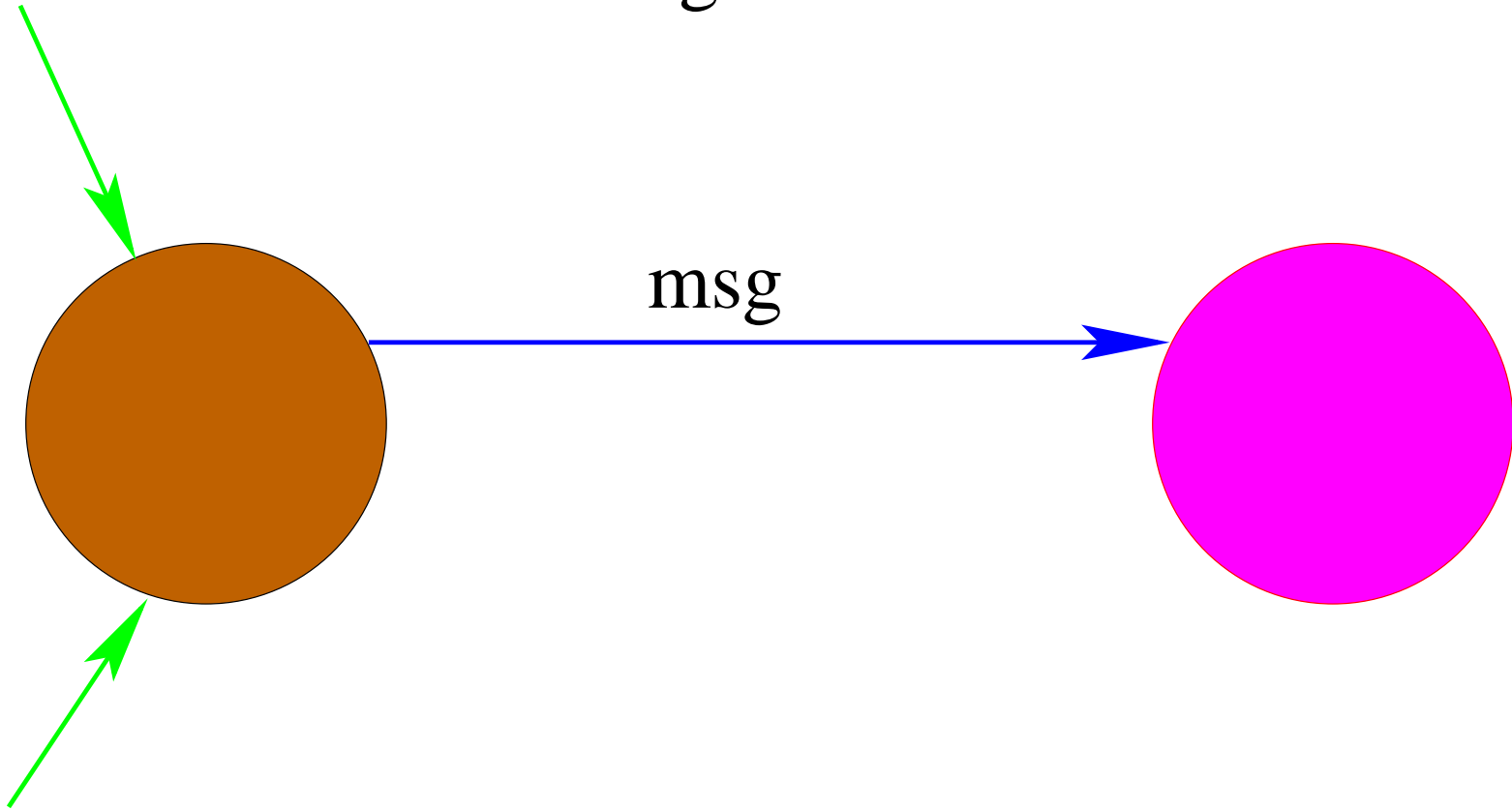
# Sending a Message



Sending another message

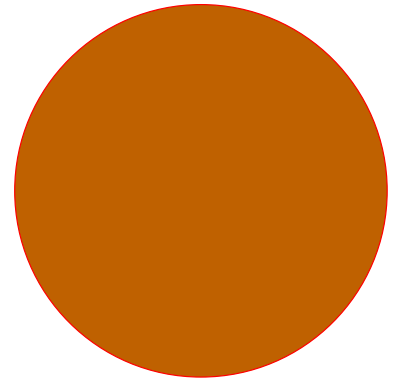
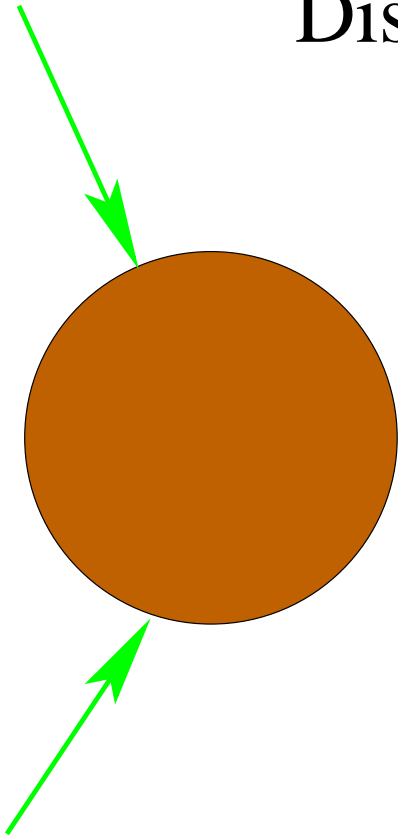


# Discovering Contention

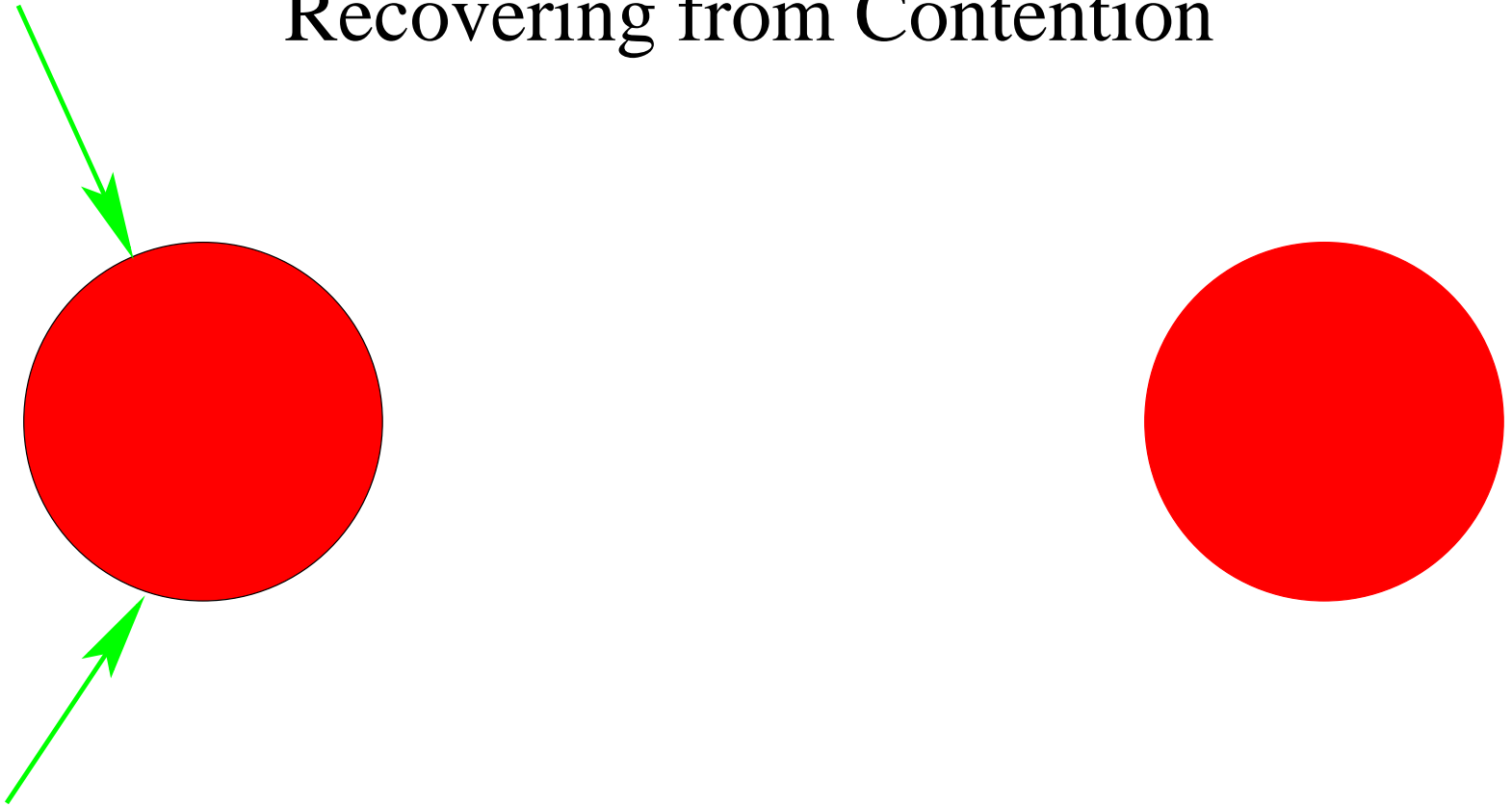




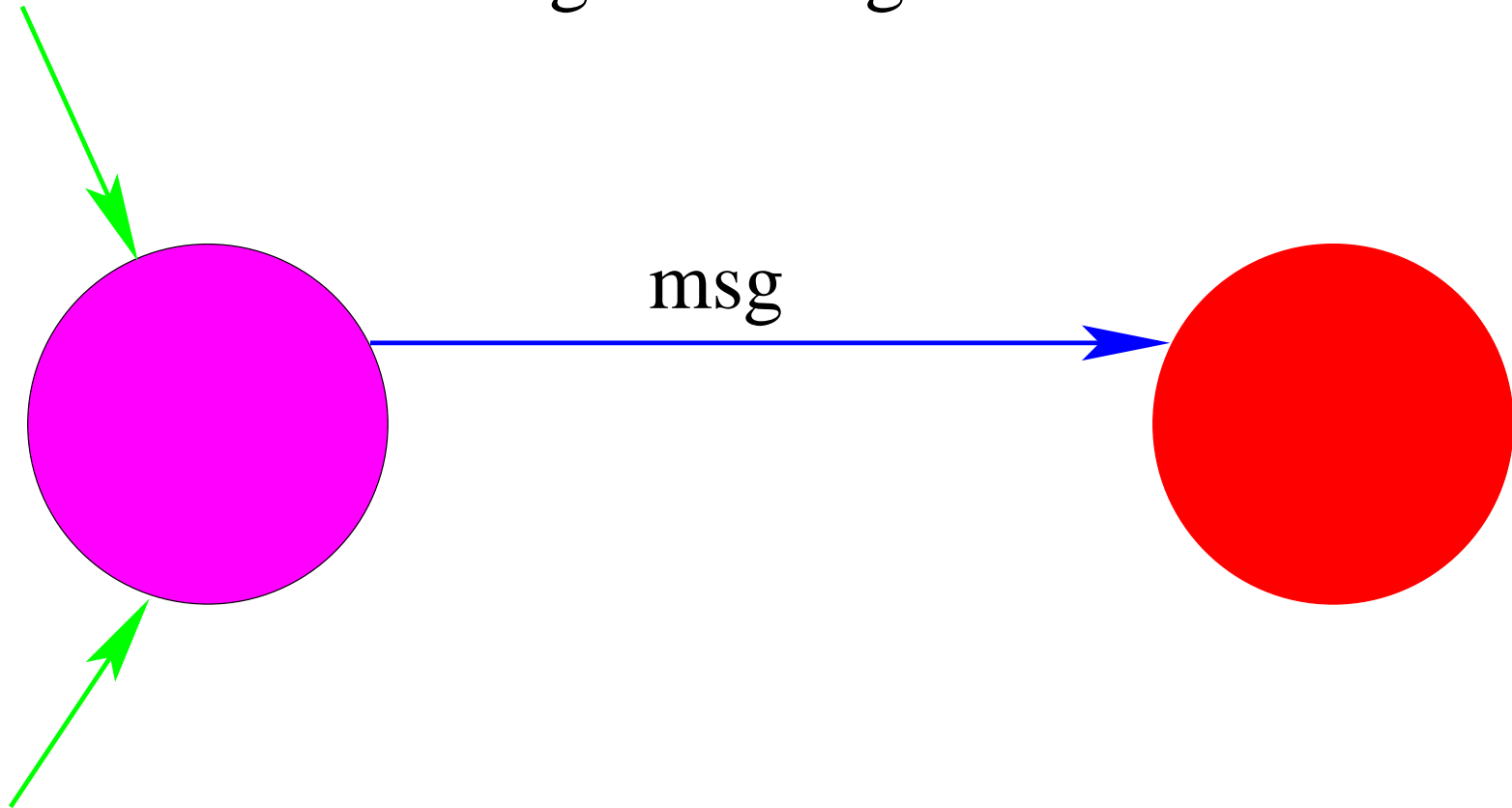
# Discovering Contention



# Recovering from Contention

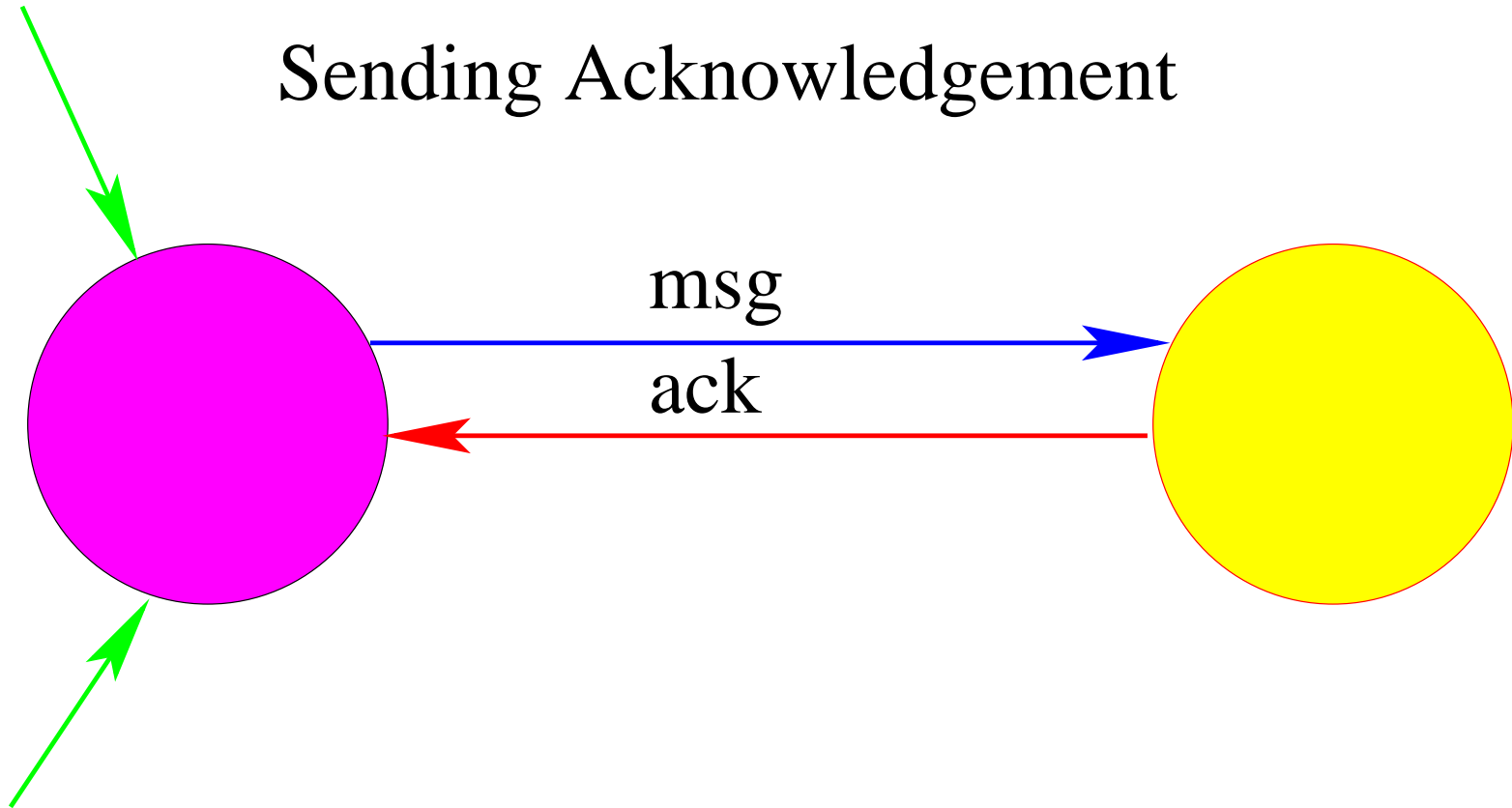


Sending a message



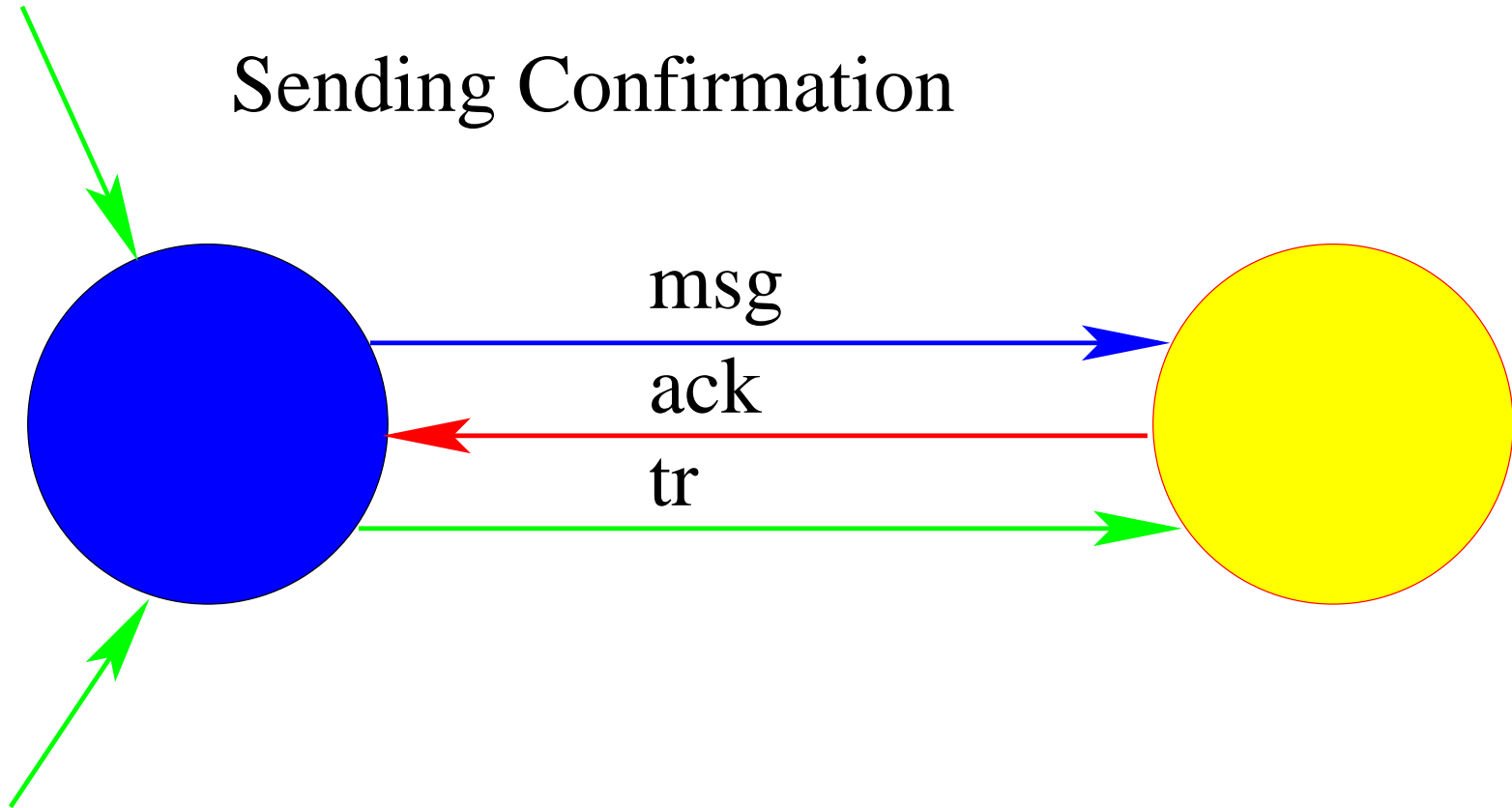
Receiving a message

Sending Acknowledgement

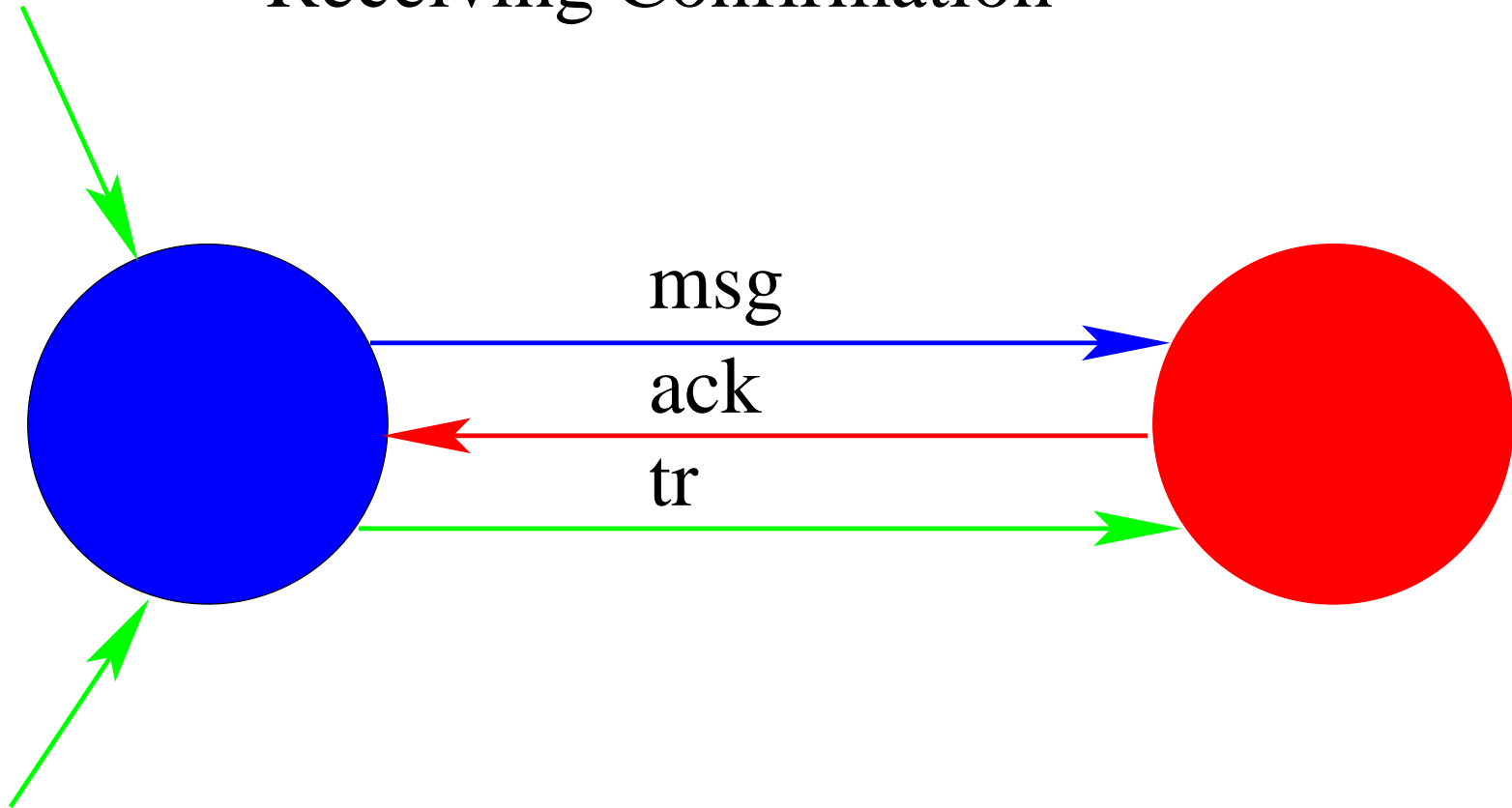


Receiving Acknowledgement

Sending Confirmation



# Receiving Confirmation



# Discovering the Contention (1)

---

- Node  $y$  **discovers the contention** with node  $x$  because:
  - It has **sent a message** to node  $x$
  - It has **not yet received** a response from node  $x$
  - It **receives instead** a message from node  $x$

## Discovering the Contention (2)

---

- Node  $x$  also **discovers the contention** with node  $y$
  - Assumption: The time **between both discoveries**  
**IS SUPPOSED TO BE BOUNDED**  
**BY  $\tau$  ms**
  - The time  $\tau$  is the **maximum transmission time**  
between 2 connected nodes
-



# A Partial Solution

---

- Each node **waits for  $\tau$  ms** after its own discovery
  - After this, each node thus **knows** that the other has also discovered the contention
  - Each node then **retries immediately**
  - **PROBLEM:** This may continue **for ever**
-

# A Better Solution (1)

---

- Each node **waits for  $\tau$  ms** after its own discovery
  - Each node then chooses **with equal probability**:
    - either to wait for a **short** delay
    - or to wait for a **large** delay
  - Each node then **retries**
-

## A Better Solution (2)

---

- **Question**: Does this solves the problem ?
  - Are we sure to **eventually** have one node winning ?
  - **Answer**: Listen carefully to **Caroll Morgan's lectures**
-

# Node $y$ discovers a contention with node $x$

---

$\text{send\_ack} \hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in \text{msg-ack} \wedge$   
 $y \notin \text{dom}(\text{msg})$

**THEN**

$\text{ack} := \text{ack} \cup \{x \mapsto y\}$

**END**

$\text{contention} \hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in \text{msg-ack} \wedge$   
 $y \in \text{dom}(\text{msg})$

**THEN**

$\text{cnt} := \text{cnt} \cup \{x \mapsto y\}$

**END**

- Introducing a dummy **contention channel**:  $\text{cnt}$

$\text{cnt} \in ND \leftrightarrow ND$

$\text{cnt} \subseteq \text{msg}$

$\text{ack} \cap \text{cnt} = \emptyset$



# Solving the contention (simulating the $\tau$ delay)

---

solve\_contention  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in cnt \cup cnt^{-1}$

**THEN**

$msg := msg - cnt$  ||

$cnt := \emptyset$

**END**

# Summary of Second Refinement

---

- 73 proofs
- Among which 34 were interactive

## Third Refinement: Localization

---

- The representation of the **graph  $gr$**  is **modified**
  - The representation of the **tree  $tr$**  is **modified**
  - Other data structures are **localized**
-



# Localization (1)

---

The graph  $gr$  and the tree  $tr$  are now **localized**

$$nb \in ND \rightarrow \mathbb{P}(ND)$$

$$\forall x \cdot (x \in ND \Rightarrow nb(x) = gr[\{x\}])$$

$$sn \in ND \rightarrow \mathbb{P}(ND)$$

$$\forall x \cdot (x \in ND \Rightarrow sn(x) \subseteq tr^{-1}[\{x\}])$$

---

## Localization (2)

---

$$bm \subseteq \underline{ND}$$

$$bm = \text{dom}(msg)$$

$$bt \subseteq \underline{ND}$$

$$bt = \text{dom}(tr)$$

$$ba \in ND \rightarrow \mathbb{P}(ND)$$

$$\forall x \cdot (x \in ND \Rightarrow ba(x) = ack^{-1}[\{x\}])$$

---

- Node  $x$  is elected the leader

elect  $\hat{=}$

**ANY**  $x$  **WHERE**

$x \in ND \wedge$

$nb(x) = sn(x)$

**THEN**

$rt := x$

**END**

- Node  $x$  sends a message to node  $y$  ( $y$  is unique)

send\_msg  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x \in ND - bm \wedge$

$y \in ND - (ba(x) \cup sn(x)) \wedge$

$nb(x) = sn(x) \cup \{y\}$

**THEN**

$msg := msg \cup \{x \mapsto y\} \quad ||$

$bm := bm \cup \{x\}$

**END**

- Node  $y$  sends an acknowledgement to node  $x$

send\_ack  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in msg \wedge$

$x \notin ba(y) \wedge$

$y \notin bm$

**THEN**

$ack := ack \cup \{x \mapsto y\} \quad ||$

$ba(y) := ba(y) \cup \{x\}$

**END**

- Node  $x$  sends a confirmation to node  $y$

progress  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in ack \wedge$

$x \notin bt$

**THEN**

$tr := tr \cup \{x \mapsto y\} \quad ||$

$bt := bt \cup \{x\}$

**END**

- Node  $y$  receives confirmation from node  $x$

$\text{rcv\_cnf} \hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in tr \wedge$

$x \notin sn(y)$

**THEN**

$sn(y) := sn(y) \cup \{x\}$

**END**

contention  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in cnt \cup cnt^{-1} \wedge$

$x \notin ba(y) \wedge$

$y \in bm$

**THEN**

$cnt := cnt \cup \{x \mapsto y\}$

**END**



`solve_contention`  $\hat{=}$

**ANY**  $x, y$  **WHERE**

$x, y \in cnt \cup cnt^{-1}$

**THEN**

$msg := msg - cnt$  ||

$bm := bm - \text{dom}(cnt)$  ||

$cnt := \emptyset$

**END**

# Summary of Third Refinement

---

- 29 proofs
- Among which 19 were interactive

# Main Summary

---

- 119 proofs
- Among which 63 were interactive

# Conclusion: a Systematic Approach to Distribution

---

- Establishing the **mathematical framework**



# Conclusion: a Systematic Approach to Distribution

---

- Establishing the **mathematical framework**
- Resolving the mathematical problem in **one shot**

# Conclusion: a Systematic Approach to Distribution

---

- Establishing the **mathematical framework**
  - Resolving the mathematical problem in **one shot**
  - Resolving the same problem on a **step by step basis**
-

# Conclusion: a Systematic Approach to Distribution

---

- Establishing the **mathematical framework**
  - Resolving the mathematical problem in **one shot**
  - Resolving the same problem on a **step by step basis**
  - Involving communication **by means of messages**
-



# Conclusion: a Systematic Approach to Distribution

---

- Establishing the **mathematical framework**
  - Resolving the mathematical problem in **one shot**
  - Resolving the same problem on a **step by step basis**
  - Involving communication **by means of messages**
  - Towards the **localization of data structures**
-