# Distributed algorithms
# The Leader Election Protocol (IEEE 1394)

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# This Session

- Background :-)

- An informal presentation of the protocol :-)

- Step by step formal design :-|

- Short Conclusion. :-)

# IEEE 1394 High Performance Serial Bus (FireWire)

- It is an international standard

- There exists a widespread commercial interest in its correctness

- Sun, Apple, Philips, Microsoft, Sony, etc involved in its development

- Made of three layers (physical, link, transaction)

- The protocol under study is the Tree Identify Protocol

- Situated in the Bus Reset phase of the physical layer

## The Problem (1)

- The bus is used to transport digitized video and audio signals

- It is “hot-pluggable”

- Devices and peripherals can be added and removed at any time

- Such changes are followed by a bus reset

- The leader election takes place after a bus reset in the network

- A leader needs to be chosen to act as the manager of the bus

# The Problem (2)

- After a bus reset: all nodes in the network have equal status

- A node only knows to which nodes it is directly connected

- The network is connected

- The network is acyclic

# References (1)

BASIC

- IEEE. *IEEE Standard for a High Performance Serial Bus. Std 1394-1995*. 1995

- IEEE. *IEEE Standard for a High Performance Serial Bus (supplement). Std 1394a-2000*. 2000

# References (2)

## GENERAL

- N. Lynch. *Distributed Algorithms*. Morgan Kaufmann. 1996

- R. G. Gallager et al. *A Distributed Algorithm for Minimum Weight Spanning Trees*. IEEE Trans. on Prog. Lang. and Systems. 1983.

# References (3)

MODEL CHECKING

- D.P.L. Simons et al. *Mechanical Verification of the IEE 1394a Root Contention Protocol using Uppaal2* Springer International Journal of Software Tools for Technology Transfer. 2001

- H. Toetenel et al. *Parametric verification of the IEEE 1394a Root Contention Protocol using LPMC* Proceedings of the 7th International Conference on Real-time Computing Systems and Applications. IEEE Computer Society Press. 2000

# References (4)

THEOREM PROVING


- M. Devillers et al. *Verification of the Leader Election: Formal Method Applied to IEEE 1394*. Formal Methods in System Design. 2000

- J.R. Abrial et al. *A Mechanically Proved and Incremental Development of IEEE 1394*. To be published 2002

# Informal Absract Properties of the Protocol

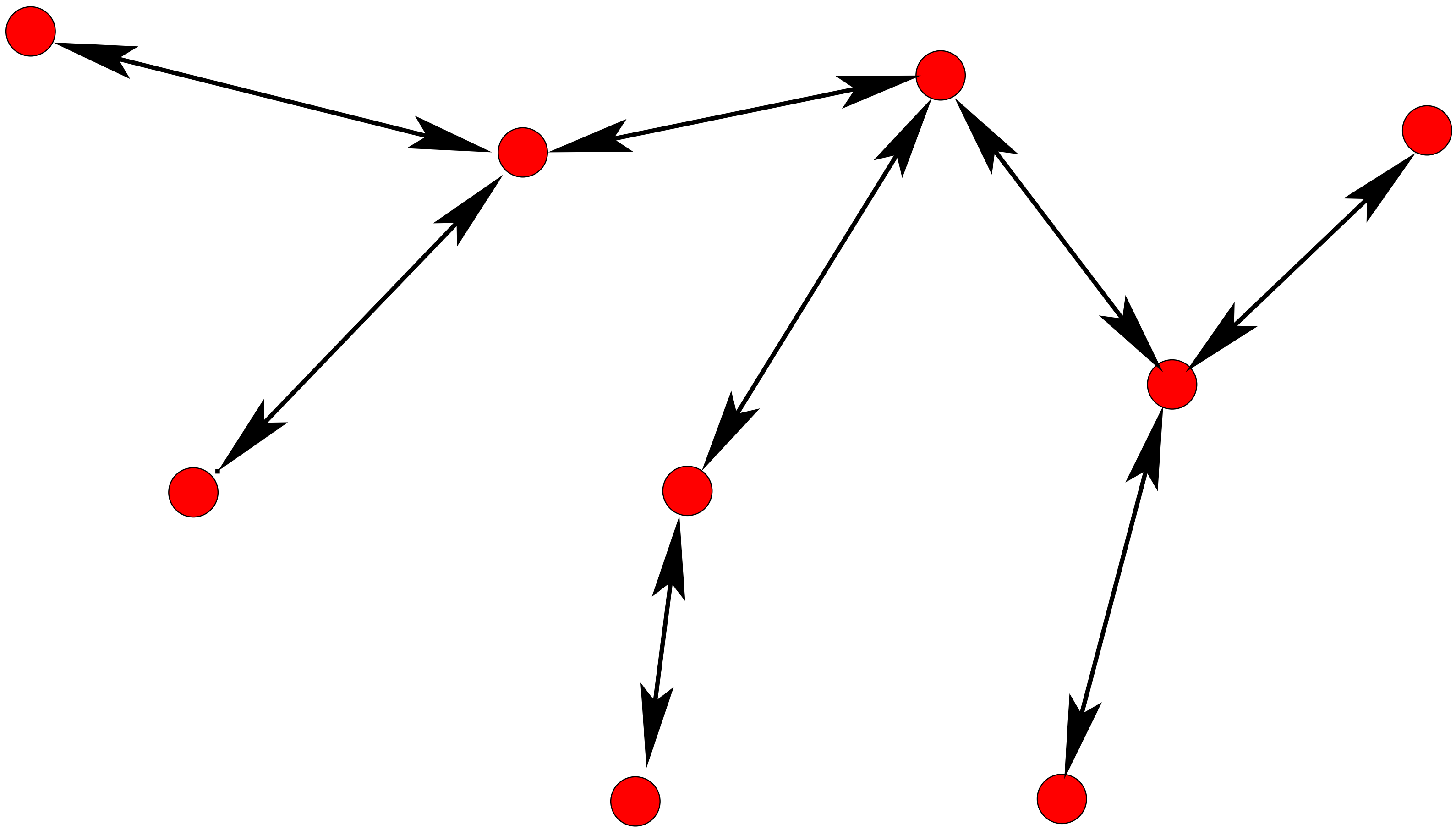
- We are given a connected and acyclic network of nodes

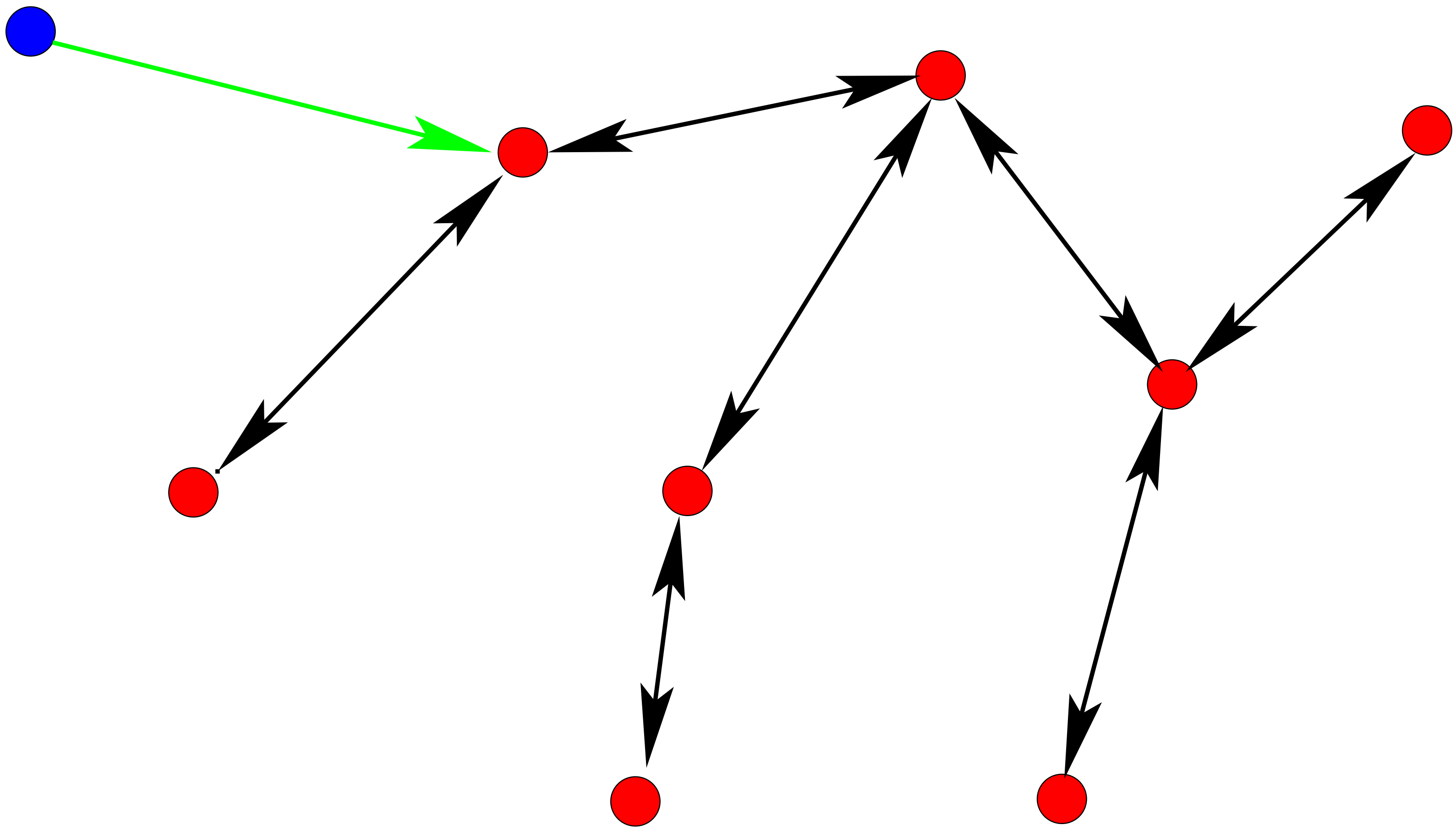
- Nodes are linked by bidirectional channels

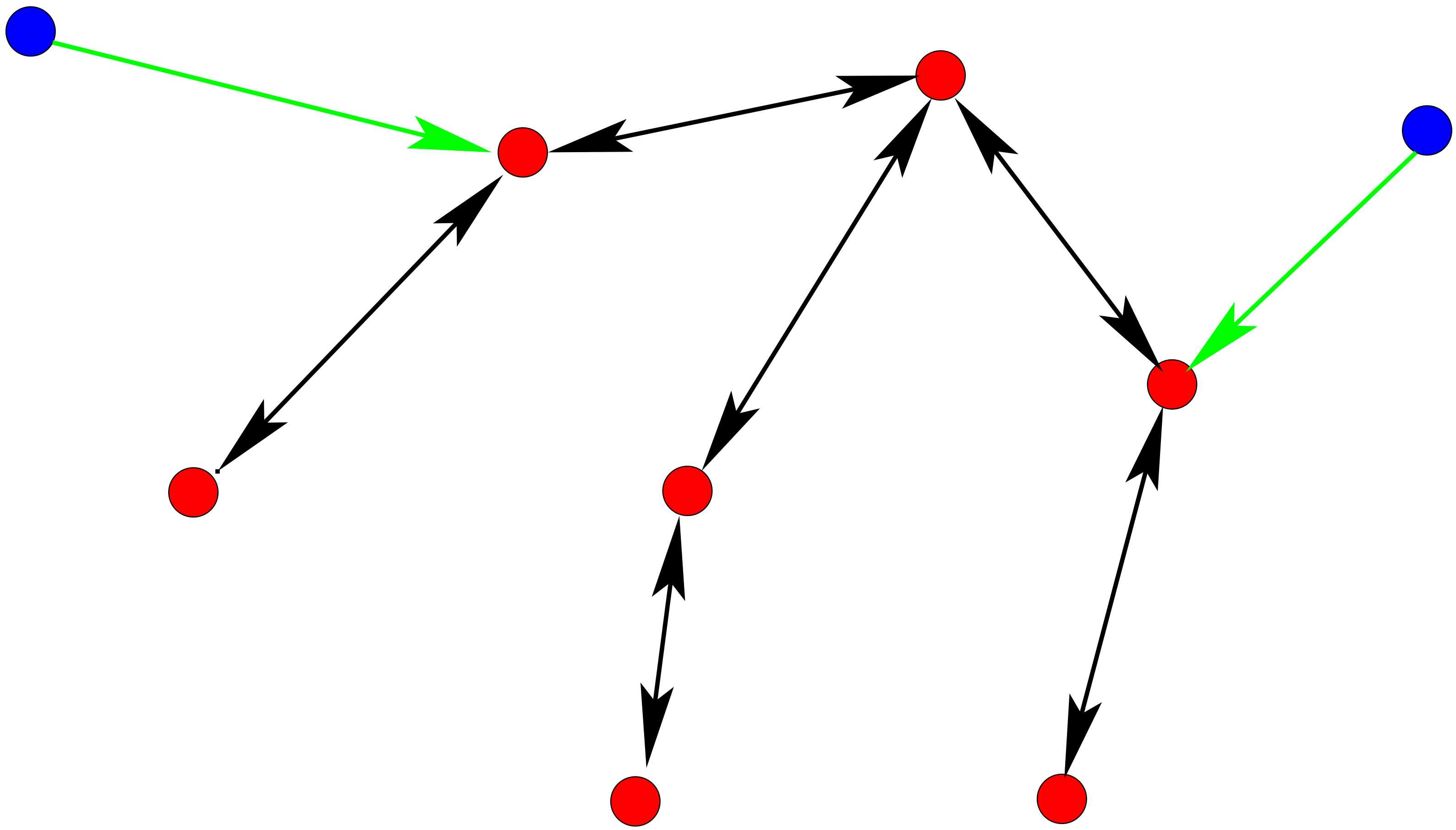
- We want to have one node being elected the leader in a finite time

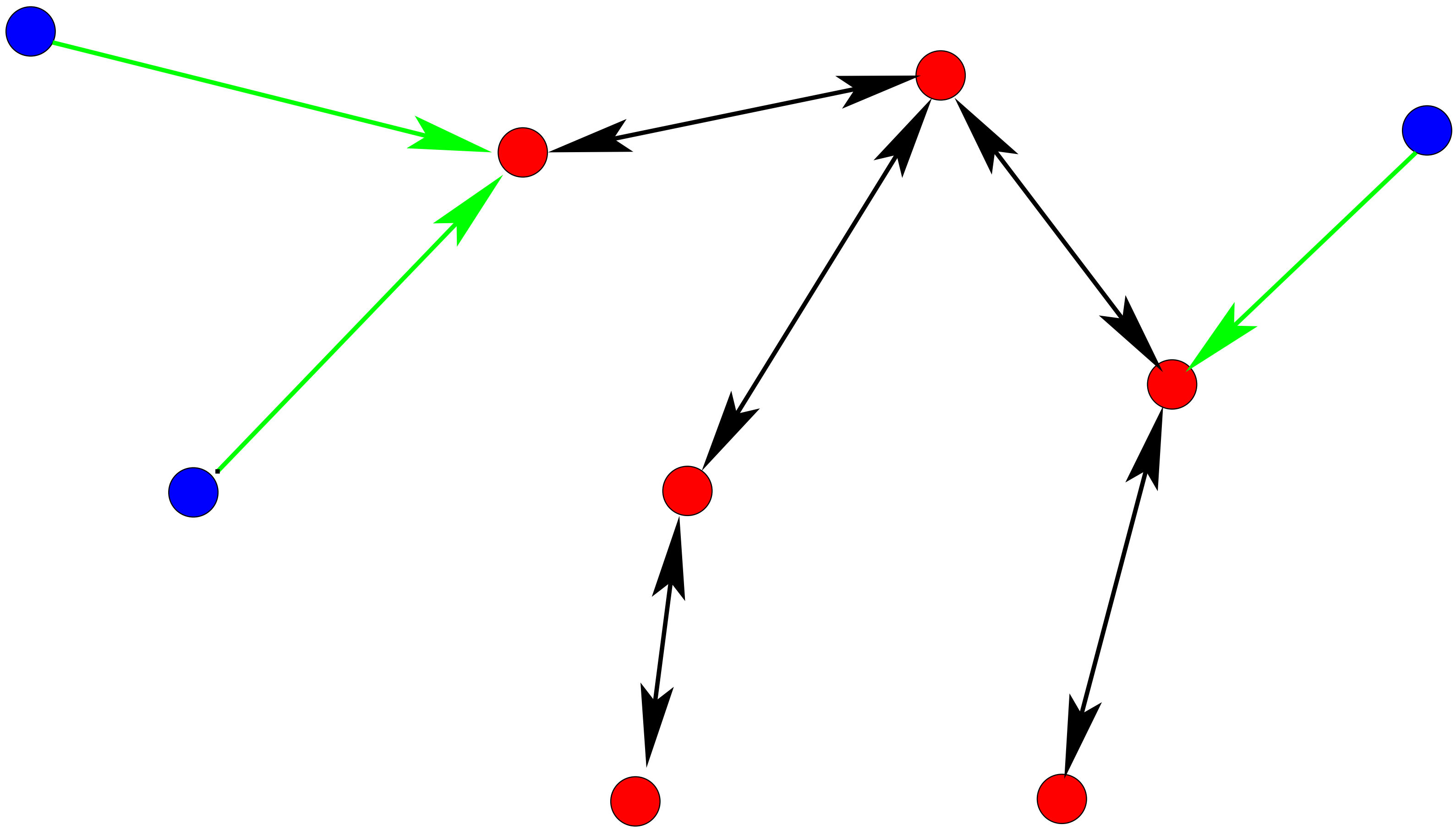
- This is to be done in a distributed and non-deterministic way

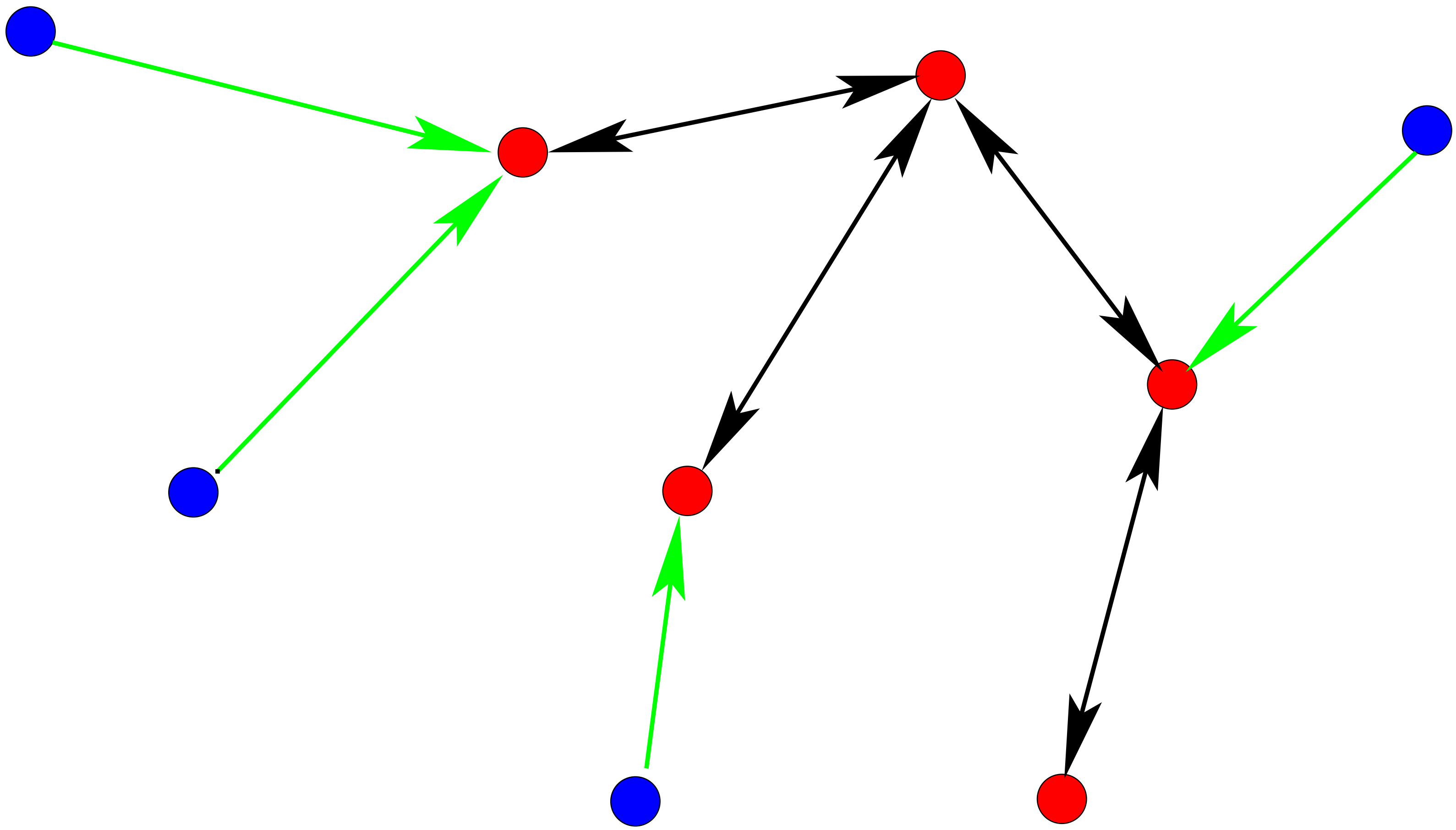
- Next are two distinct abstract animations of the protocol

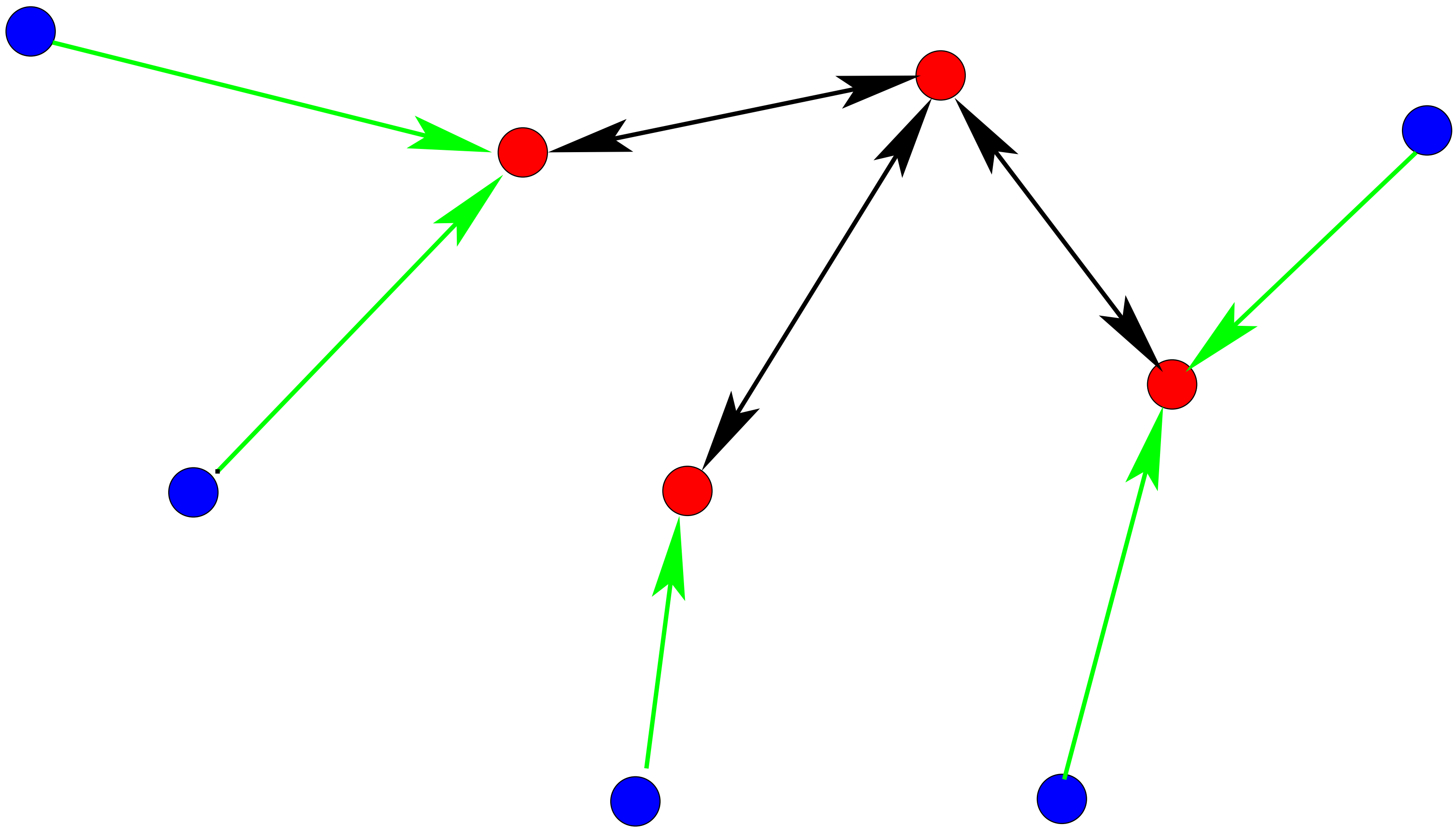


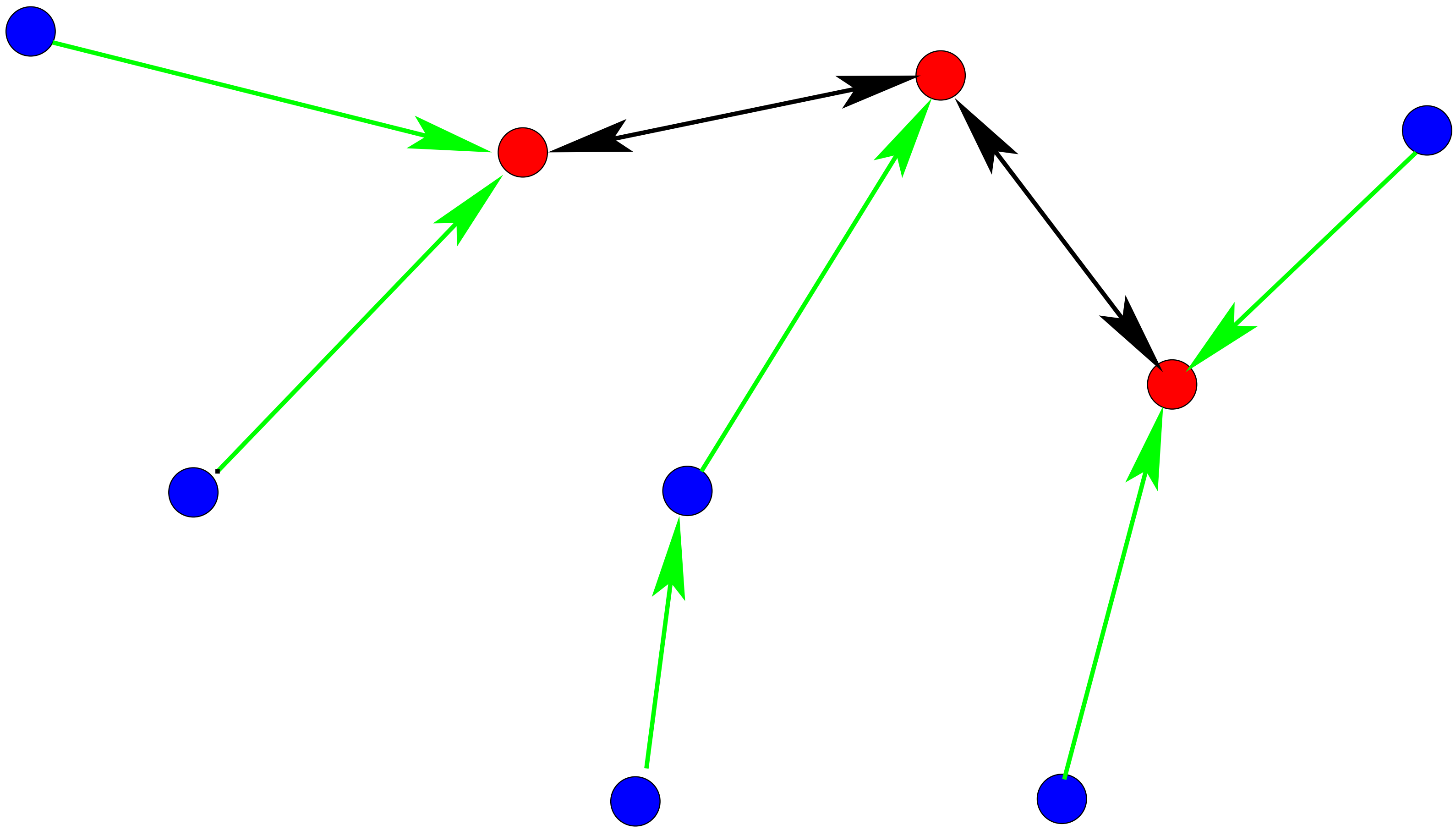


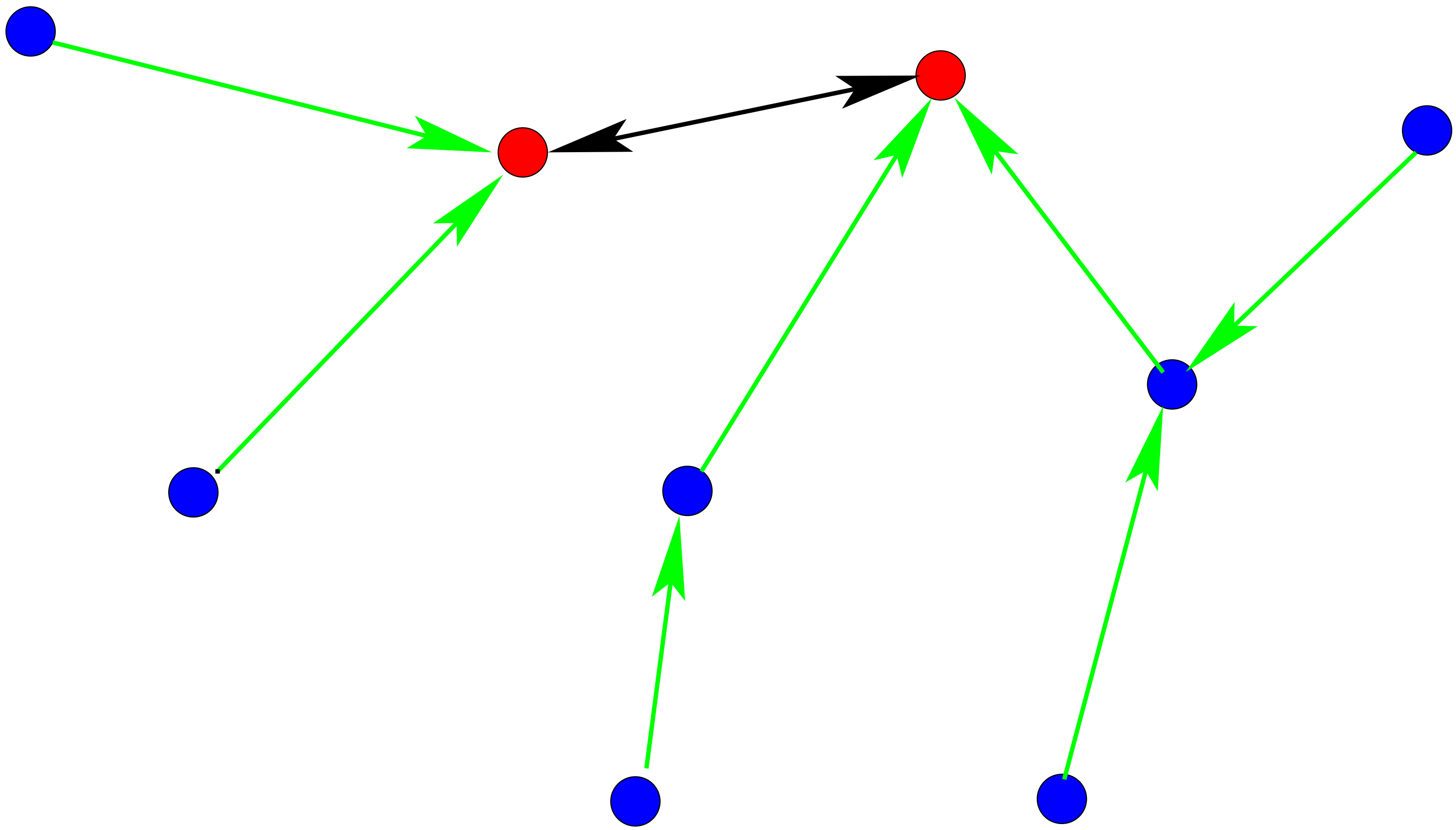


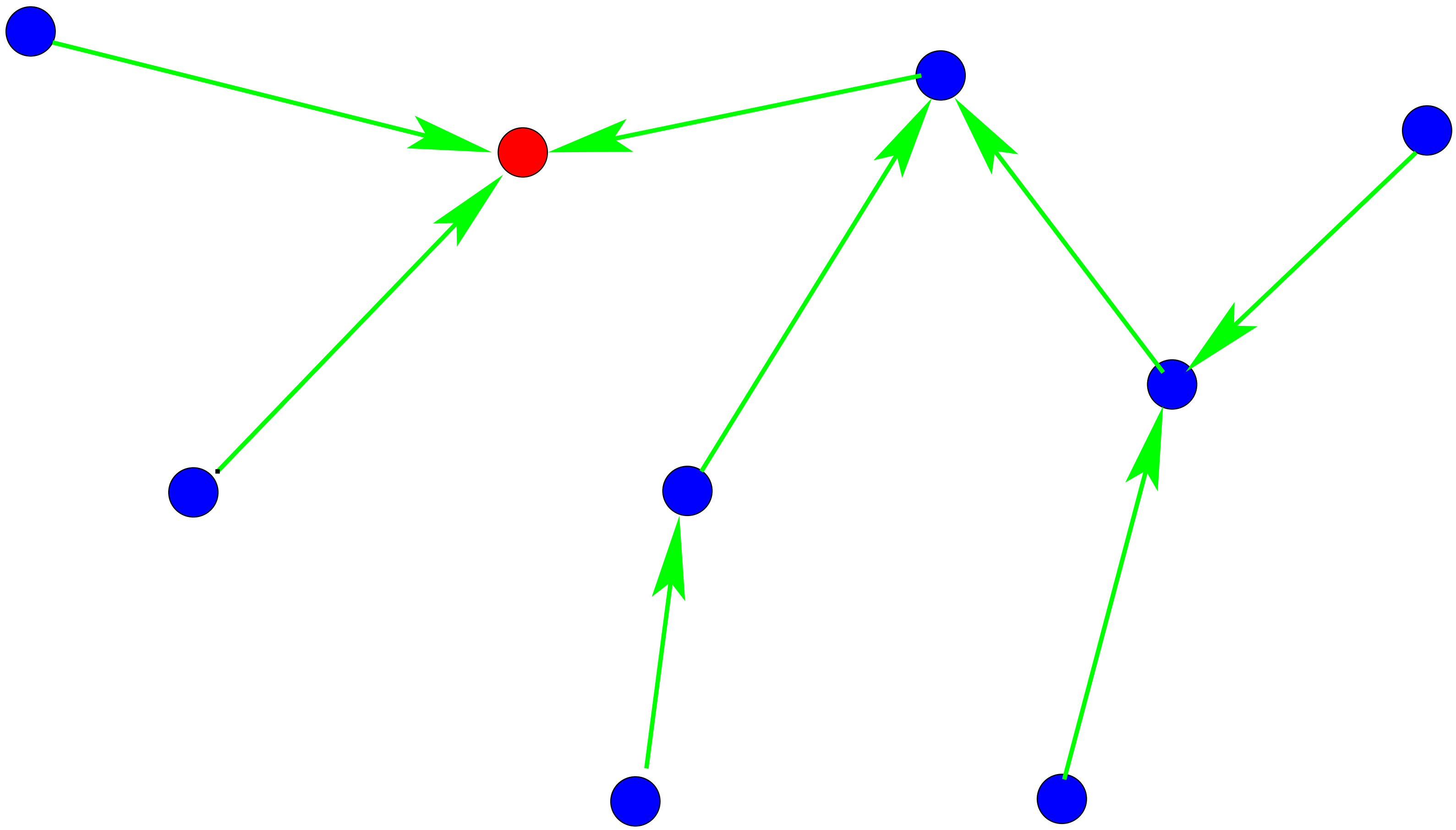


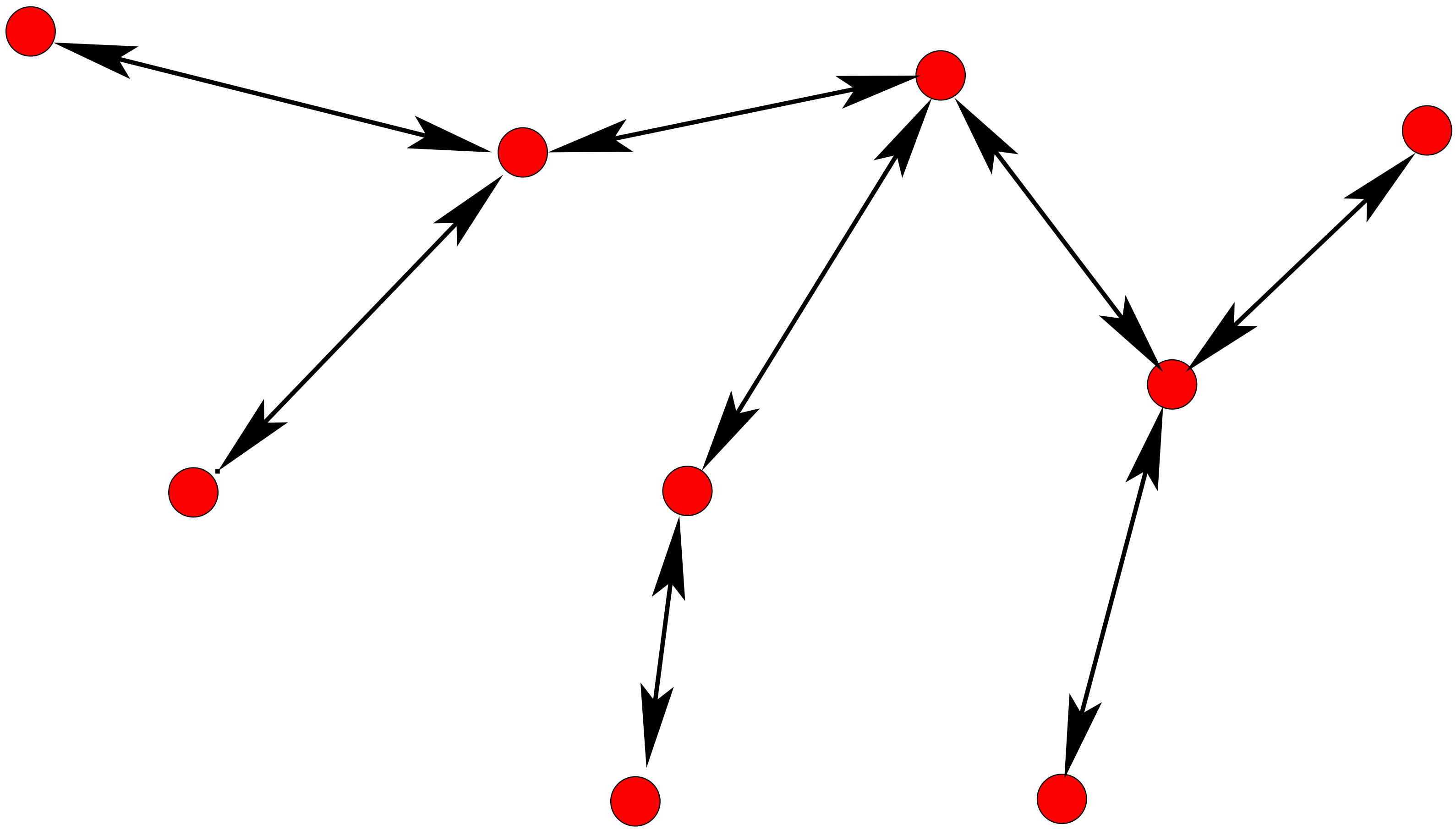


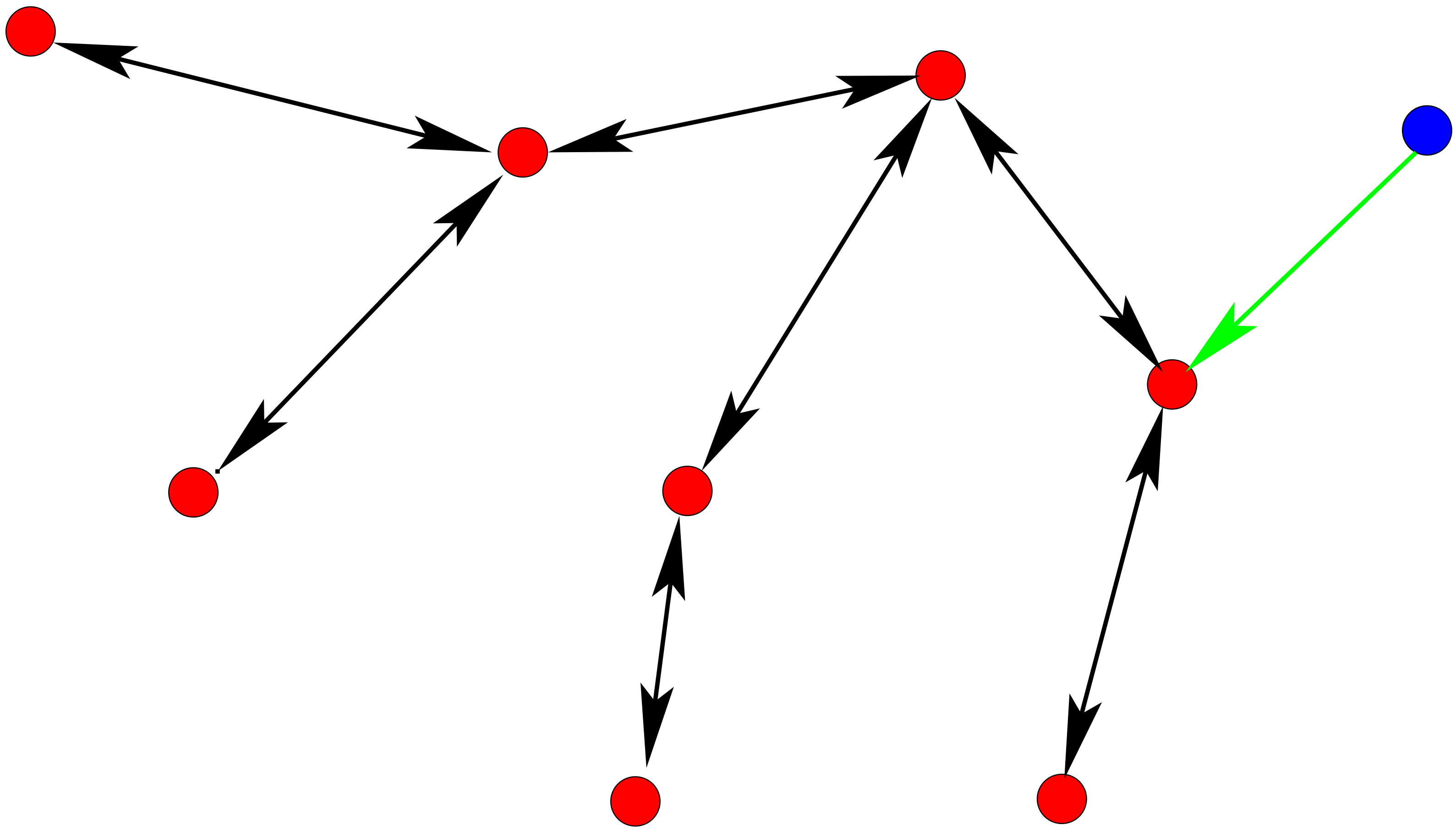


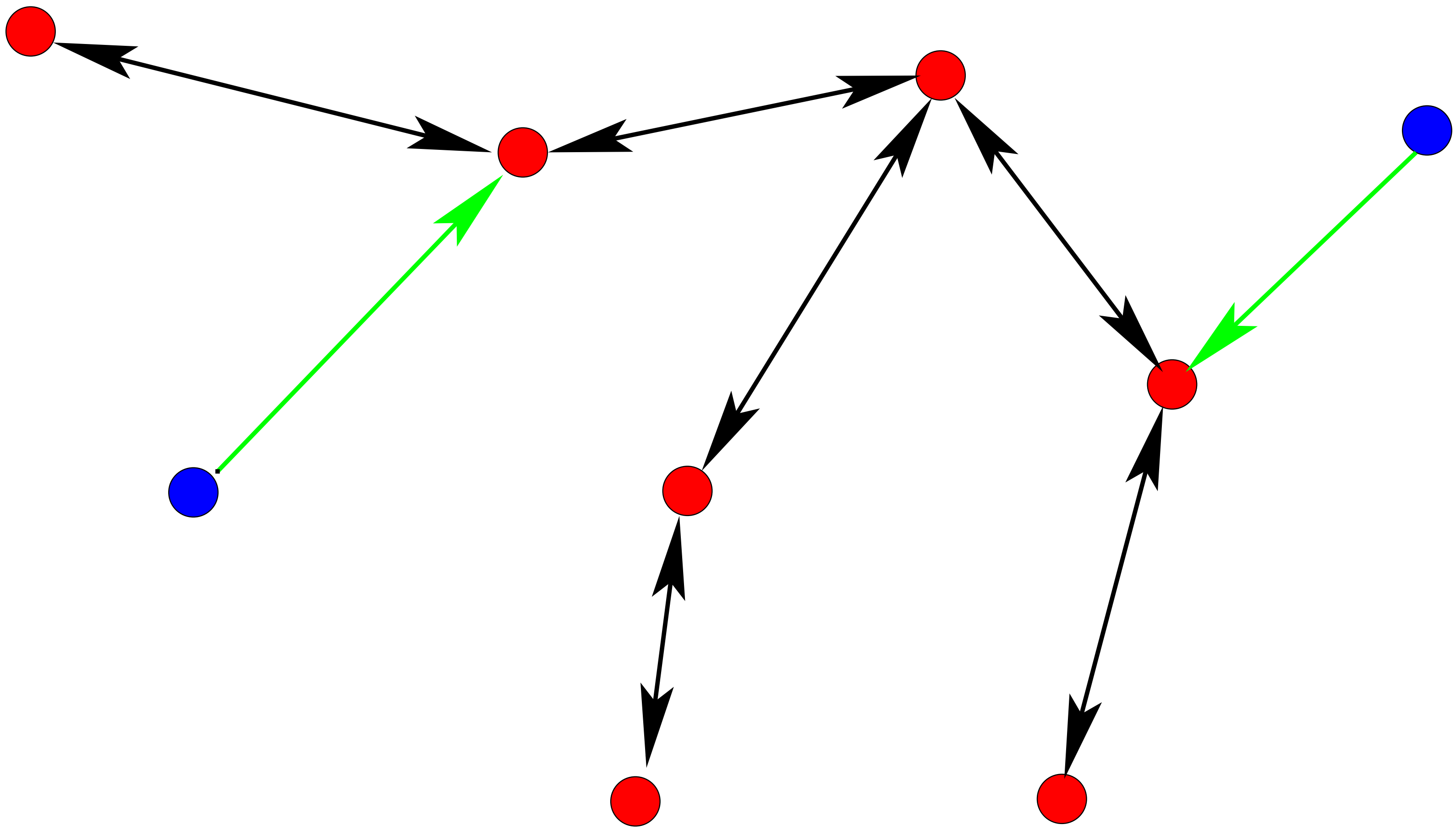


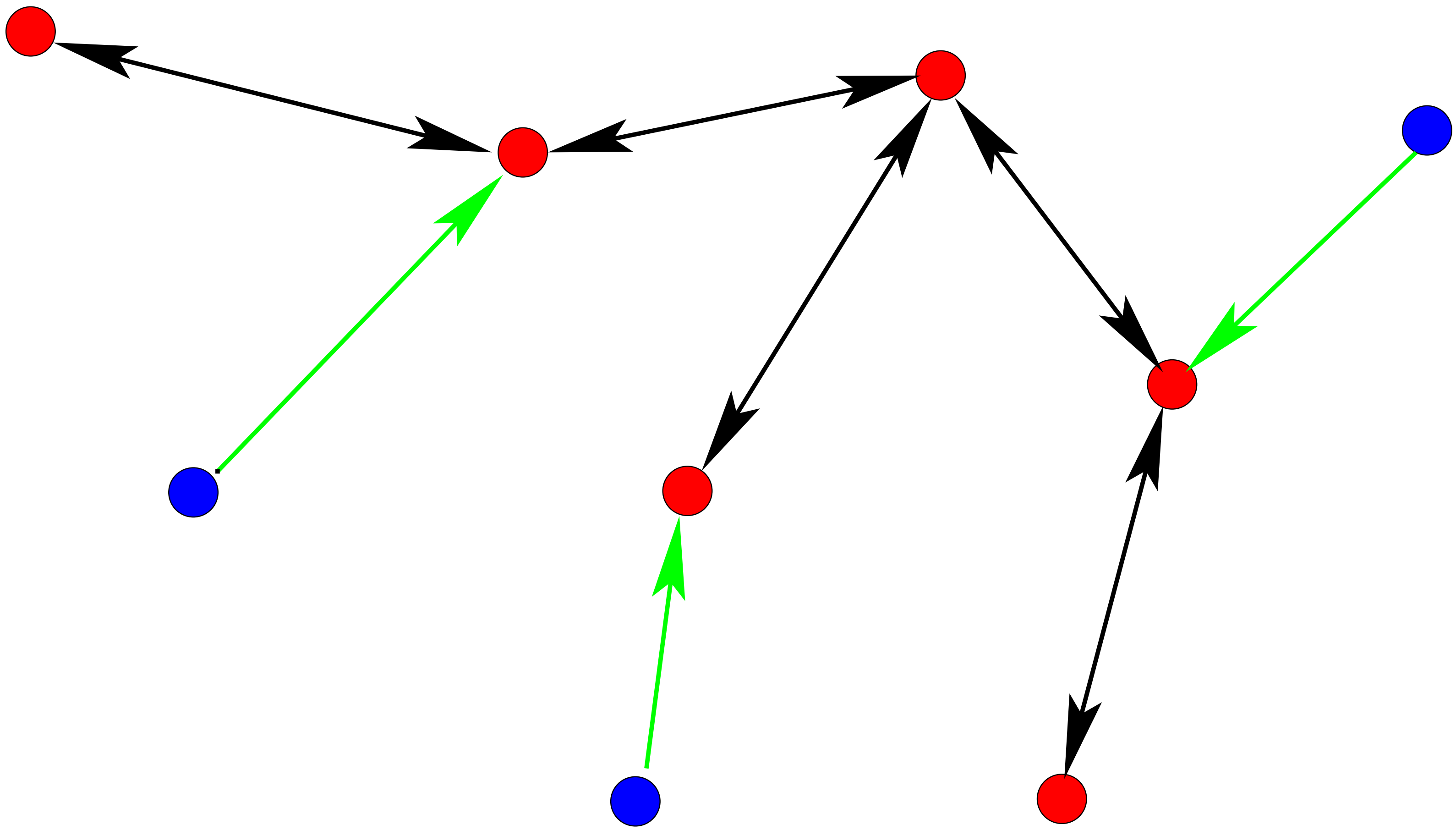


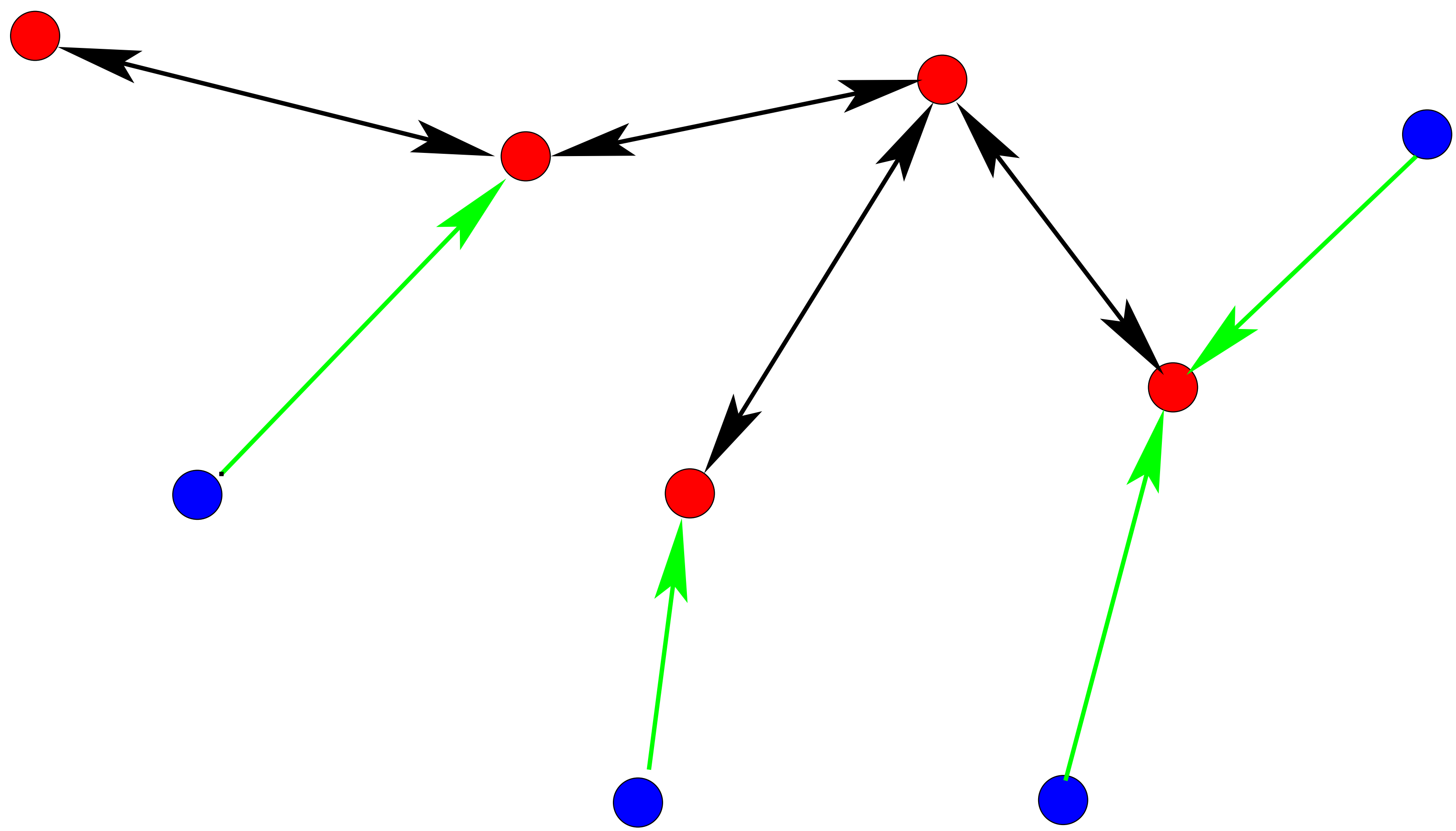


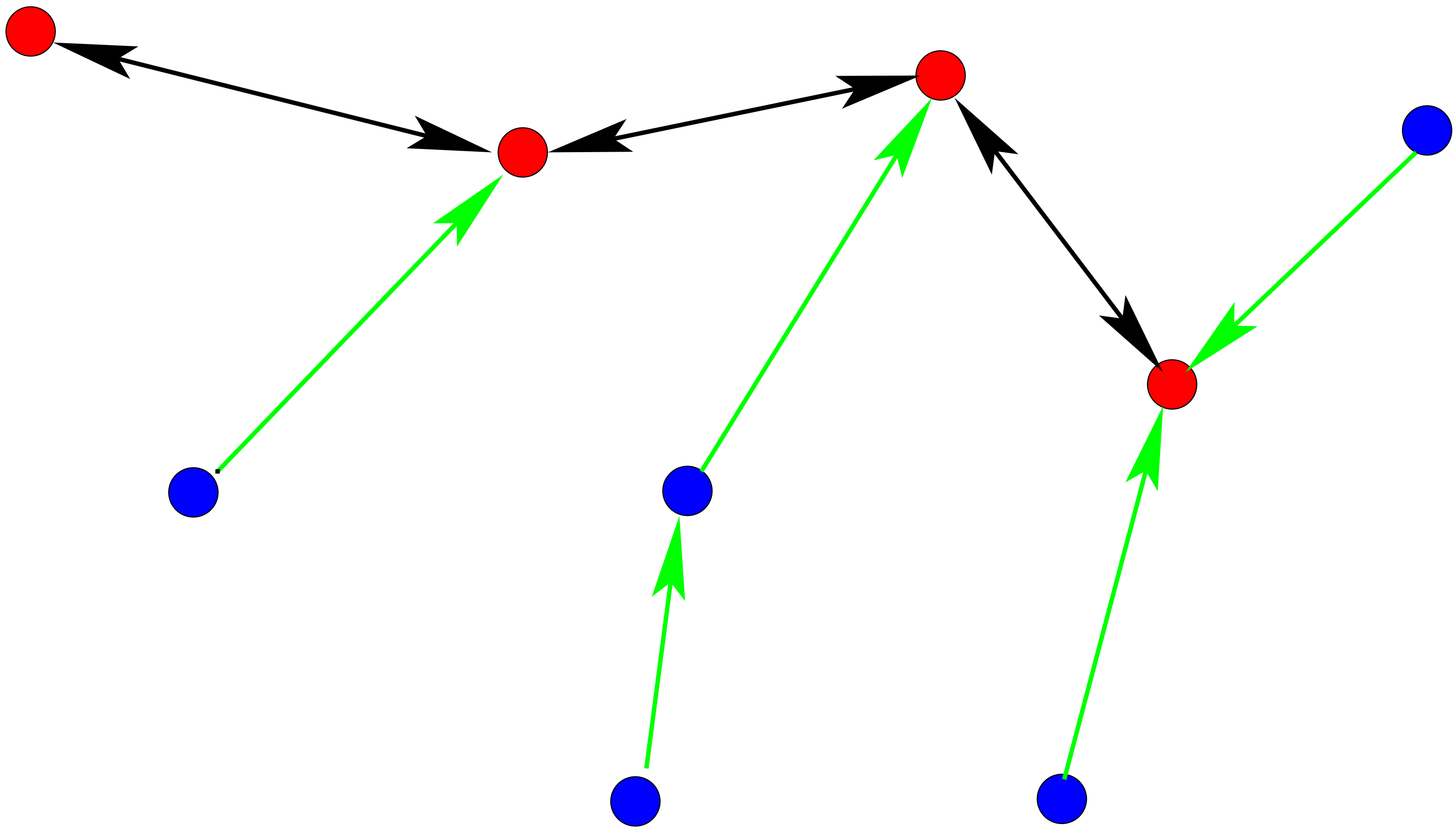


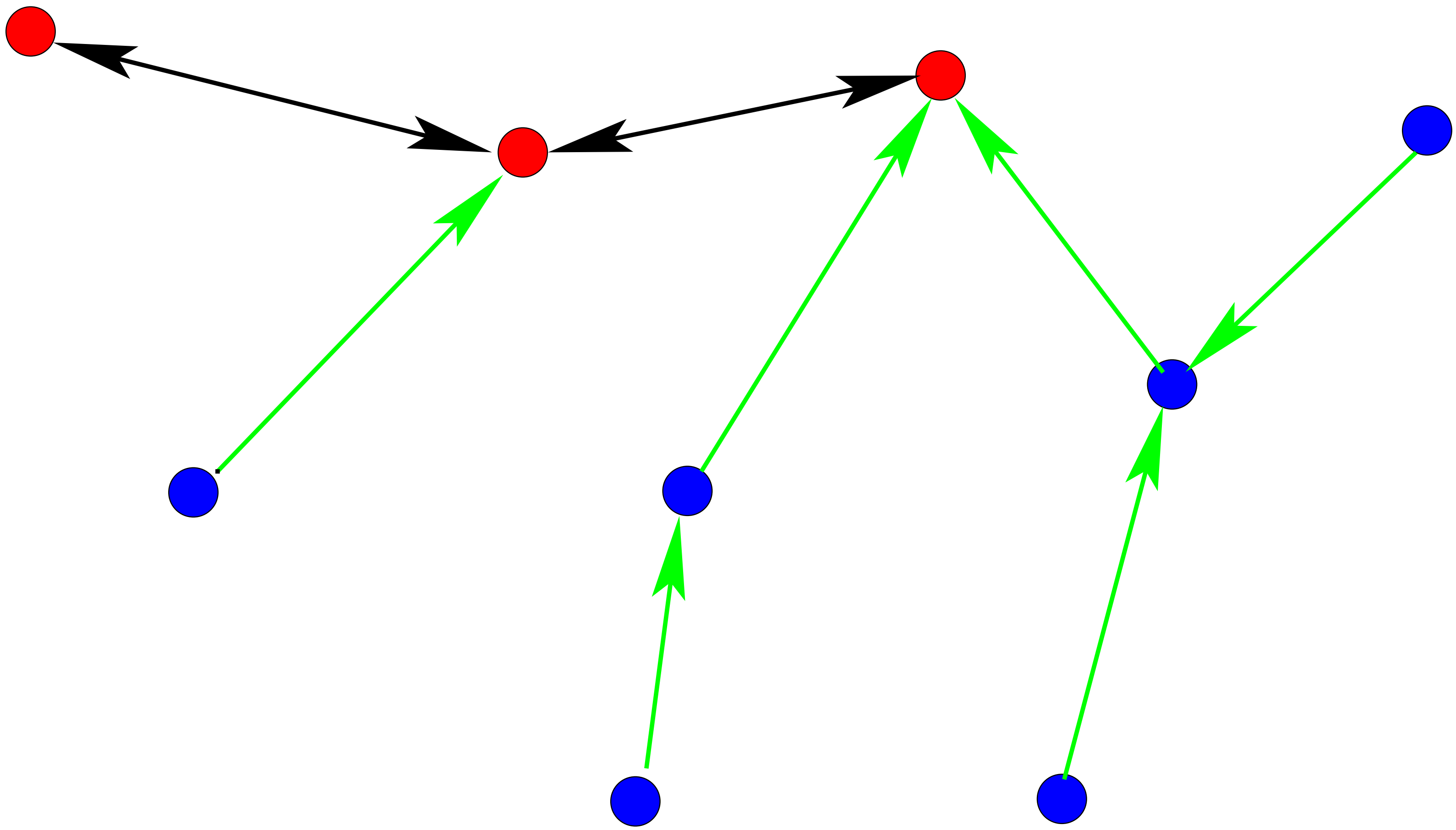


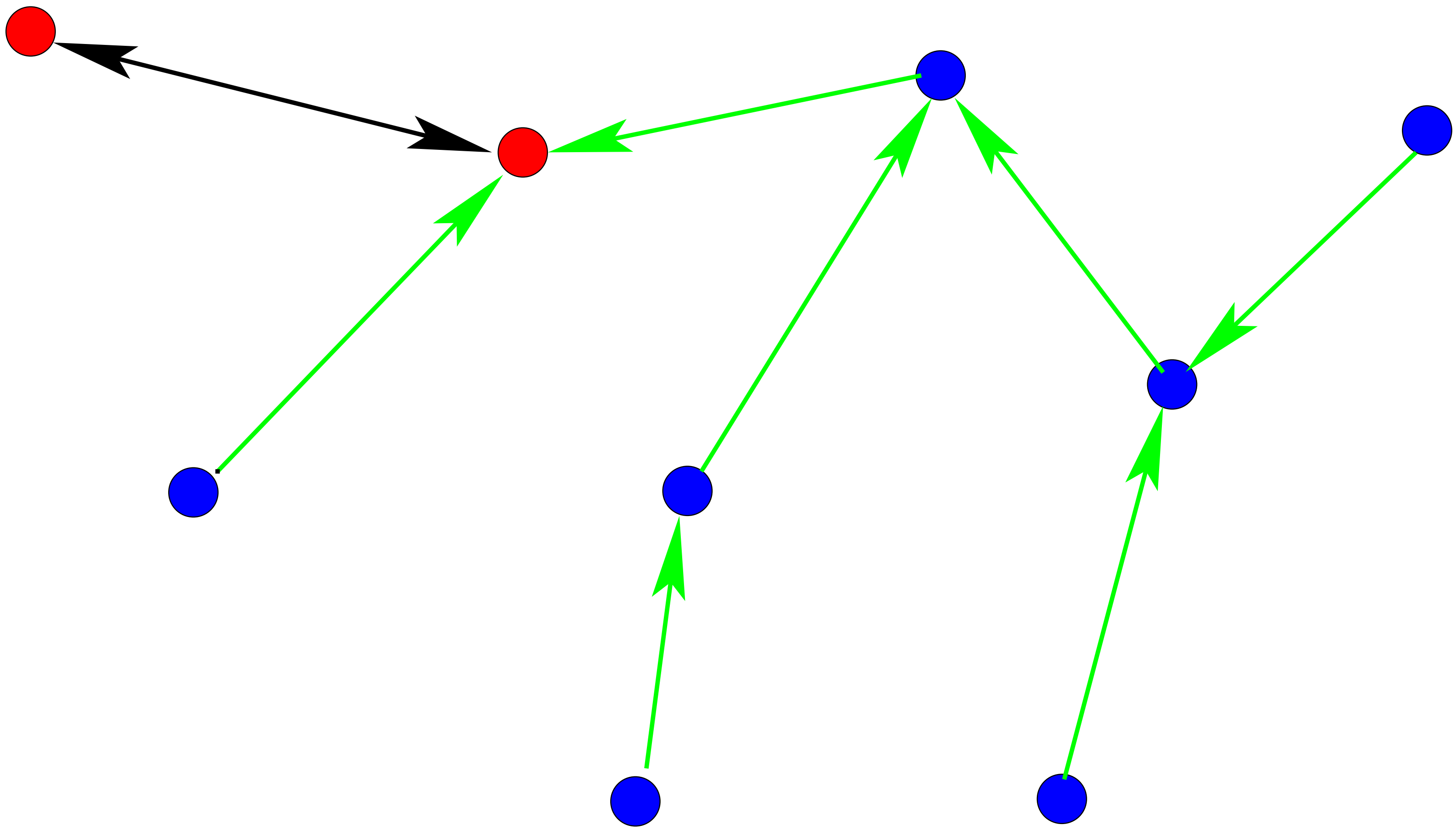


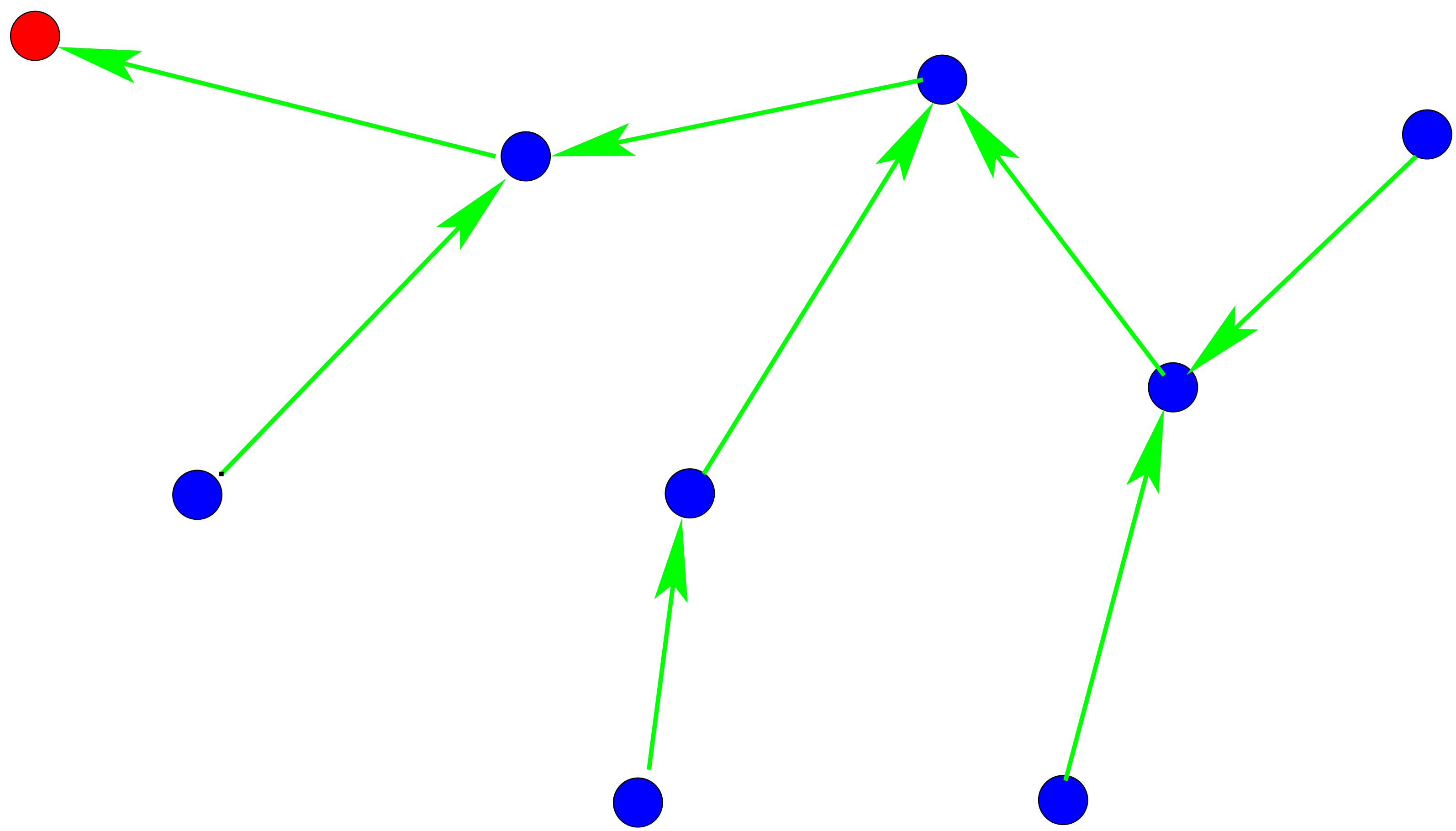












# Summary of Development Process

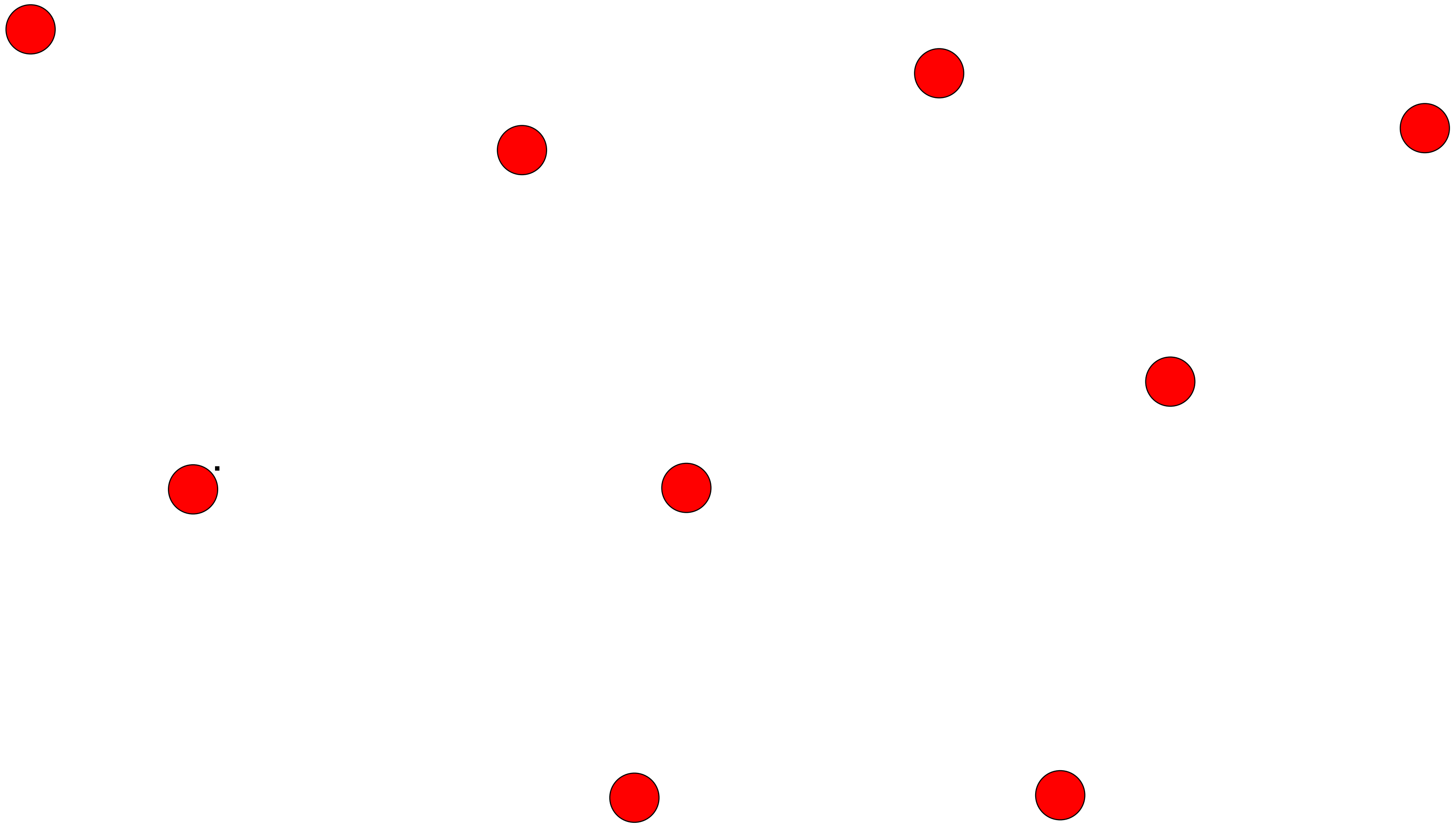
- Formal definition and properties of the network

- A one-shot abstract model of the protocol

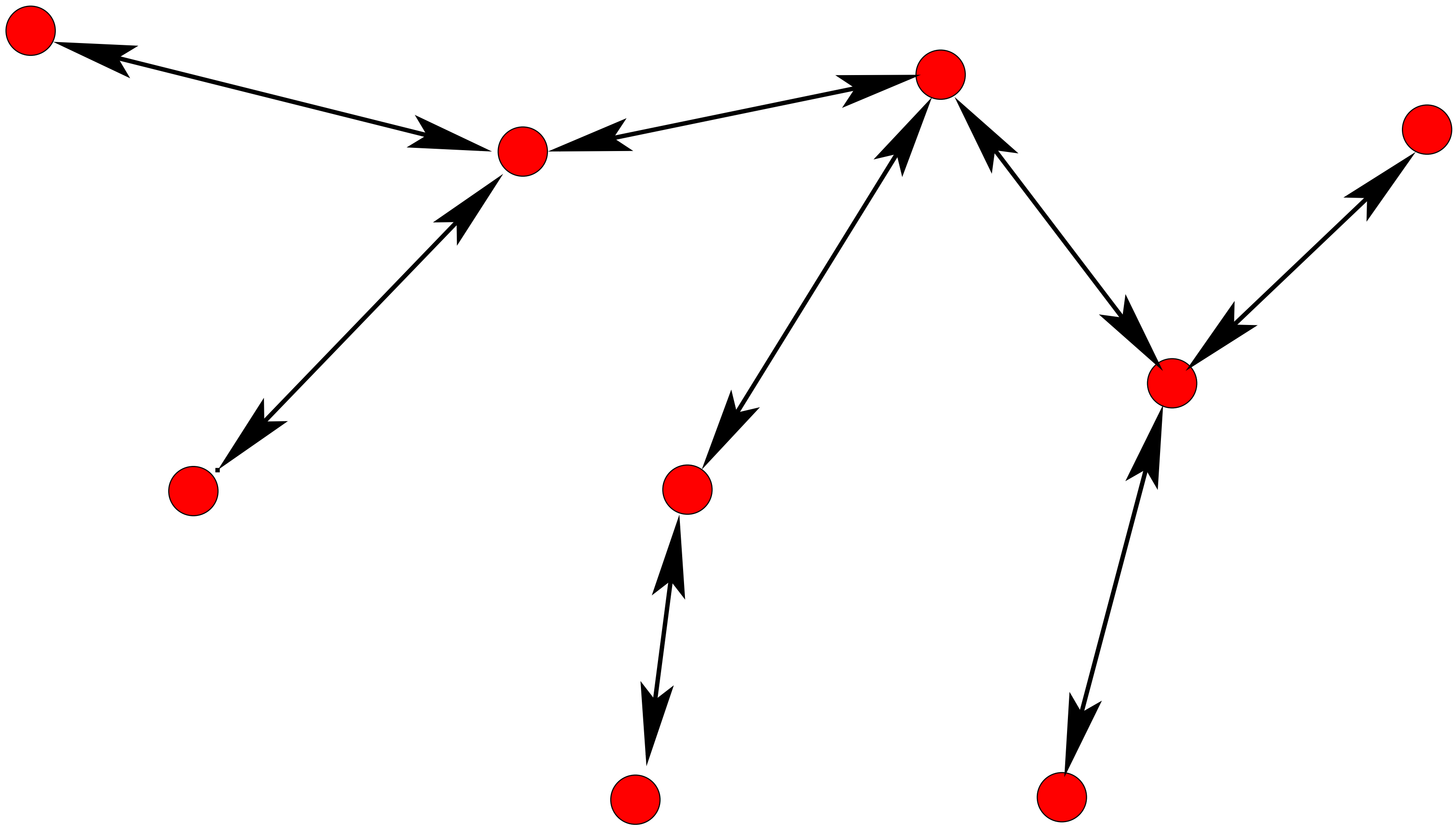
- Presenting a (still abstract) loop-like centralized solution

- Introducing message passing between the nodes (delays)

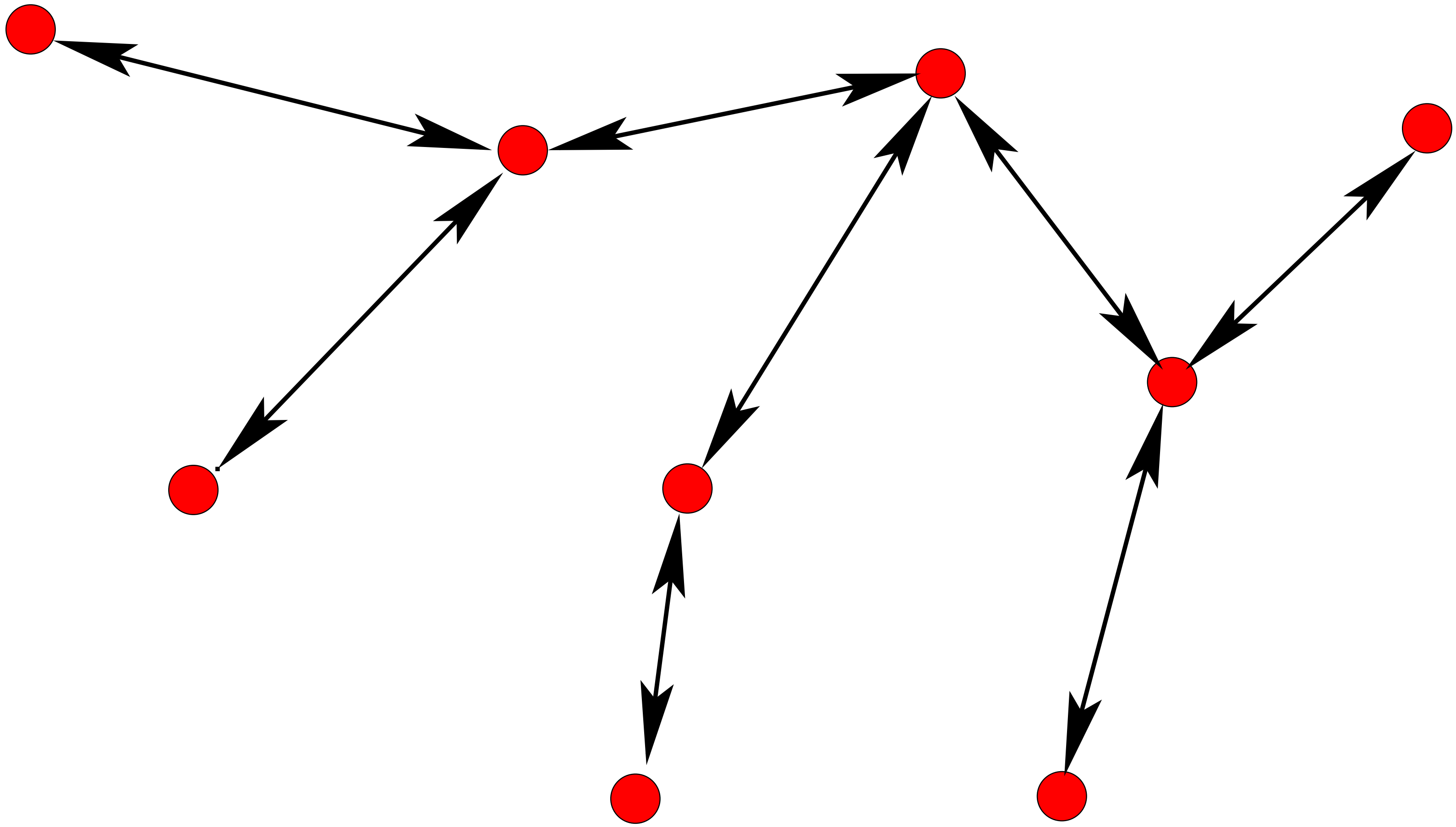
- Modifying the data structure in order to distribute the protocol



Let ND be a set of nodes (with at least 2 nodes)



Let gr be a graph built and defined on ND



gr is a symmetric and irreflexive graph

$gr$ is a graph built on $ND$ $\qquad gr \subseteq ND \times ND$

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$gr$ is defined on $ND$ $\qquad \mathsf{dom}\,(gr) = ND$

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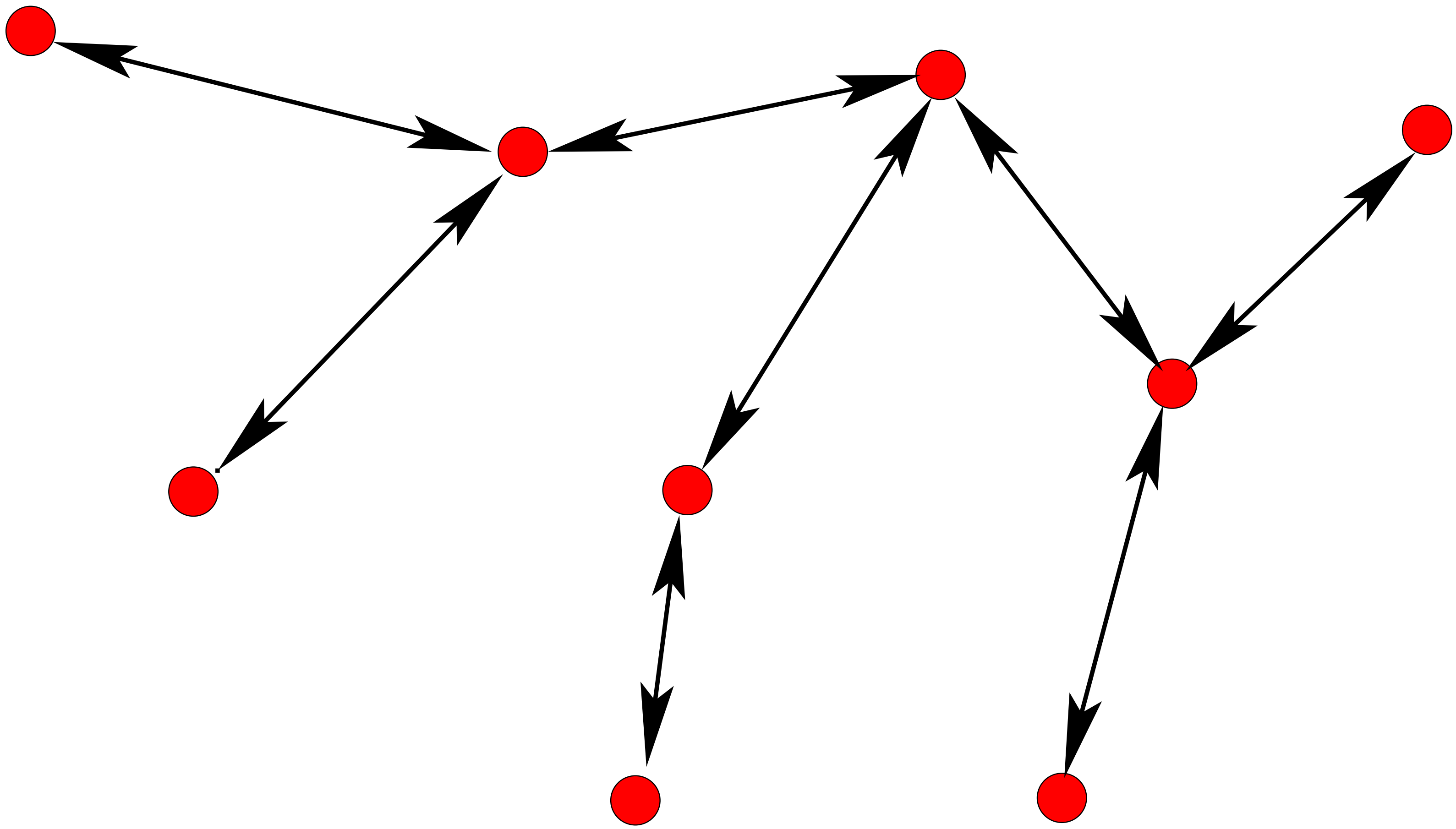
$gr$ is defined on $ND$ $\quad \mathsf{dom}\,(gr) = ND$

$gr$ is symmetric $\quad gr = gr^{-1}$

| | |
|---|---|
| $gr$ is a graph built on $ND$ | $gr \subseteq ND \times ND$ |
| $gr$ is defined on $ND$ | $\mathsf{dom}\,(gr) = ND$ |
| $gr$ is symmetric | $gr = gr^{-1}$ |
| $gr$ is irreflexive | $\mathsf{id}\,(ND) \cap gr = \emptyset$ |



gr is connected and acyclic

# A Little Detour Through Trees

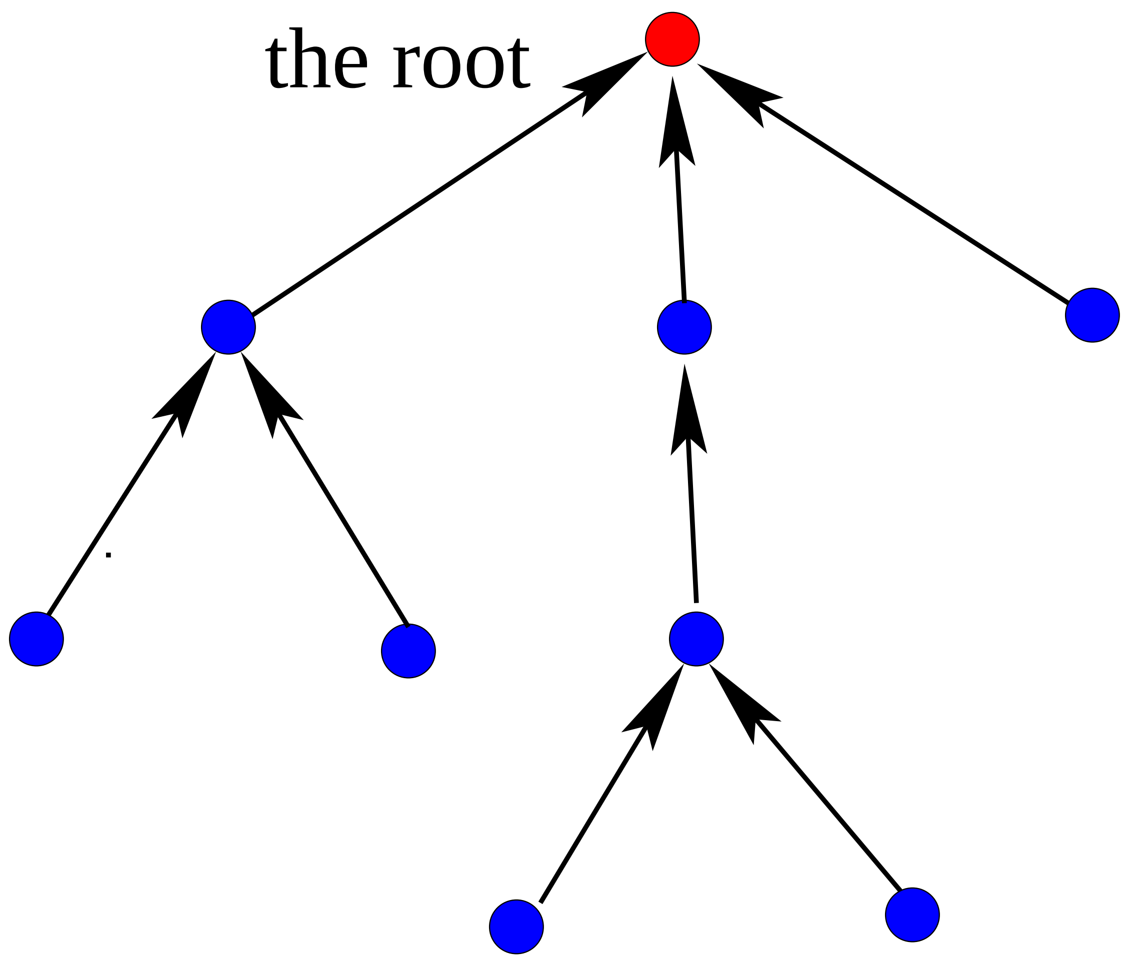
- A tree is a special graph

- A tree has a root

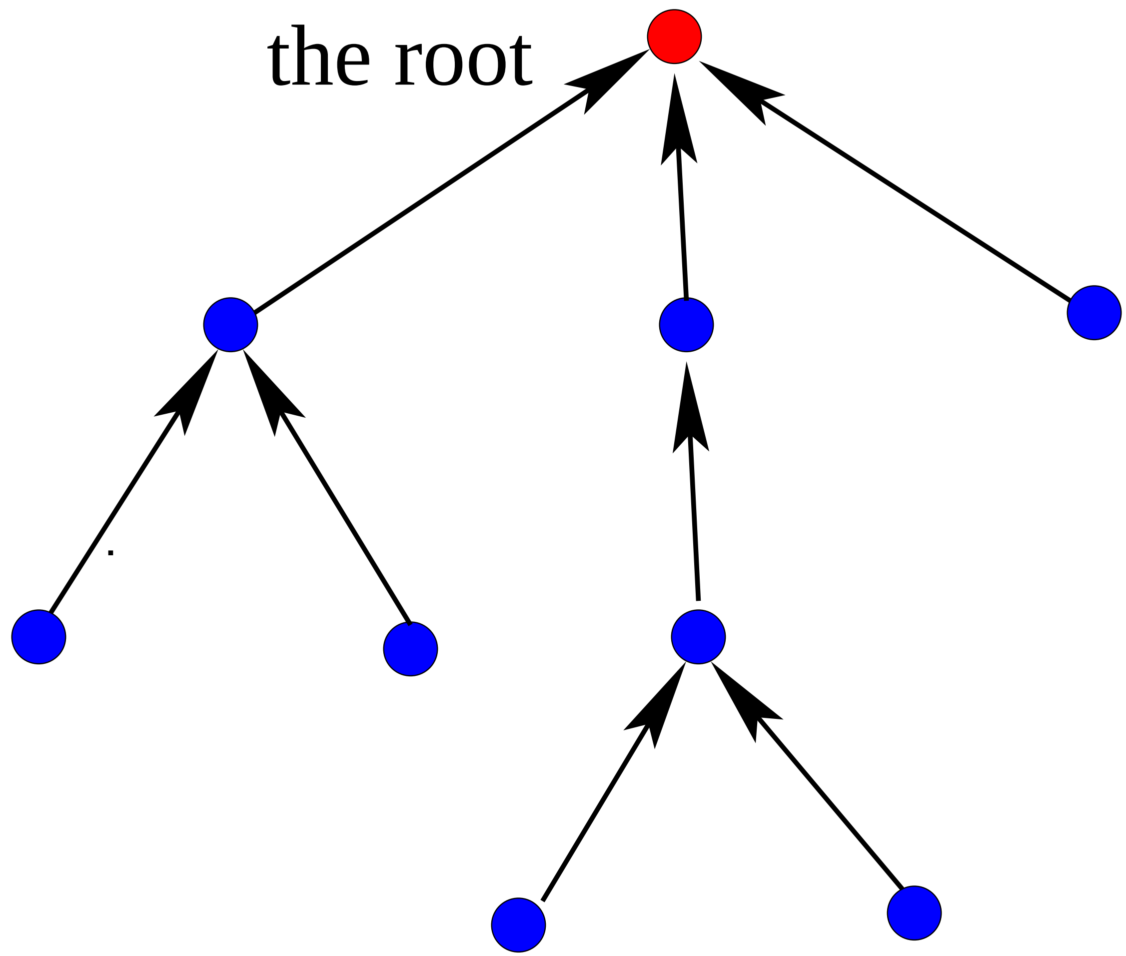
- A tree has a, so-called, father function

- A tree is acyclic

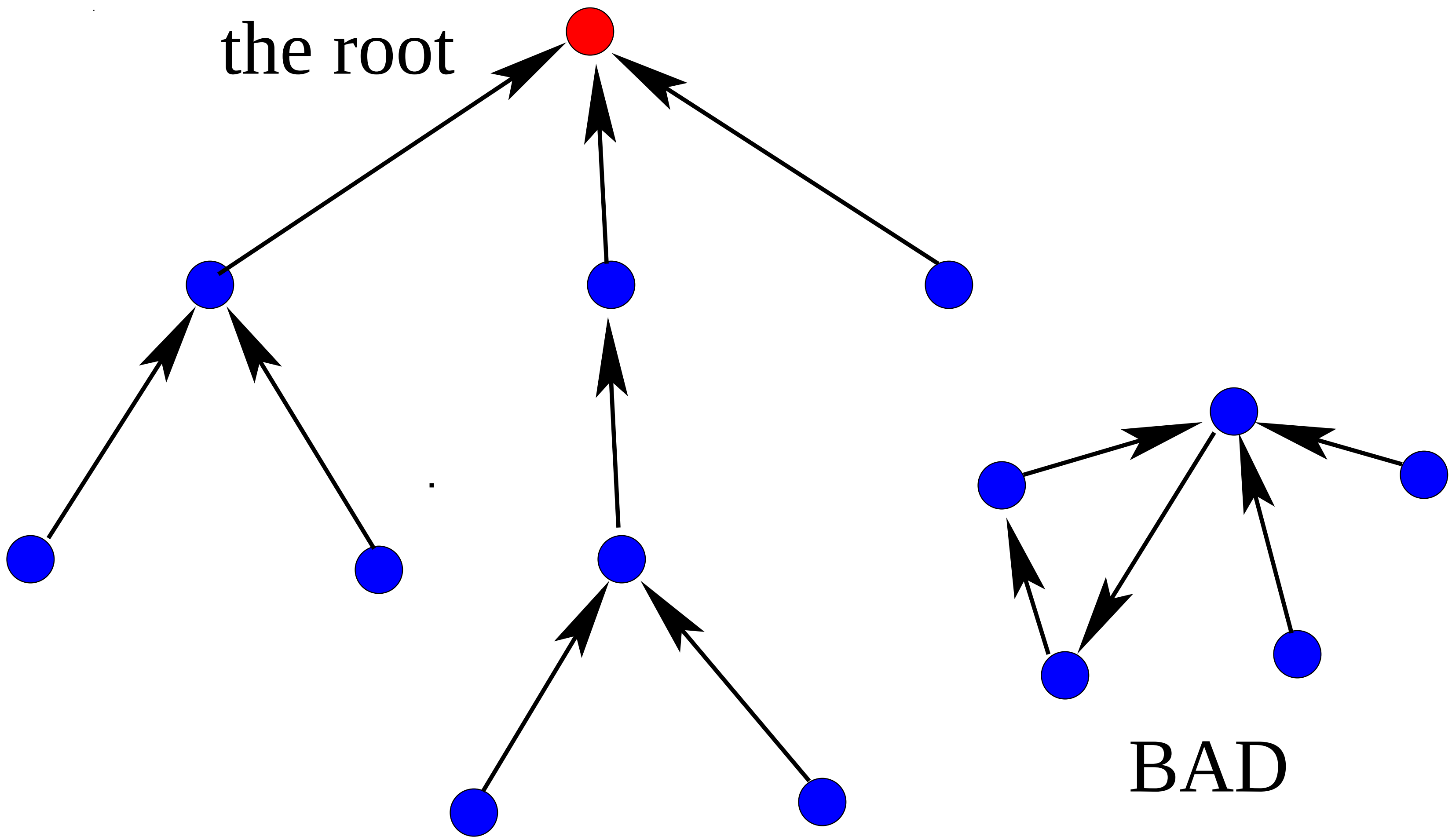
- A tree is connected from the root



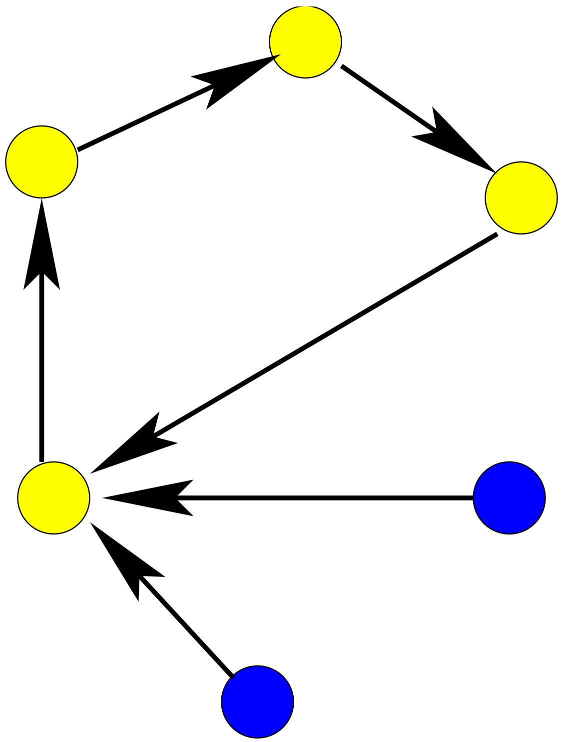

A tree t built on a set of nodes



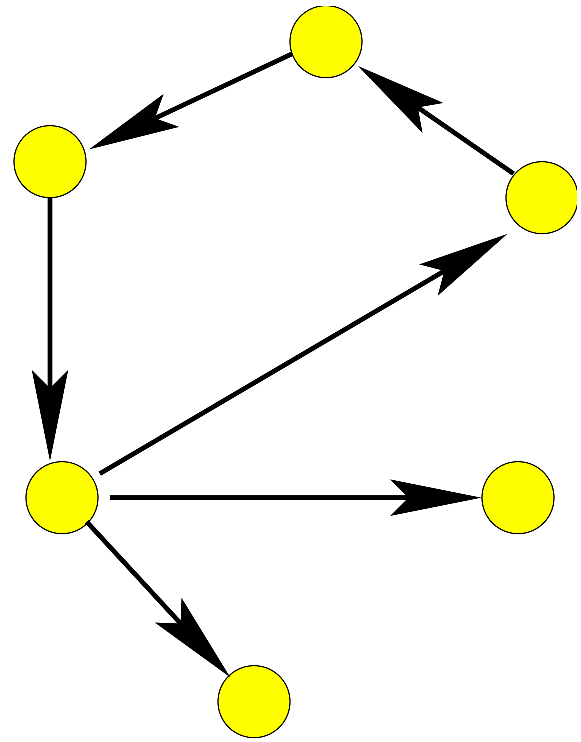

t is a function defined on ND except at the root




Avoidind cycles



A cycle



Its inverse image

The nodes of a cycle are included in their inverse image

- Given
  - a set $ND$
  - a subset $p$ of $ND$
  - a binary relation $t$ built on $ND$
- The inverse image of $p$ under $t$ is denoted by $t^{-1}[p]$

$$t^{-1}[p] \;\widehat{=}\; \{\, x \mid x \in ND \;\wedge\; \exists\, y \cdot (\, y \in p \;\wedge\; (x, y) \in t) \,\}$$

- When $t$ is a partial function, this reduces to

$$\{\, x \mid x \in \mathsf{dom}\,(t) \;\wedge\; t(x) \in p \,\}$$

- If $p$ is included in its inverse image, we have then:

$$\forall x \cdot ( x \in p \Rightarrow x \in \mathsf{dom}\,(t) \ \wedge \ t(x) \in p )$$

- Notice that the empty set enjoys this property

$$\emptyset \subseteq t^{-1}[\emptyset]$$

- The property of having no cycle is thus equivalent to:

The only subset $p$ of $ND$ s.t. $p \subseteq t^{-1}[p]$ is EMPTY

$$\forall p \cdot \begin{pmatrix} \quad p \subseteq ND \;\; \wedge \\ \quad p \subseteq t^{-1}[p] \\ \Rightarrow \\ \quad p = \emptyset \end{pmatrix}$$

The predicate tree $(r, t)$

The predicate tree $(r, t)$

$r$ is a member of $ND$ $\quad r \in ND$

## The predicate tree $(r, t)$

$r$ is a member of $ND$ $\quad r \in ND$

$t$ is a function $\quad t \in ND - \{r\} \longrightarrow ND$

# The predicate tree $(r, t)$

| | |
|---|---|
| $r$ is a member of $ND$ | $r \in ND$ |
| $t$ is a function | $t \in ND - \{r\} \rightarrow ND$ |
| $t$ is acyclic | $\forall p \cdot \left( \begin{array}{l} \quad p \subseteq ND \;\; \wedge \\ \quad p \subseteq t^{-1}[p] \\ \Rightarrow \\ \quad p = \emptyset \end{array} \right)$ |

# $t$ is acyclic: equivalent formulations

$$\forall p \cdot \begin{pmatrix} \quad p \subseteq ND \;\wedge \\ \quad p \subseteq t^{-1}[p] \\ \Rightarrow \\ \quad p = \emptyset \end{pmatrix} \qquad \Leftrightarrow \qquad \forall q \cdot \begin{pmatrix} \quad q \subseteq ND \;\wedge \\ \quad r \in q \;\wedge \\ \quad t^{-1}[q] \subseteq q \\ \Rightarrow \\ \quad ND \subseteq q \end{pmatrix}$$

# This gives an Induction Rule

$$\forall q \cdot \left( \begin{array}{l} \quad q \subseteq ND \;\; \wedge \\ \quad r \in q \;\; \wedge \\ \quad \forall x \cdot (\, x \in ND - \{r\} \;\wedge\; t(x) \in q \Rightarrow x \in q \,) \\ \Rightarrow \\ \quad ND \subseteq q \end{array} \right)$$

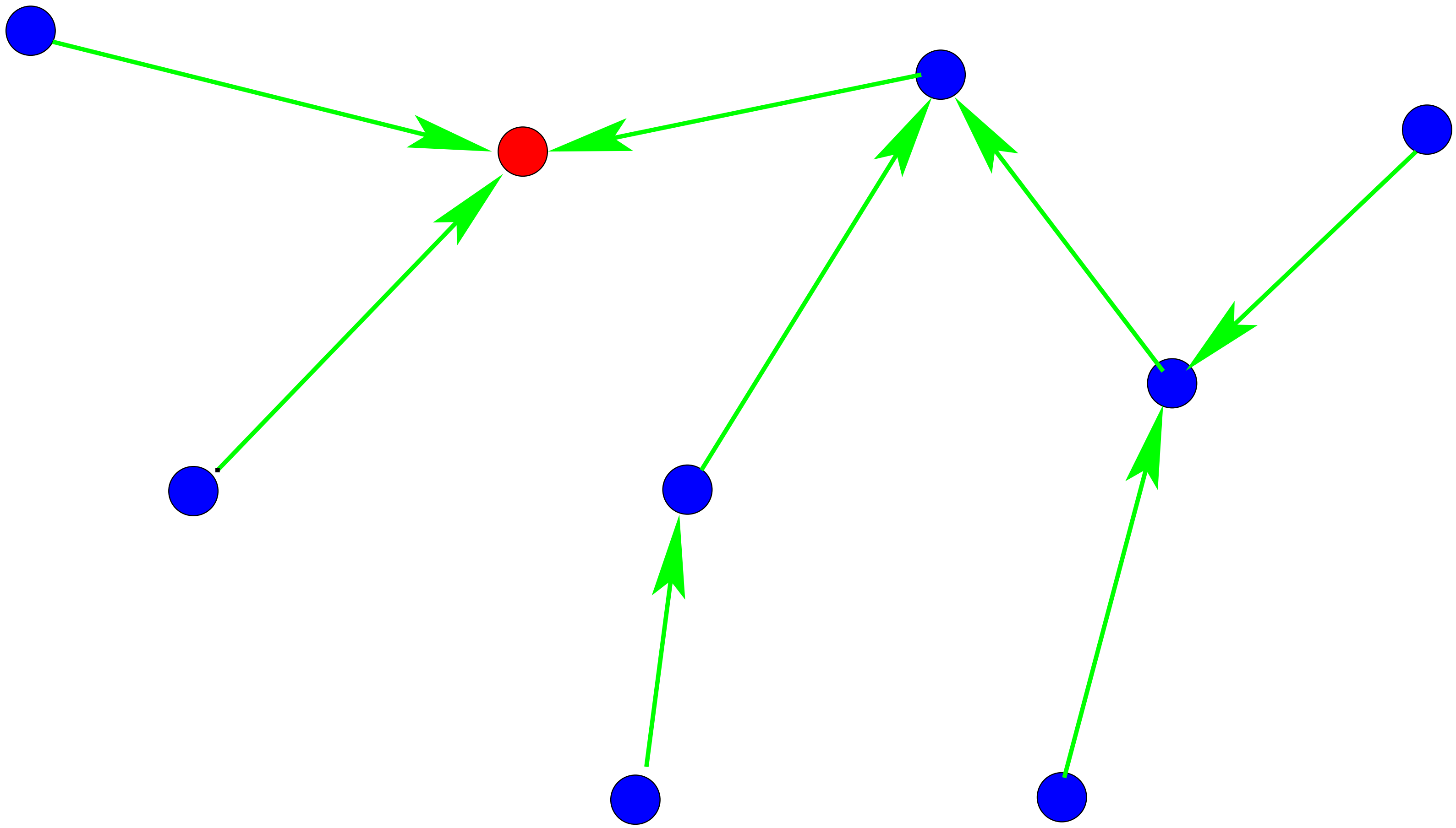
# The predicate tree $(r, t)$

$r$ is a member of $ND$ $\quad r \in ND$

$t$ is a function $\quad t \in ND - \{r\} \rightarrow ND$

$t$ is acyclic

$$\forall q \cdot \left( \begin{array}{l} \quad q \subseteq ND \;\; \wedge \\ \quad r \in q \;\; \wedge \\ \quad t^{-1}[q] \subseteq q \\ \Rightarrow \\ \quad ND \subseteq q \end{array} \right)$$



A spanning tree t of the graph gr

The predicate spanning $(r, t, gr)$

| | |
|---|---|
| $r, t$ is a tree | tree $(r, t)$ |
| $t$ is included in $gr$ | $t \subseteq gr$ |

# The graph $gr$ is connected and acyclic (1)

- Defining a relation $fn$ linking a node to the possible spanning trees of $gr$ having that node as a root:

$$fn \;\subseteq\; ND \times (ND \mathrel{+\!\!\!\rightarrow} ND)$$

$$\forall (r,t) \cdot \left( \begin{array}{l} \quad r \in ND \;\; \wedge \\ \quad t \in ND \mathrel{+\!\!\!\rightarrow} ND \\ \Rightarrow \\ \quad (r,t) \in fn \;\Leftrightarrow\; \text{spanning}\,(r,t,gr) \end{array} \right)$$

## The graph $gr$ is connected and acyclic (2)

Totality of relation $fn \Rightarrow$ Connectivity of $gr$

Functionality of relation $fn \Rightarrow$ Acyclicity of $gr$

# Summary of constants $gr$ and $fn$

$$gr \subseteq ND \times ND$$
$$\mathrm{dom}\,(gr) = ND$$
$$gr = gr^{-1}$$
$$\mathrm{id}\,(ND) \cap gr \; = \; \emptyset$$

$$fn \in ND \to (ND \twoheadrightarrow ND)$$
$$\forall (r,t) \cdot \left( \begin{array}{l} \quad r \in ND \;\; \land \\ \quad t \in ND \twoheadrightarrow ND \\ \Rightarrow \\ \quad t = fn(r) \;\; \Leftrightarrow \;\; \mathrm{spanning}\,(r, t, gr) \end{array} \right)$$

# Election in One Shot: Building a Spanning Tree

- Variables $rt$ and $ts$

$$rt \in ND$$
$$ts \in ND \leftrightarrow ND$$

elect $\widehat{=}$

**BEGIN**

$rt, ts$ : spanning $(rt, ts, gr)$

**END**

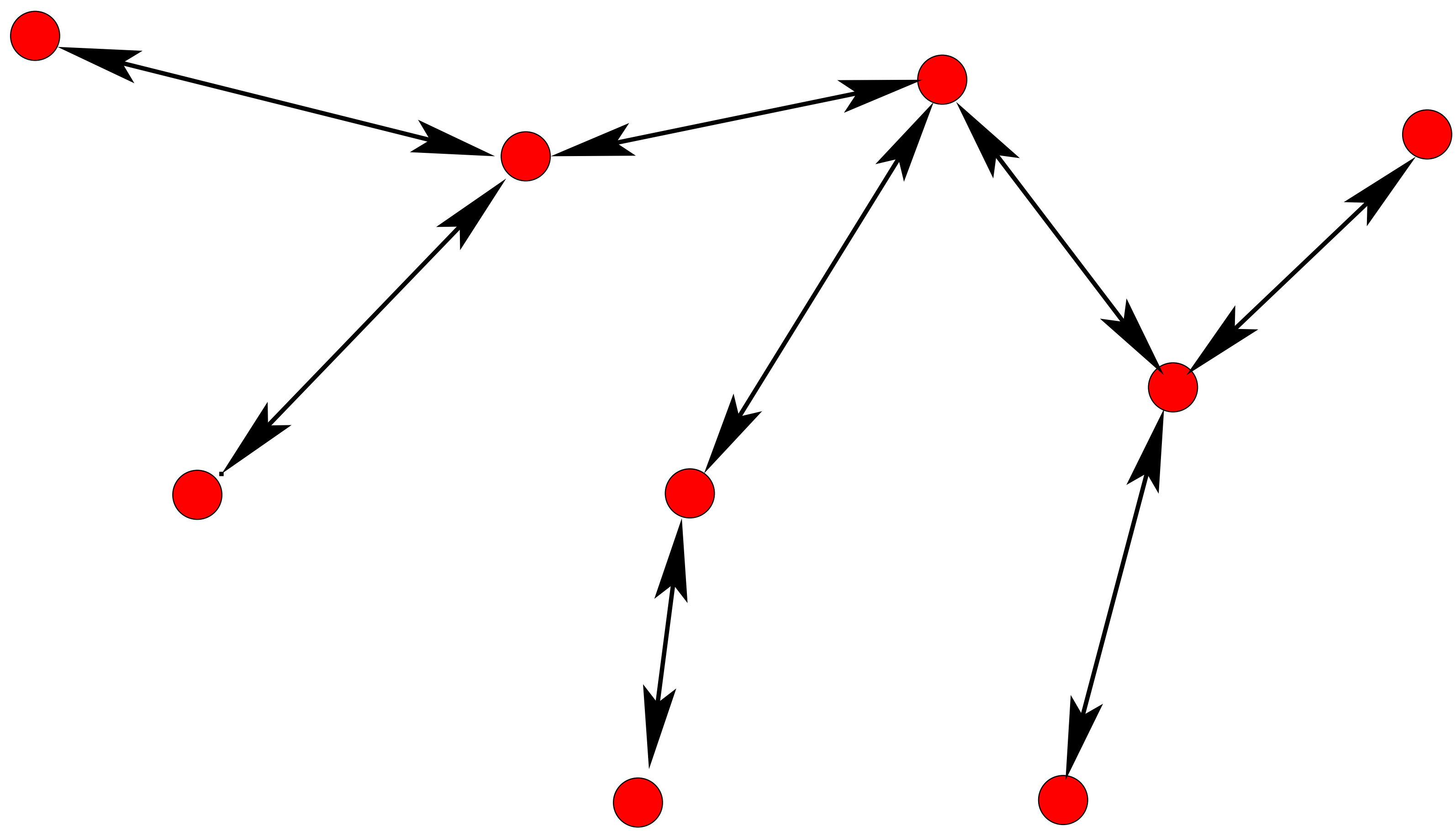
# First Refinement (1)

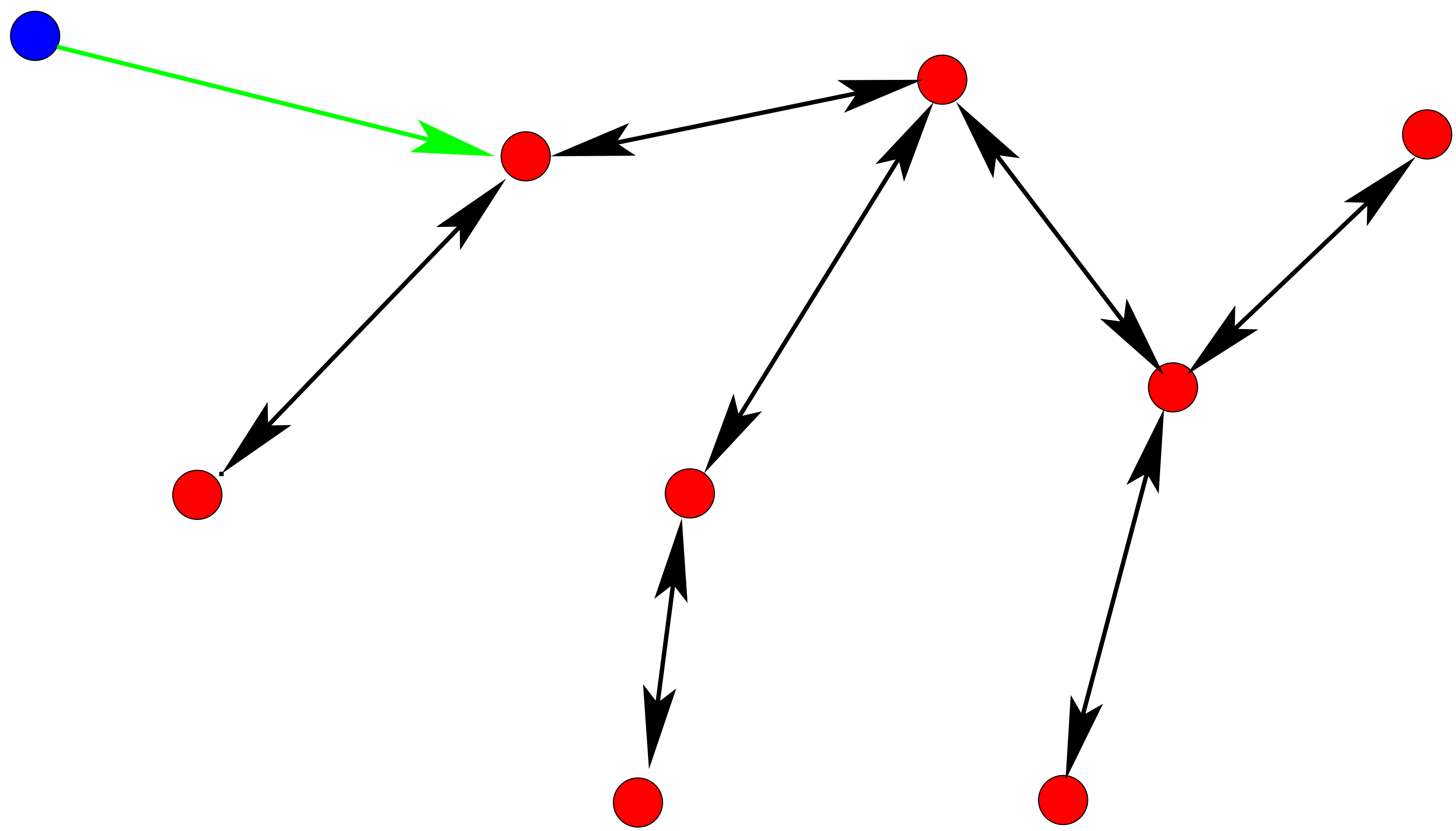
- Introducing a new variable, $tr$, corresponding to the "tree" in construction

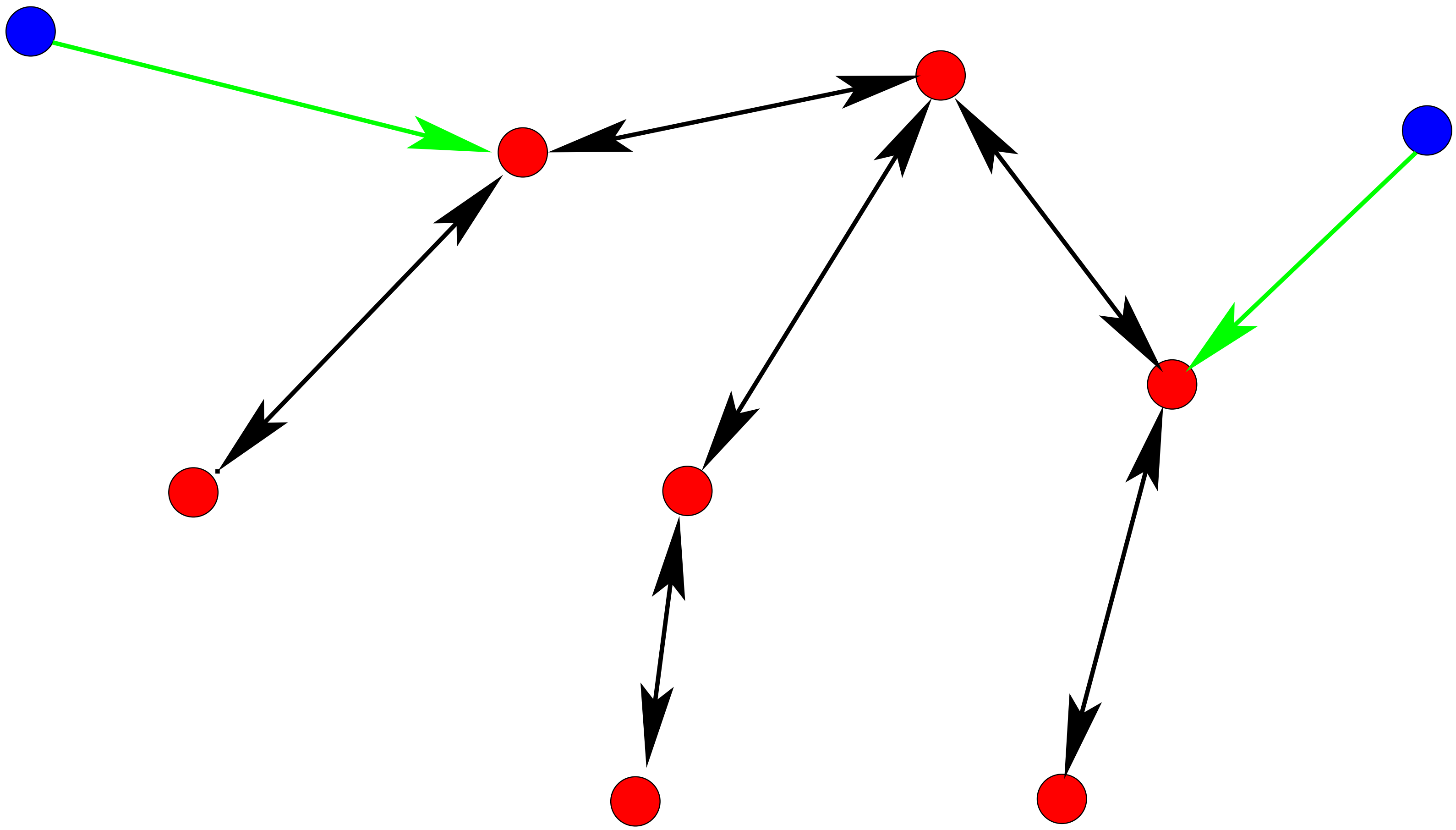
- Introducing a new event: the progression event

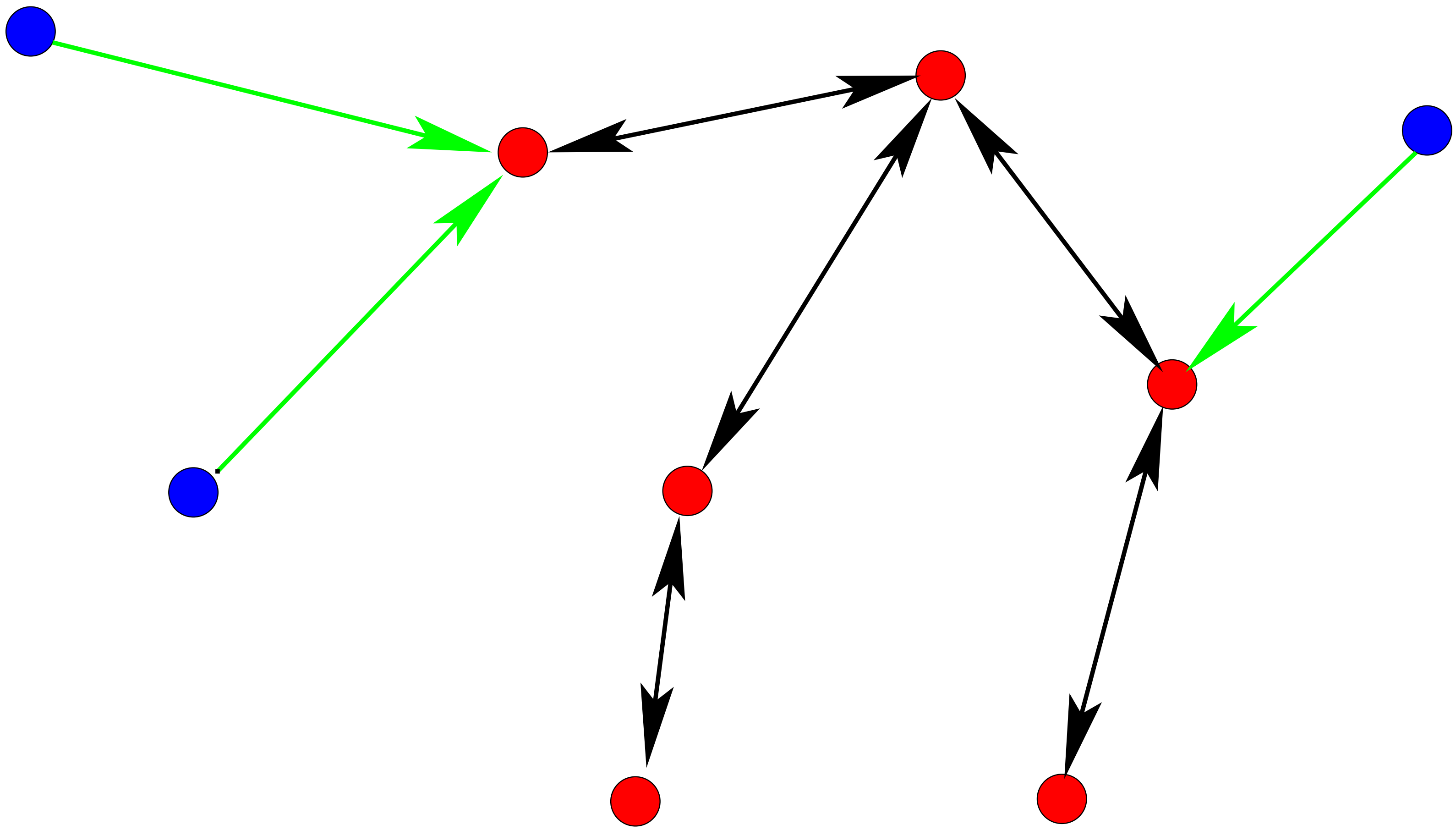
- Defining the invariant

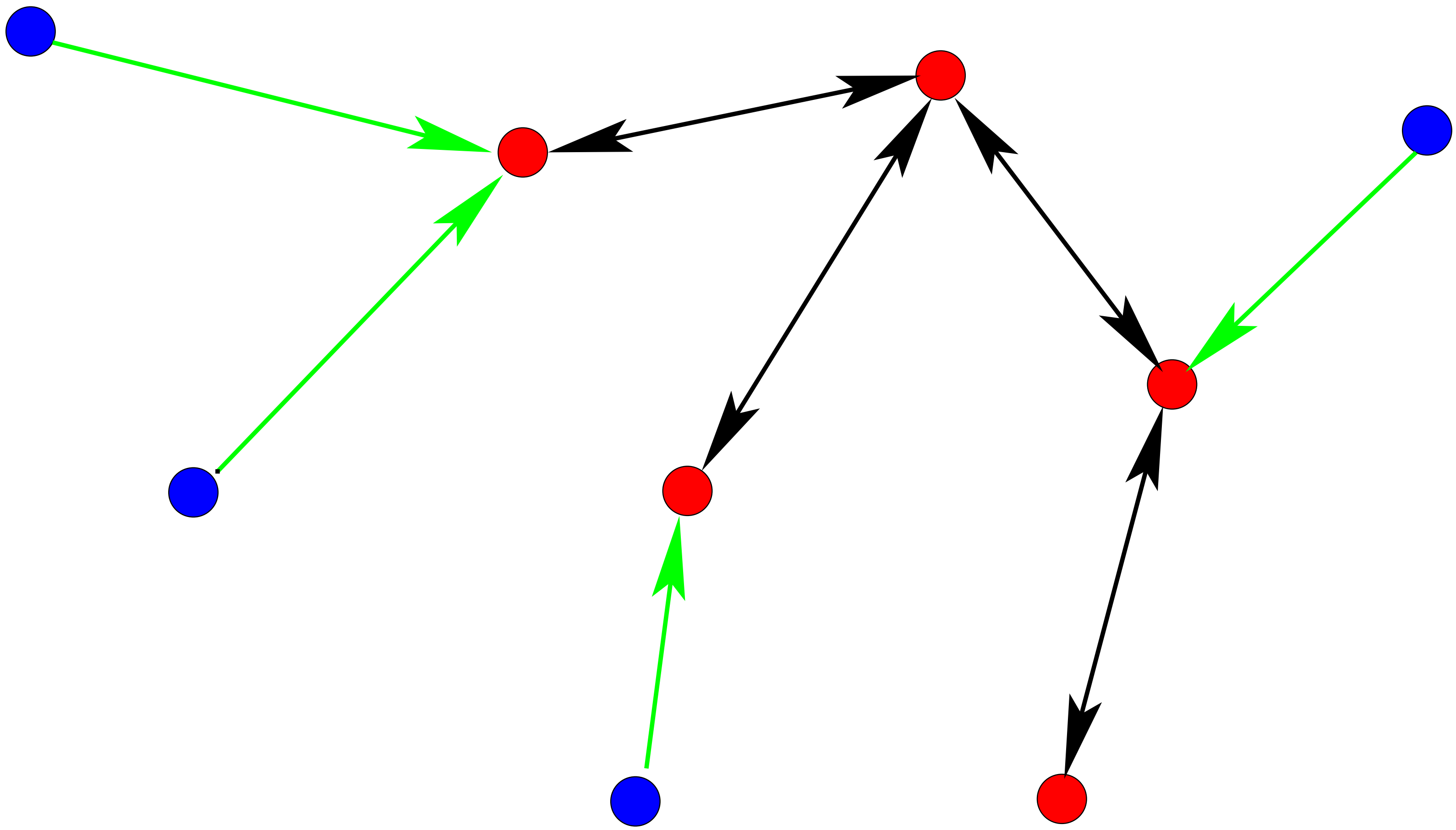
- Back to the animation : Observe the construction of the tree

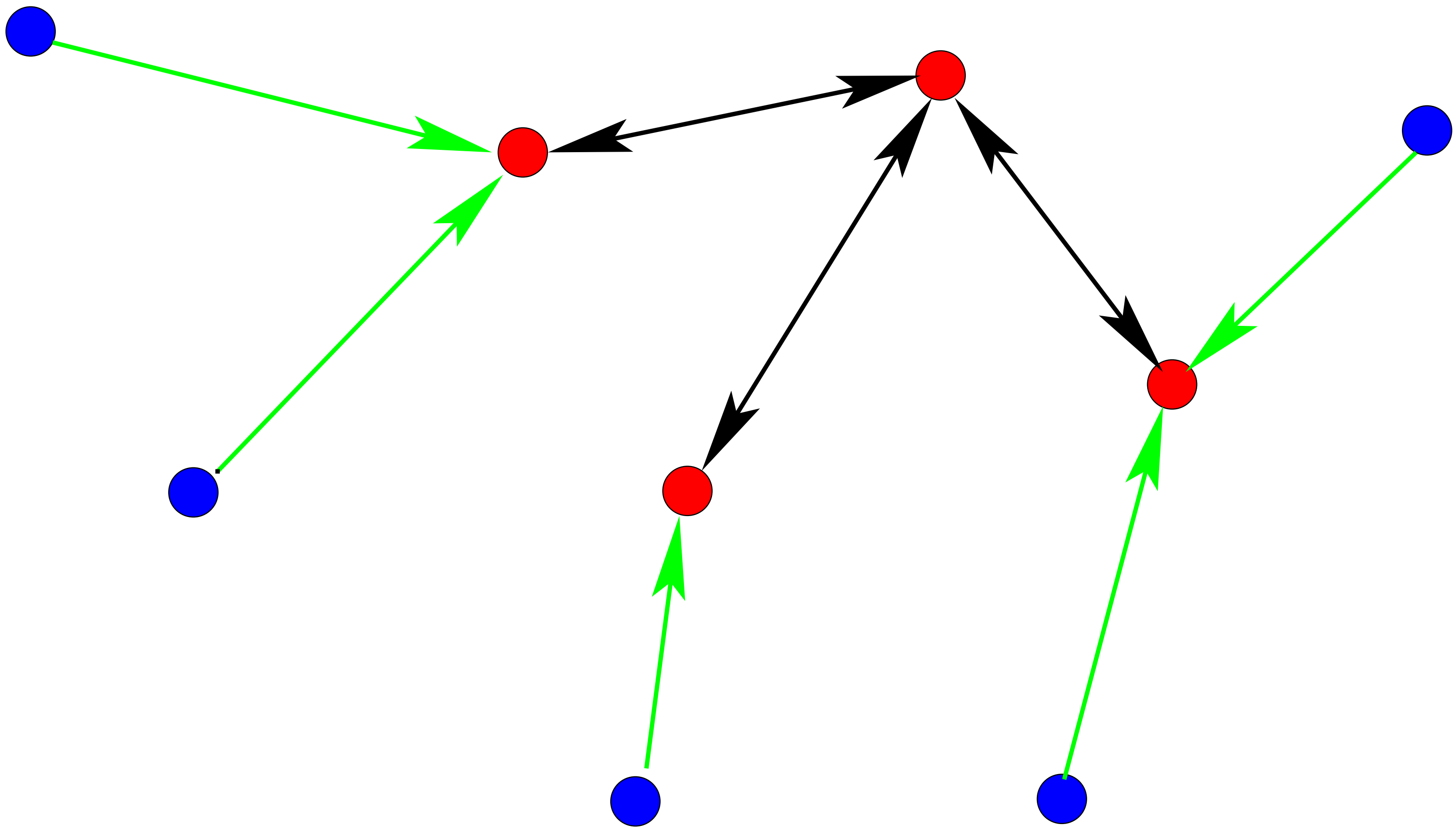


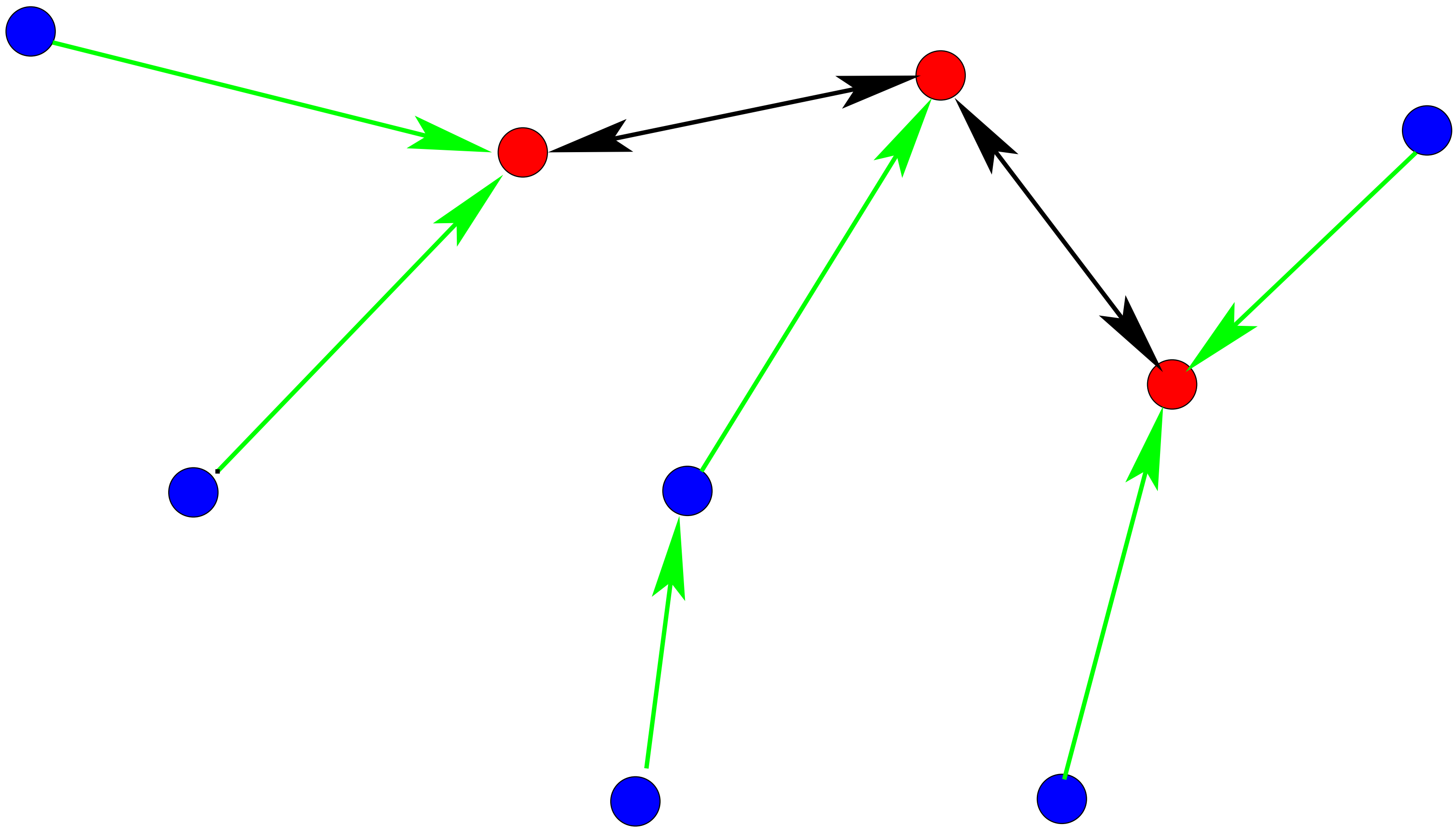


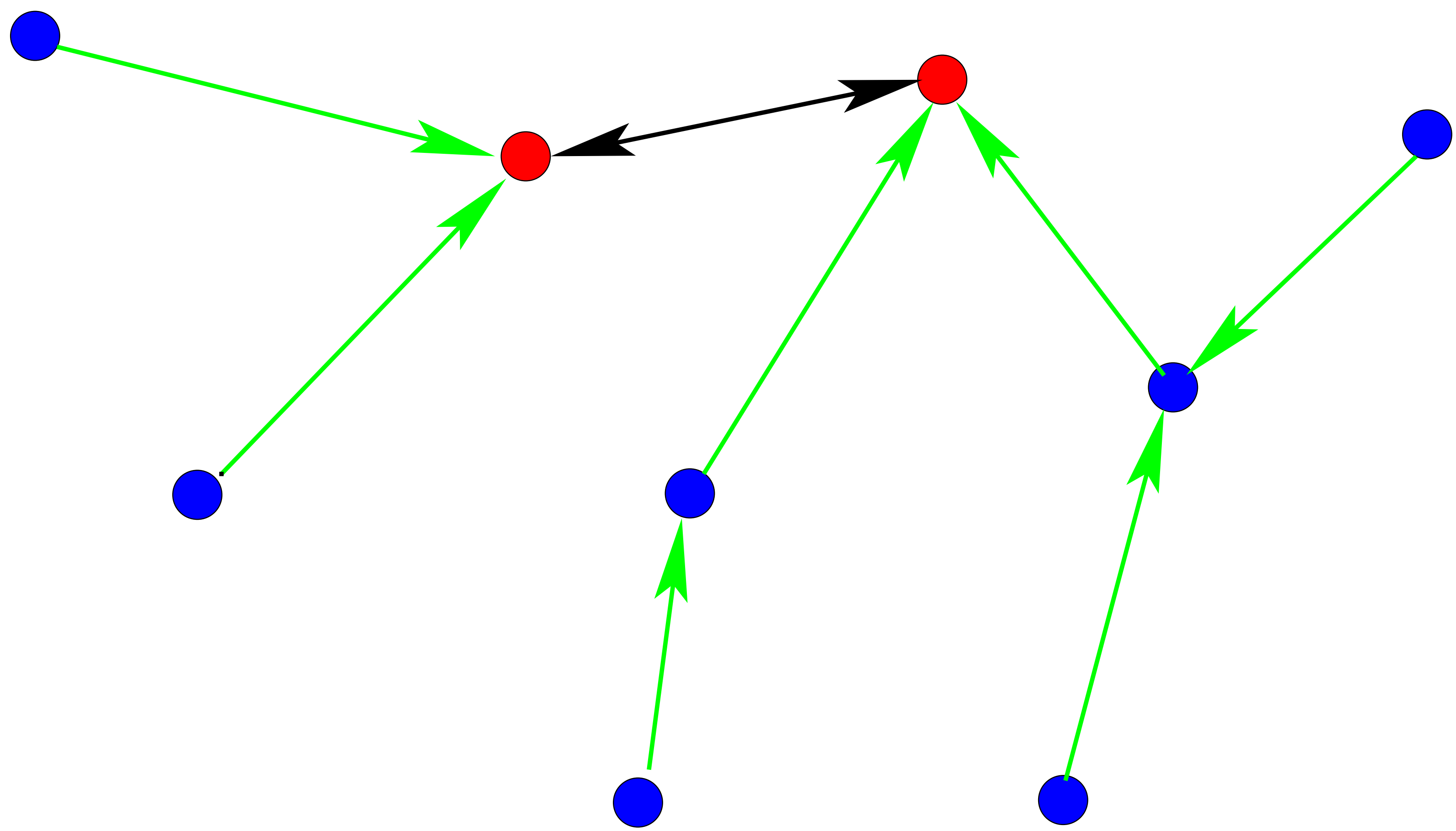


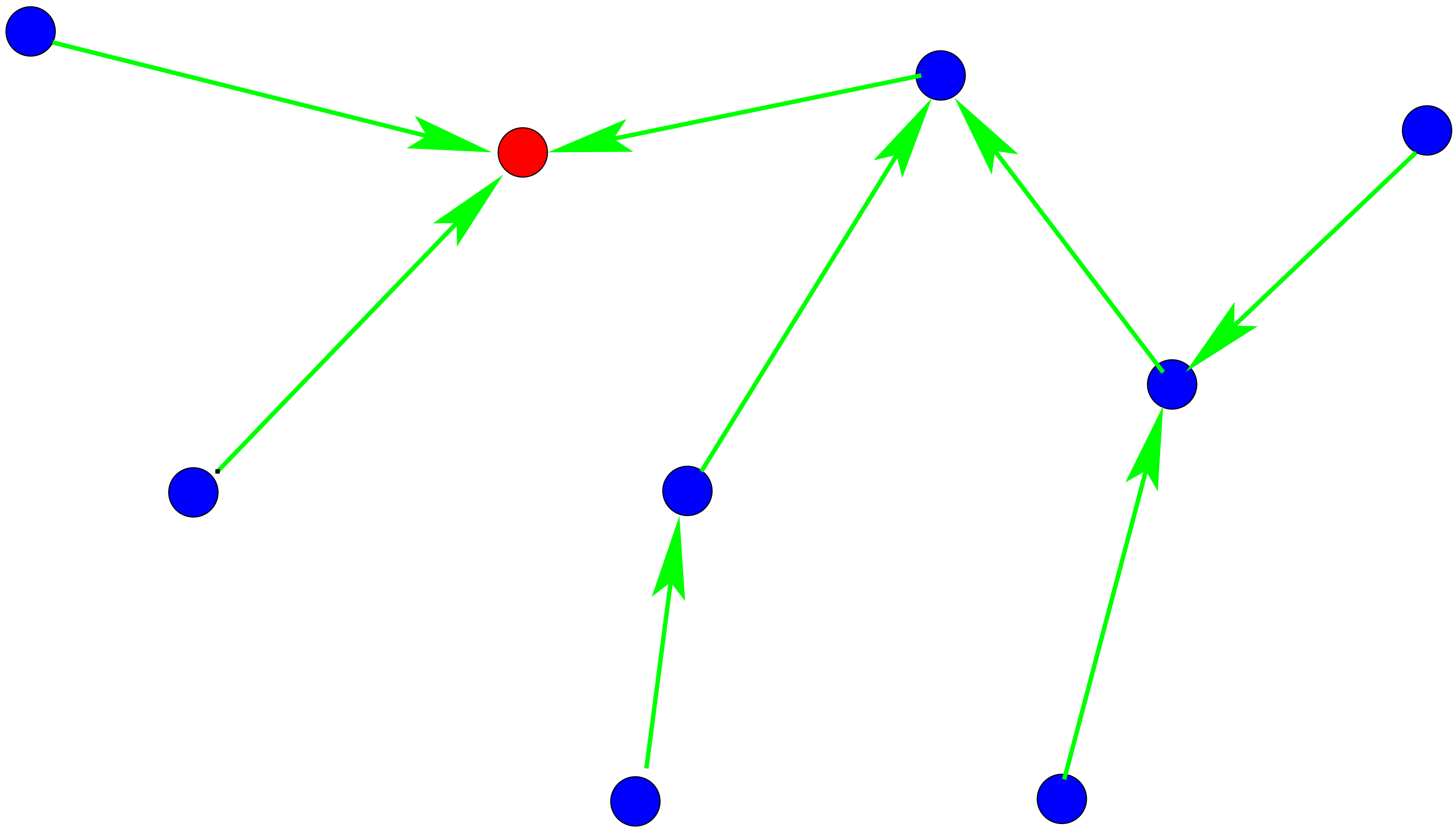












- The green arrows correspond to the $tr$ function

- The blue nodes are the domain of $tr$

- The function $tr$ is a forest (multi-tree) on nodes

- The red nodes are the roots of these trees

## The predicate invariant $(tr)$

$$tr \in ND \nrightarrow ND$$

# The predicate invariant $(tr)$

$$tr \in ND \leftrightarrow ND$$

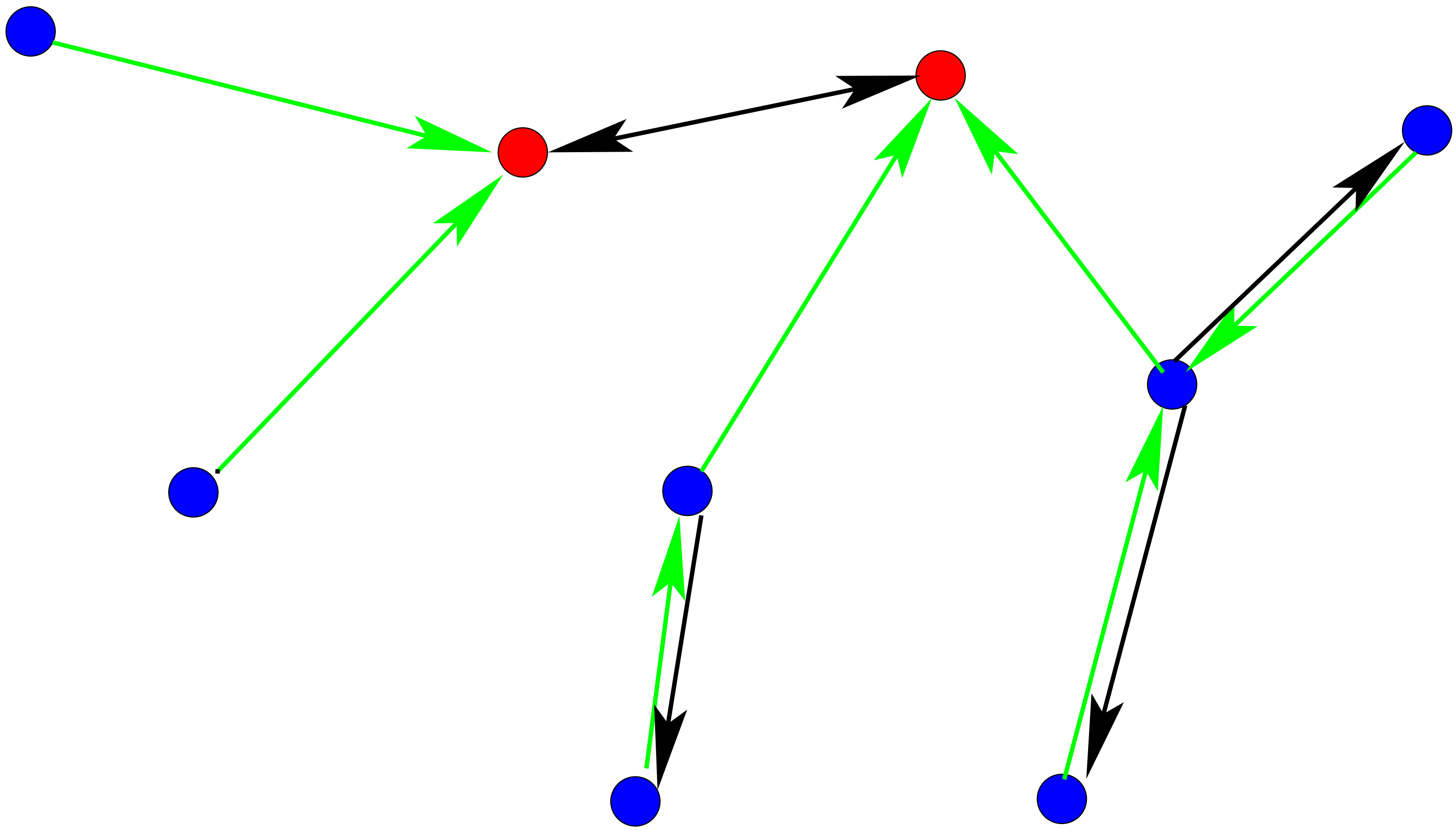
$$\forall p \cdot \left( \begin{array}{l} \quad p \subseteq ND \qquad \wedge \\ \quad ND - \mathrm{dom}\,(tr) \subseteq p \qquad \wedge \\ \quad tr^{-1}[p] \subseteq p \\ \Rightarrow \\ \quad ND \subseteq p \end{array} \right)$$

# The predicate $\text{invariant}\,(tr)$

$$tr \in ND \leftrightarrow ND$$

$$\forall p \cdot \begin{pmatrix} \quad p \subseteq ND \qquad \wedge \\ \quad ND - \mathrm{dom}\,(tr) \subseteq p \qquad \wedge \\ \quad tr^{-1}\,[p] \subseteq p \\ \Rightarrow \\ \quad ND \subseteq p \end{pmatrix}$$

$$\mathrm{dom}\,(tr) \lhd (tr \cup tr^{-1}) \;=\; \mathrm{dom}\,(tr) \lhd gr$$

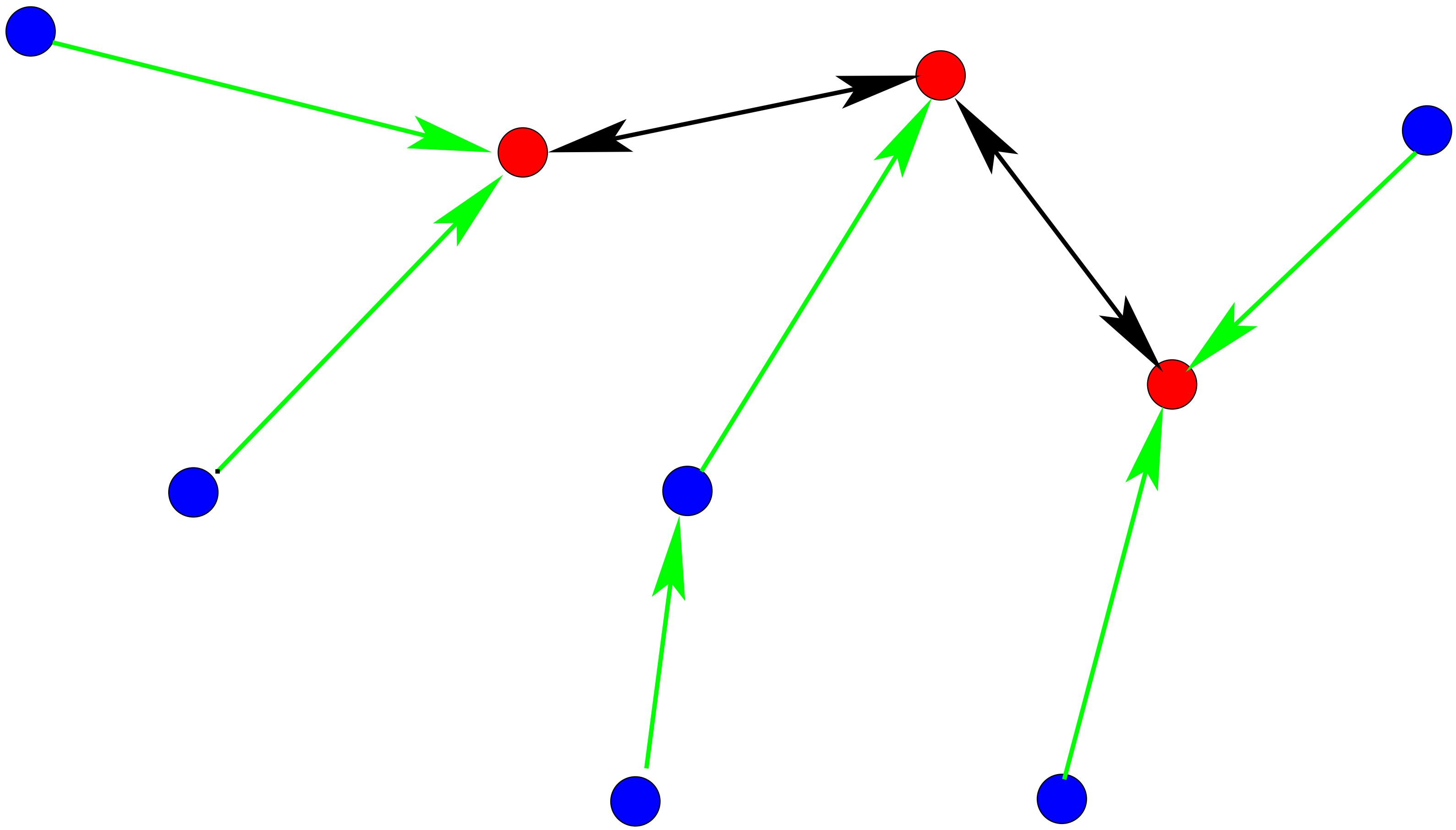


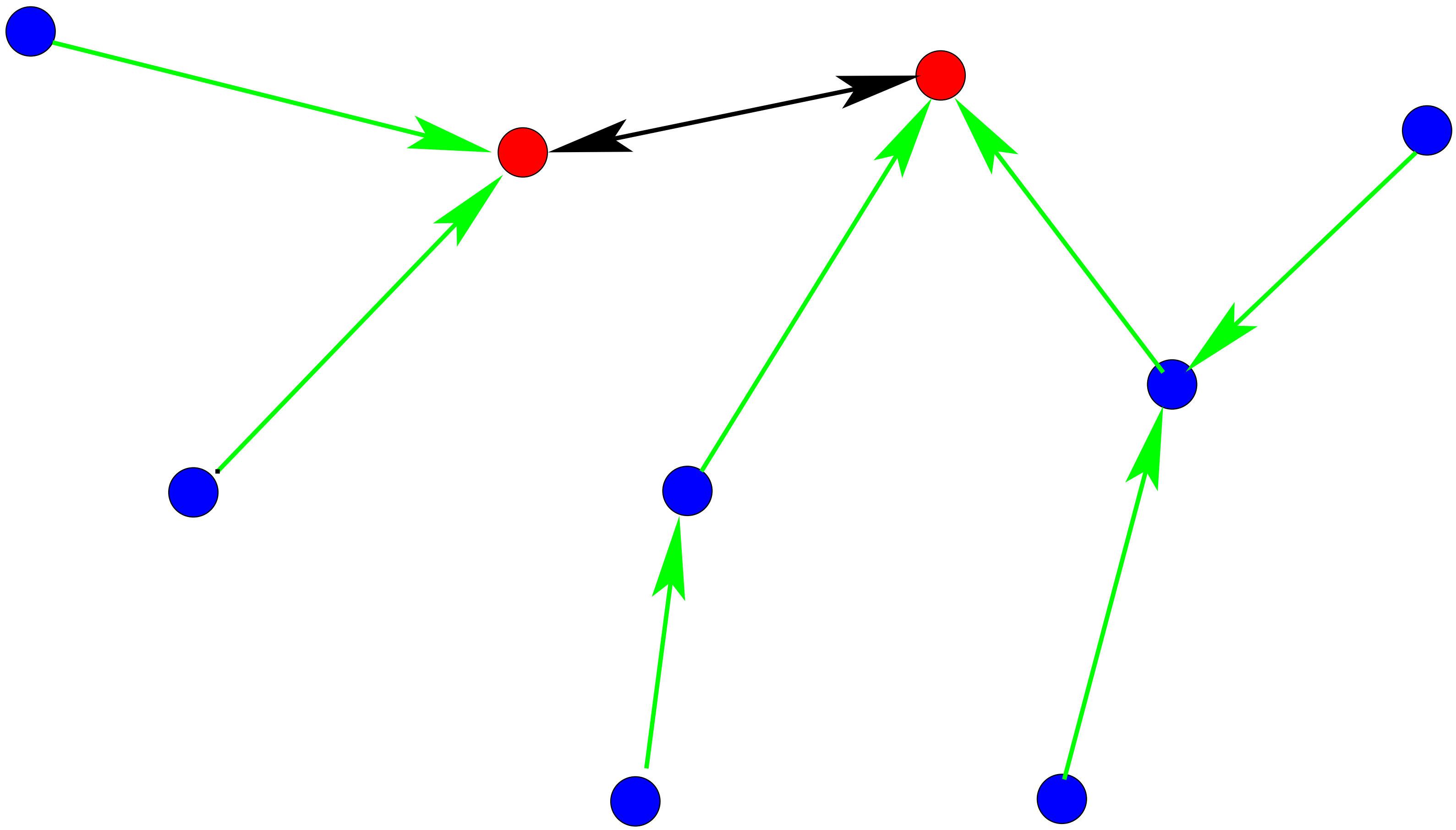
## First Refinement (2)

- Introducing the new event "progress"

- Refining the abstract event "elect"

- Back to the animation : Observe the "guard" of progress



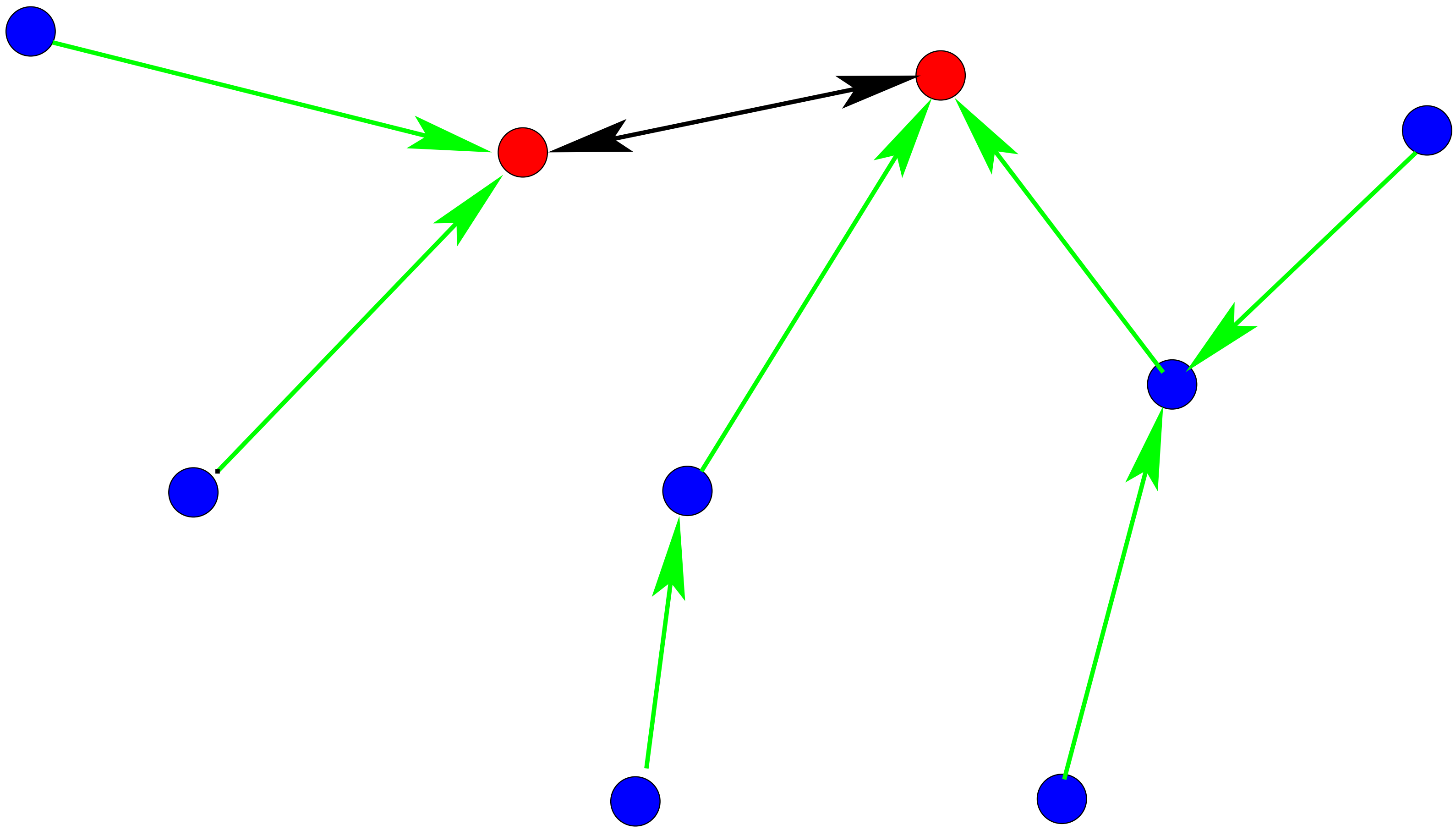


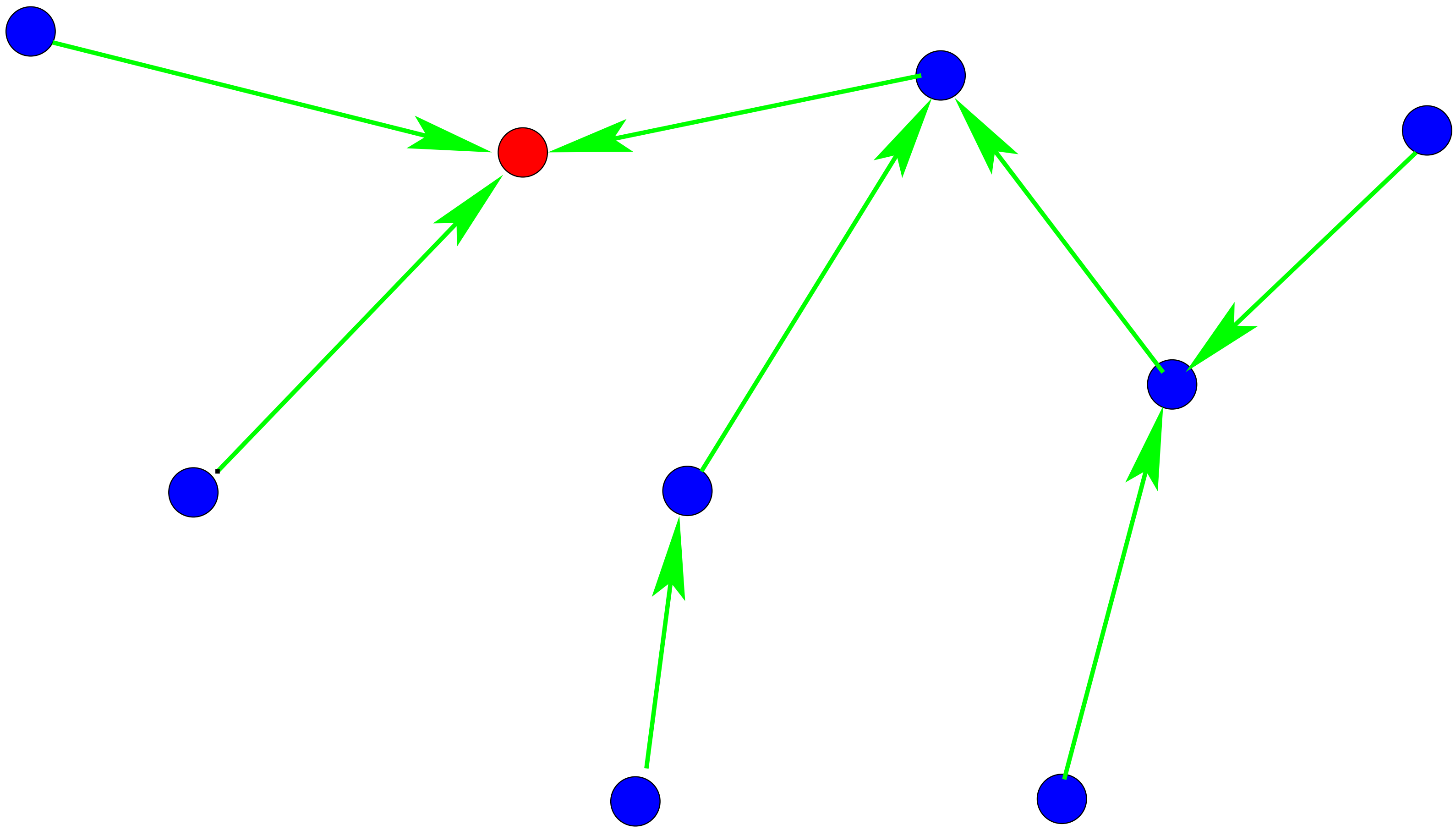
When a red node $x$ is connected to AT MOST one other red node $y$ then event "progress" can take place

$$
\begin{array}{l}
\text{progress} \;\;\widehat{=} \\
\quad \textbf{ANY } x, y \textbf{ WHERE} \\
\qquad x, y \in gr \;\; \wedge \\
\qquad x \notin \operatorname{dom}(tr) \;\; \wedge \\
\qquad y \notin \operatorname{dom}(tr) \;\; \wedge \\
\qquad gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\} \\
\quad \textbf{THEN} \\
\qquad tr := tr \cup \{x \mapsto y\} \\
\quad \textbf{END}
\end{array}
$$

# To be proved

$$
\begin{array}{l}
\quad \text{invariant}(tr) \quad \wedge \\
\quad x, y \in gr \quad \wedge \\
\quad x \notin tr \quad \wedge \\
\quad y \notin tr \quad \wedge \\
\quad gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\} \\
\Rightarrow \\
\quad \text{invariant}(tr \cup \{x \mapsto y\})
\end{array}
$$





When a red node $x$ is ONLY connected to blue nodes then event "elect" can take place

elect $\;\widehat{=}$

**ANY** $x$ **WHERE**

$x \in ND \;\wedge$

$gr[\{x\}] = tr^{-1}[\{x\}]$

**THEN**

$rt, ts := x, tr$

**END**

elect $\widehat{=}$

**BEGIN**

$rt, ts$ : spanning $(rt, ts, gr)$

**END**

elect $\widehat{=}$

**ANY** $x$ **WHERE**

$x \in ND \;\; \wedge$

$gr[\{x\}] = tr^{-1}[\{x\}]$

**THEN**

$rt, ts := x, tr$

**END**

# To be proved

$$
\begin{aligned}
&\text{invariant}(tr) \quad \wedge \\
&x \in ND \quad \wedge \\
&gr[\{x\}] = tr^{-1}[\{x\}] \\
&ts = tr \\
\Rightarrow \\
&\text{spanning}(x, ts, gr)
\end{aligned}
$$

# Summary of First Refinement

- 15 proofs

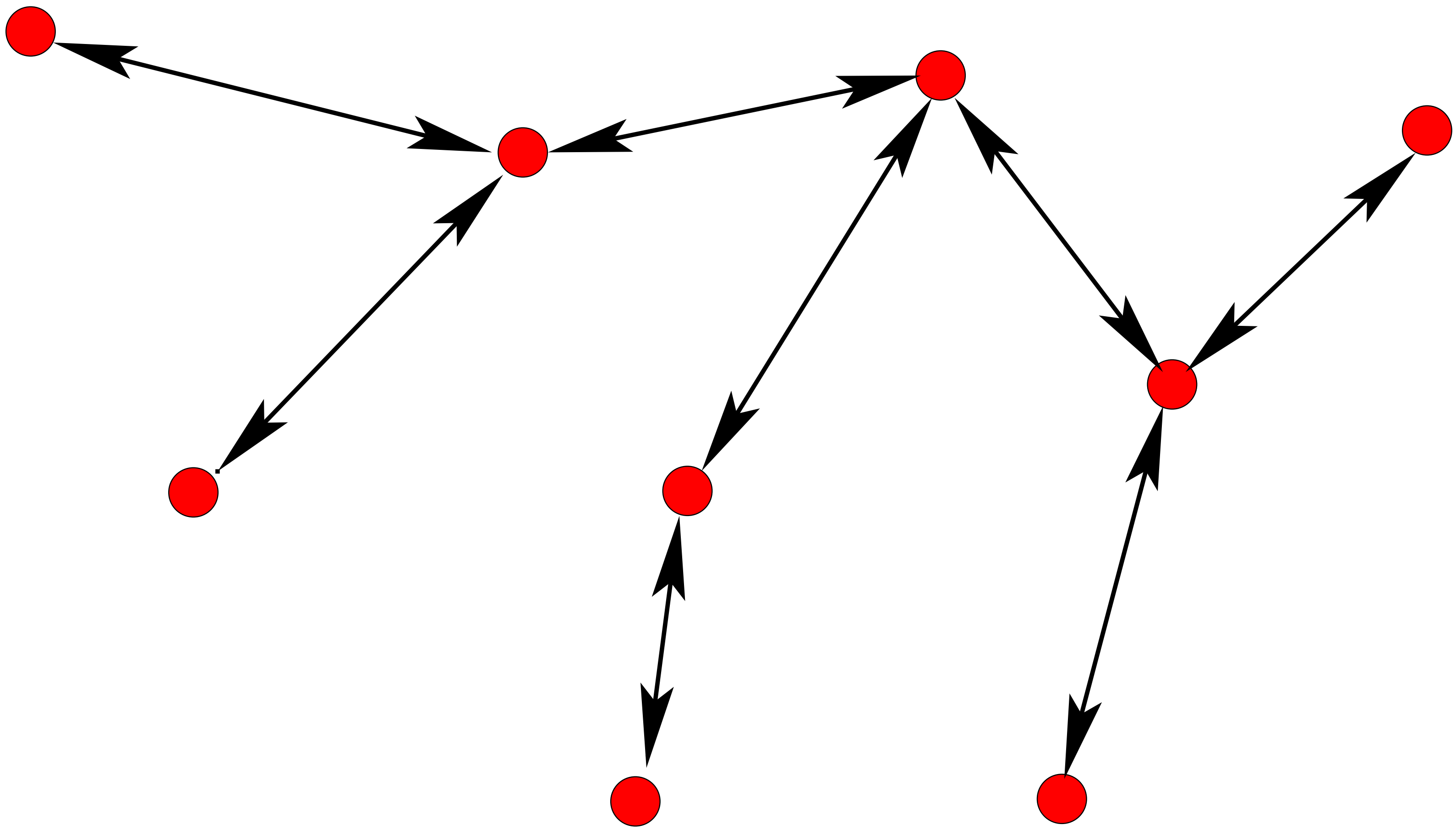
- Among which 9 were interactive (one is a bit difficult !)

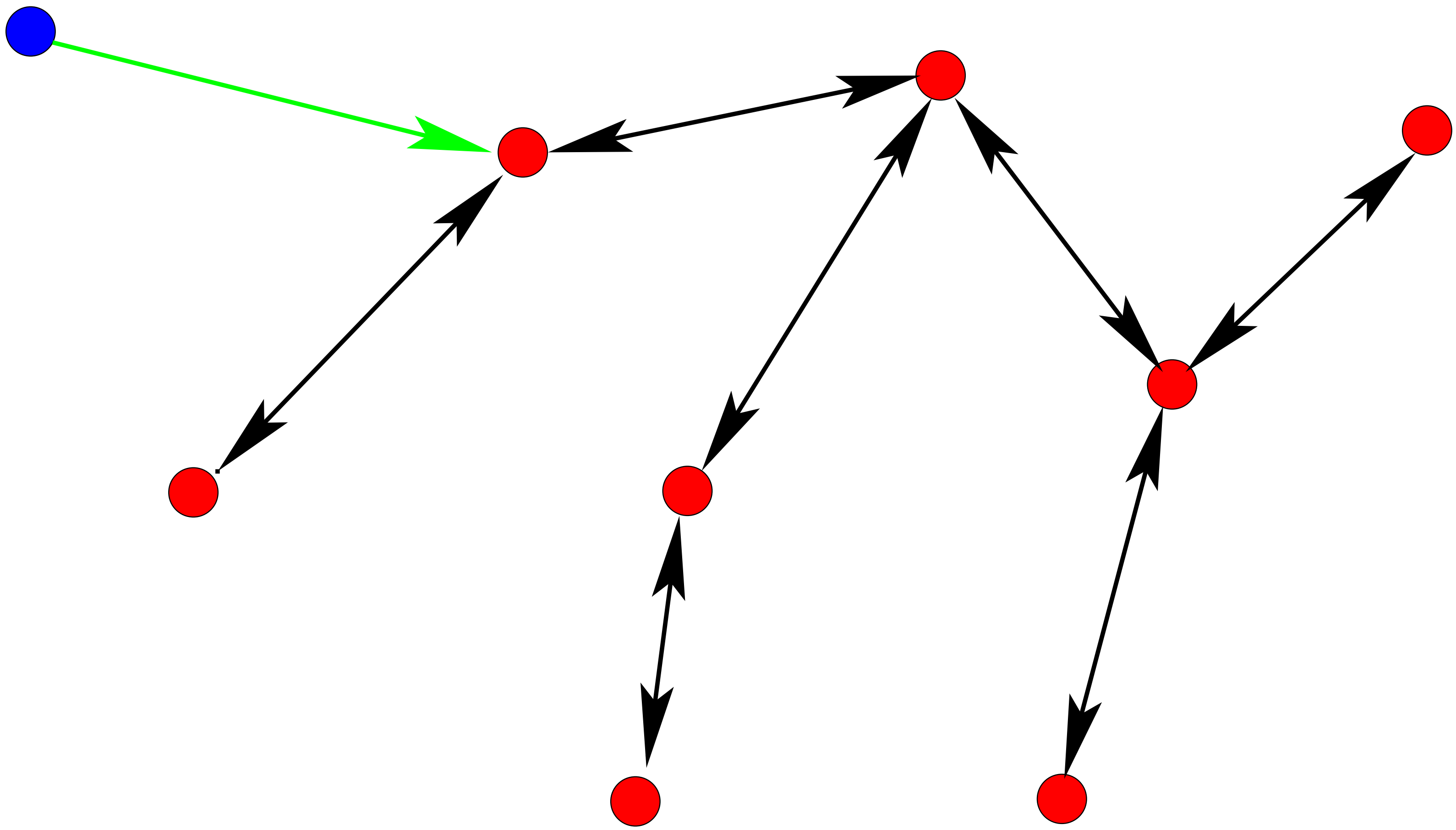
# Current state of the model

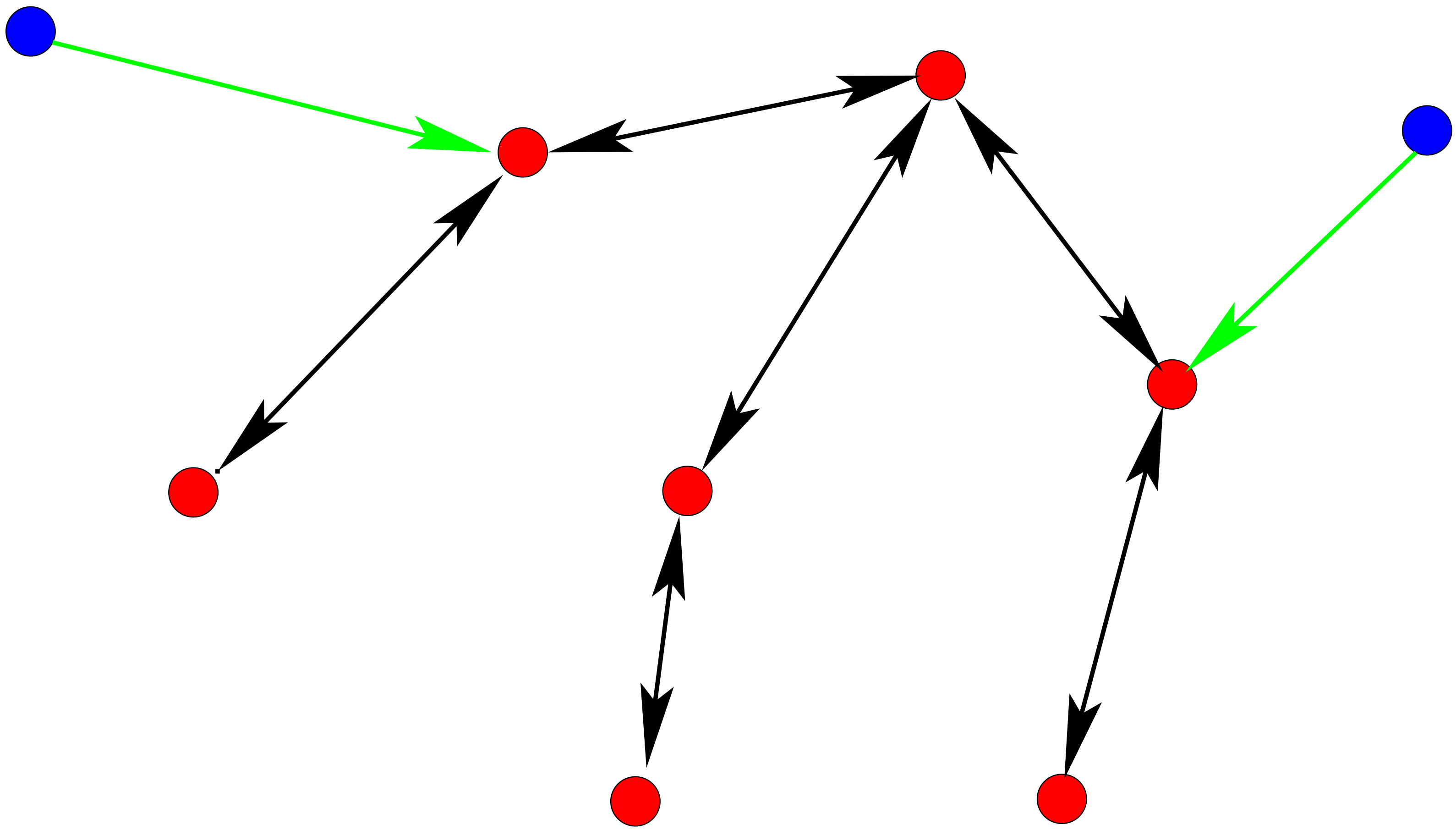
- 12 proofs

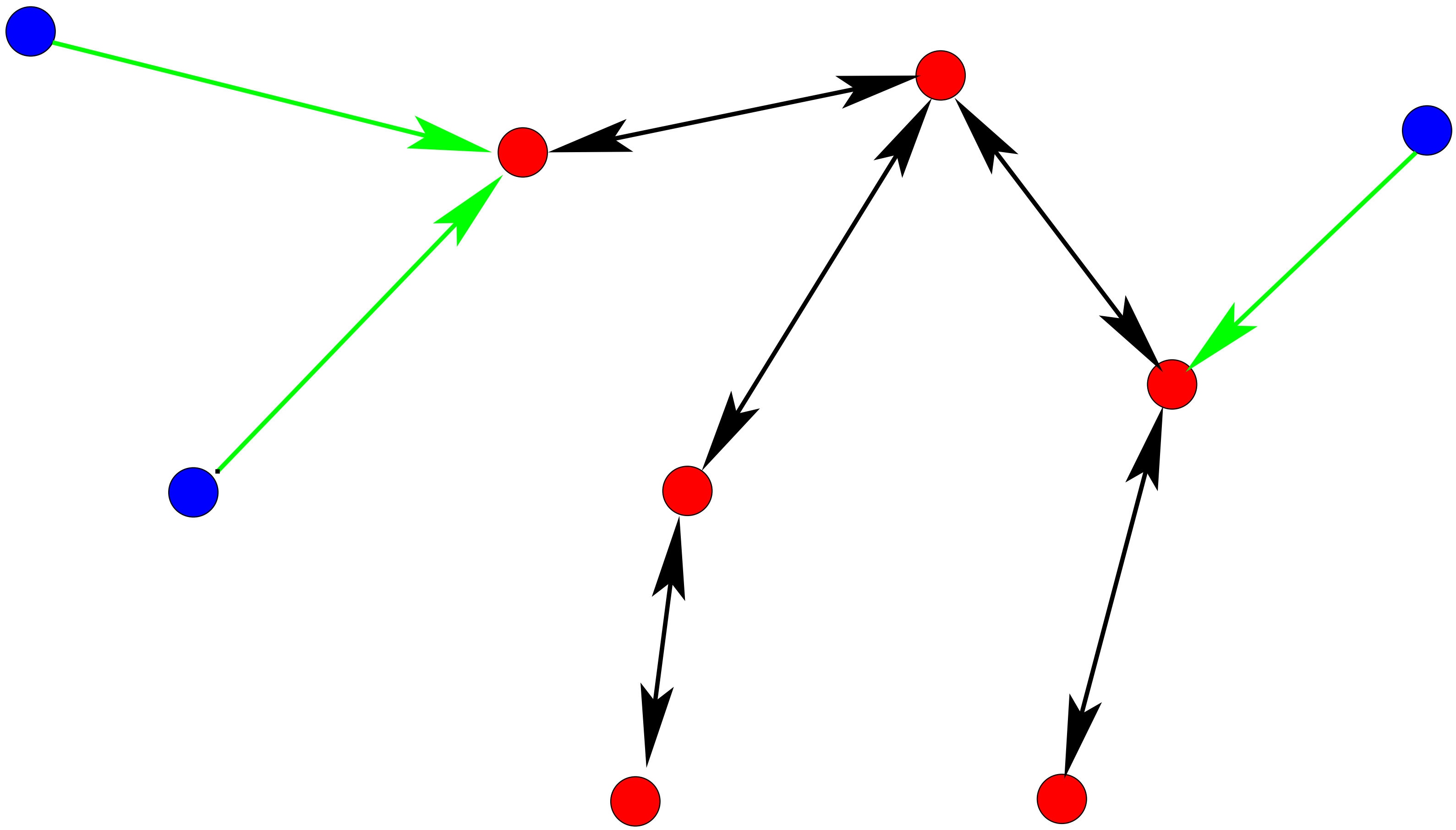
- Among which 5 were interactive (one is a bit difficult !)

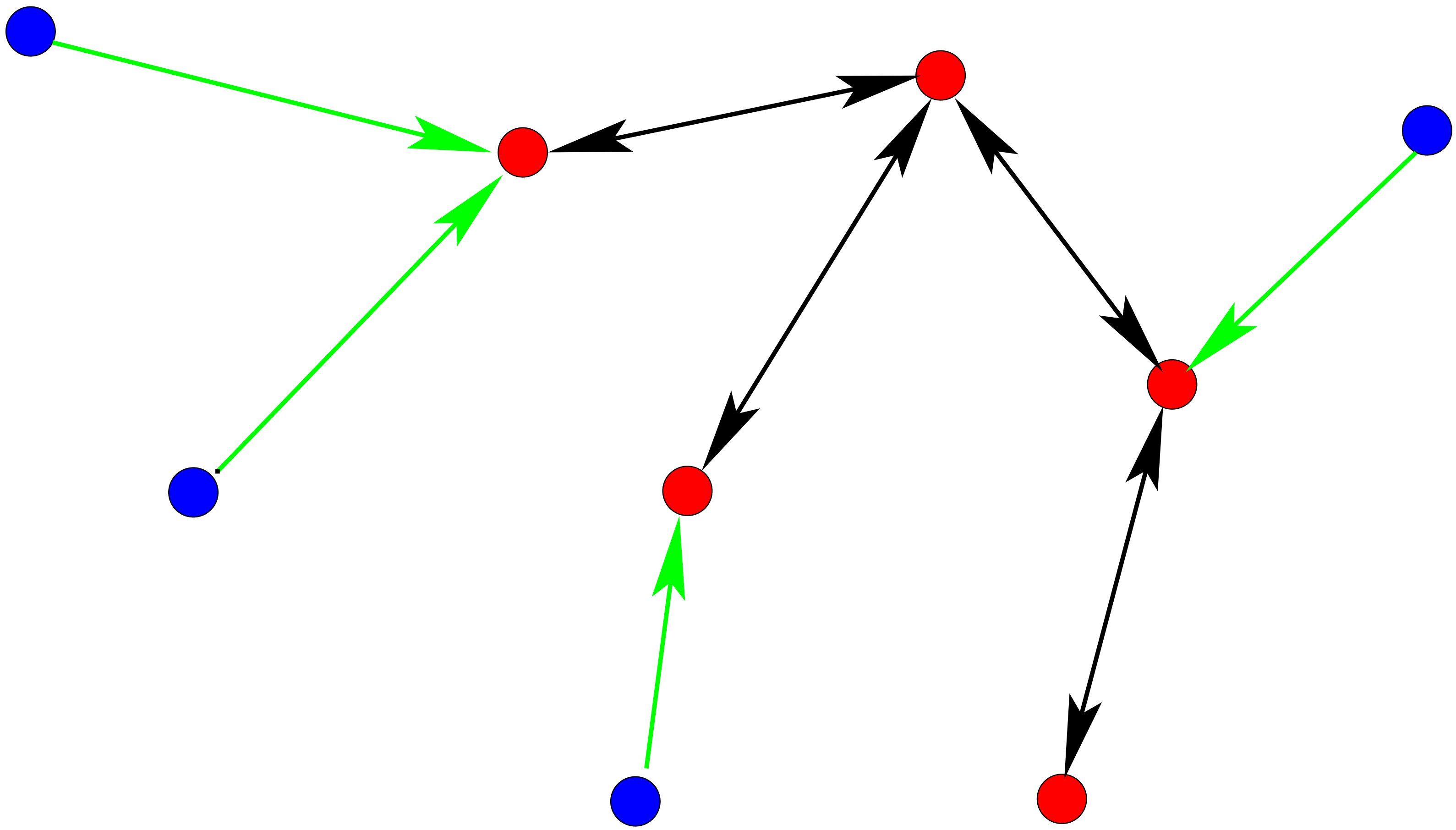
- Animation of the current model

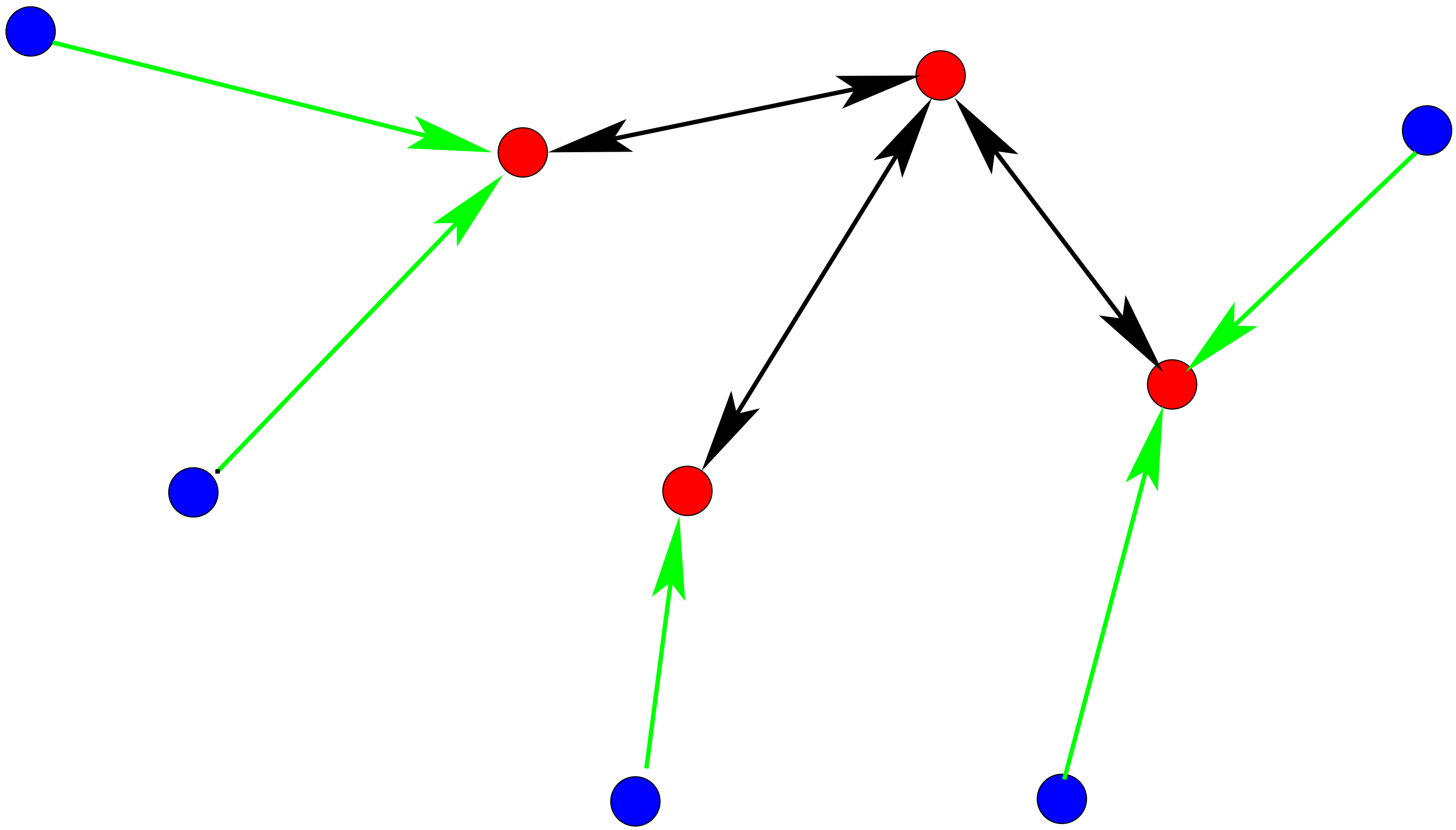


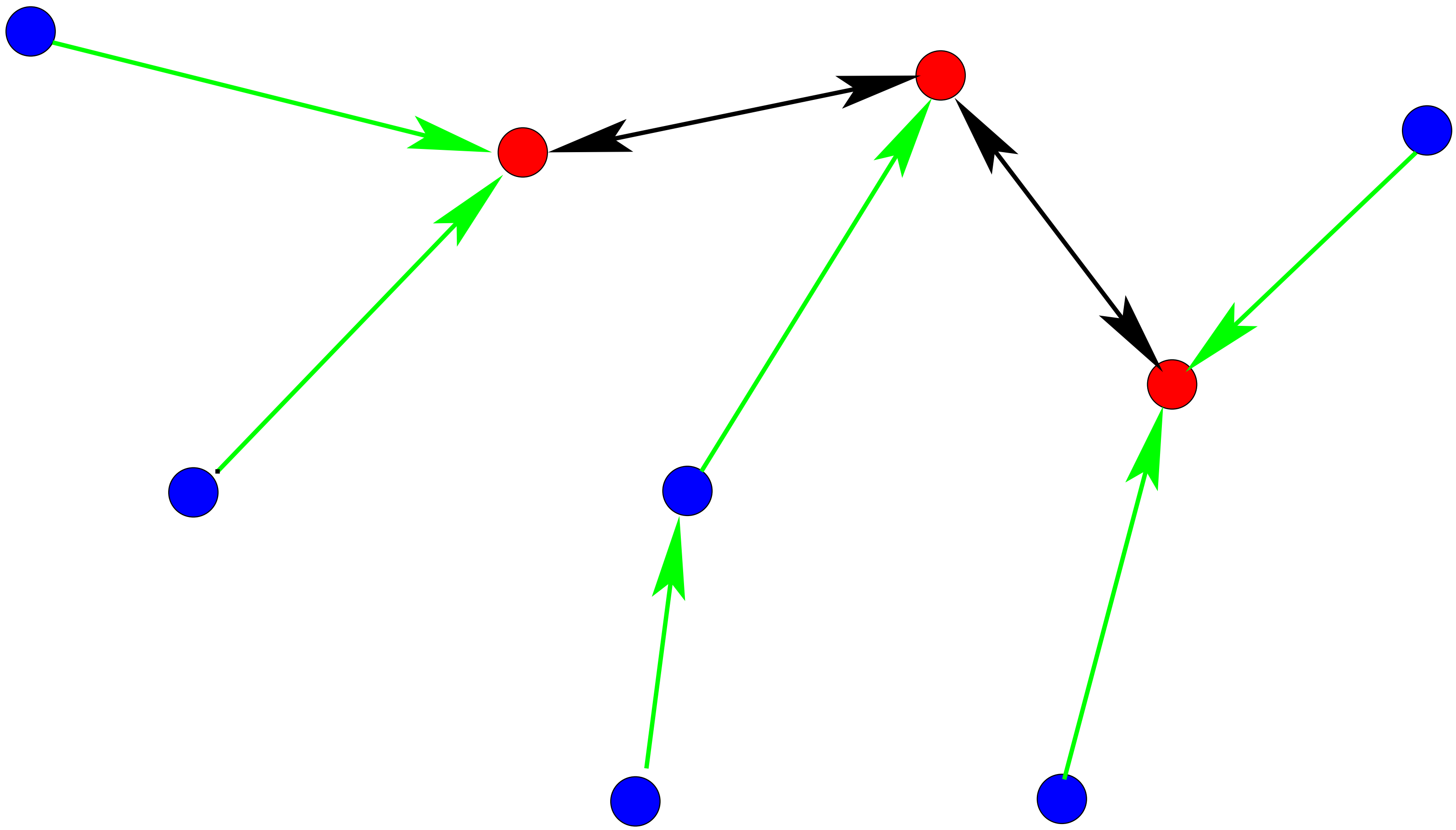


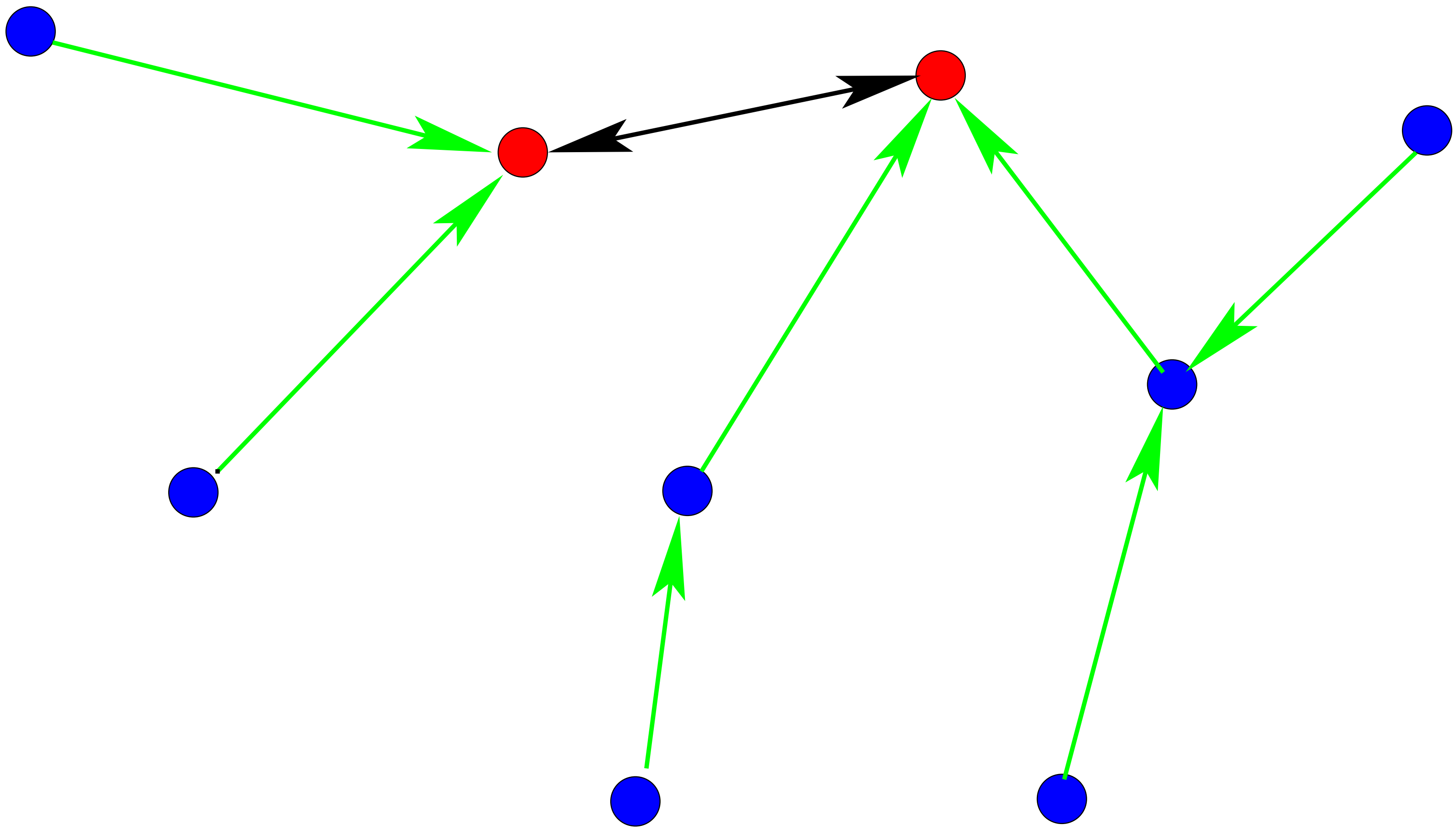


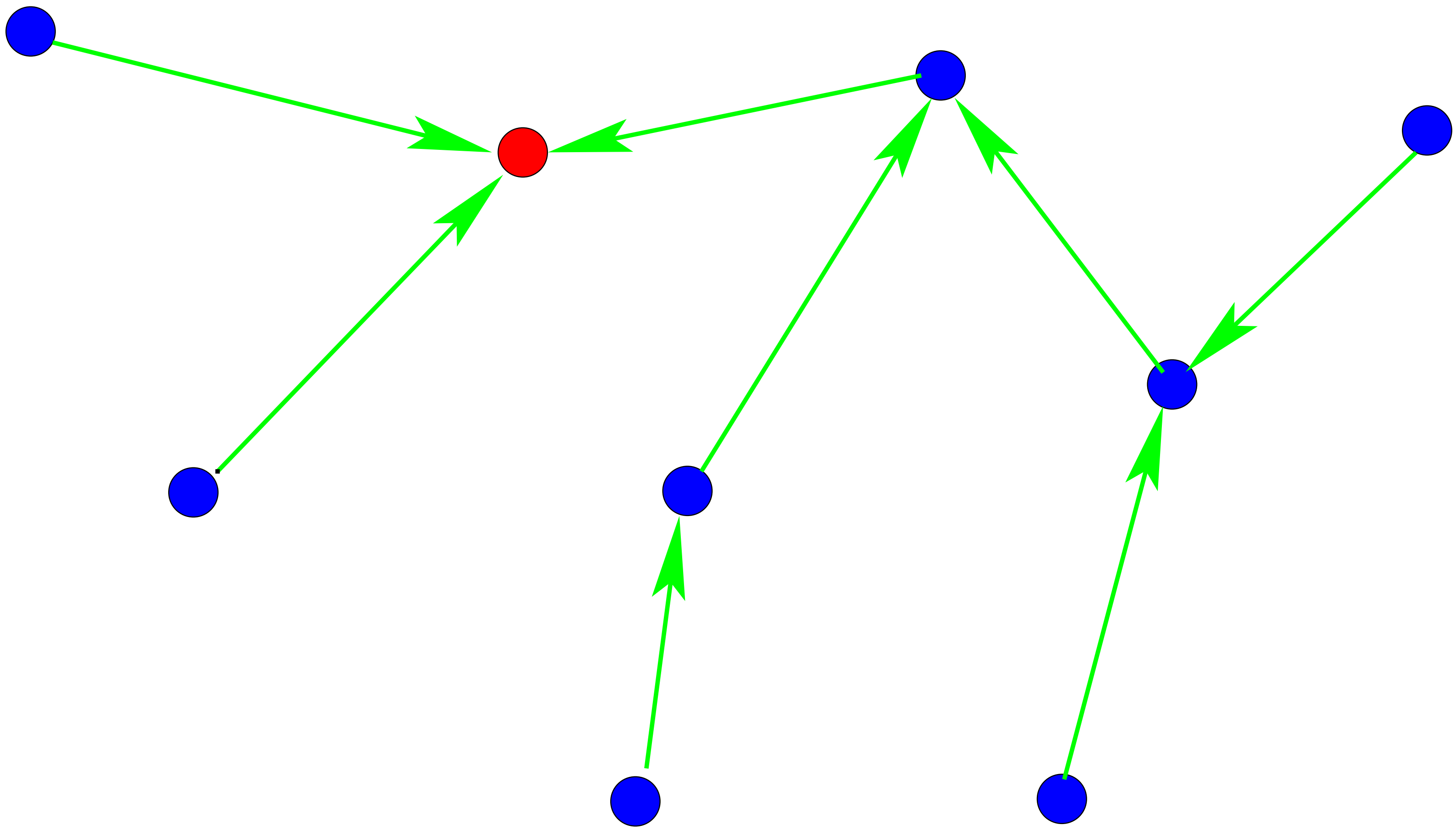


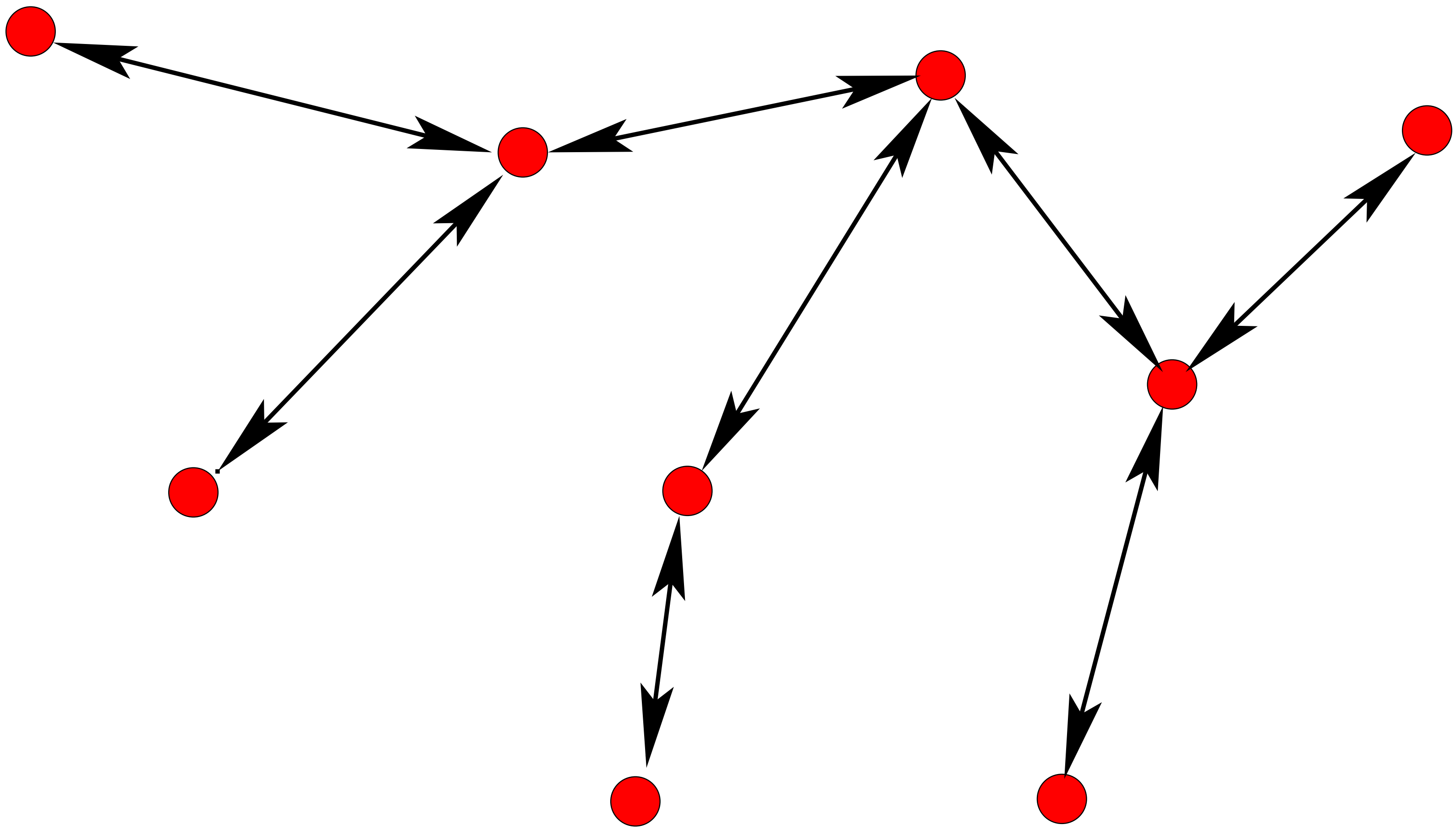


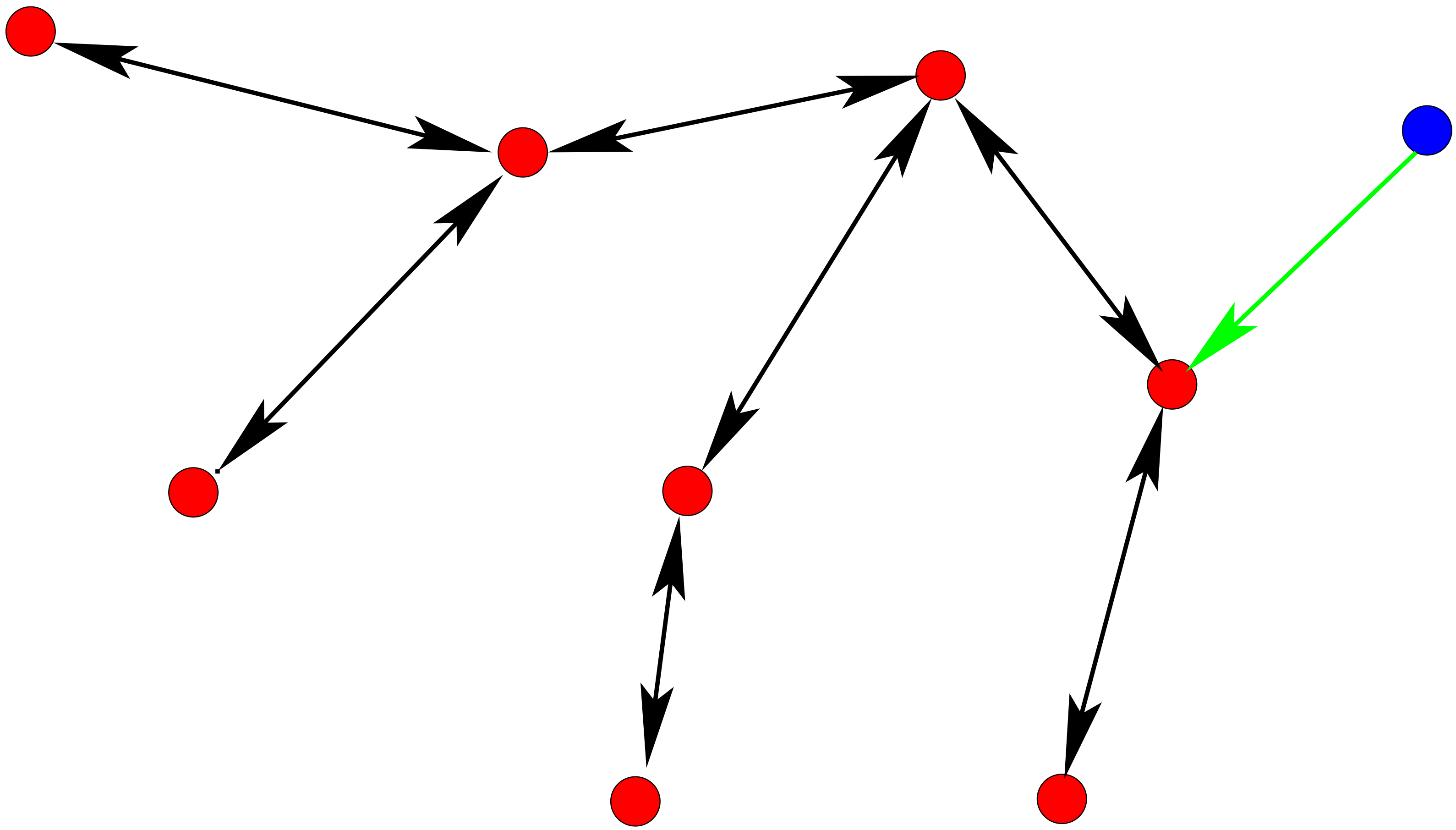


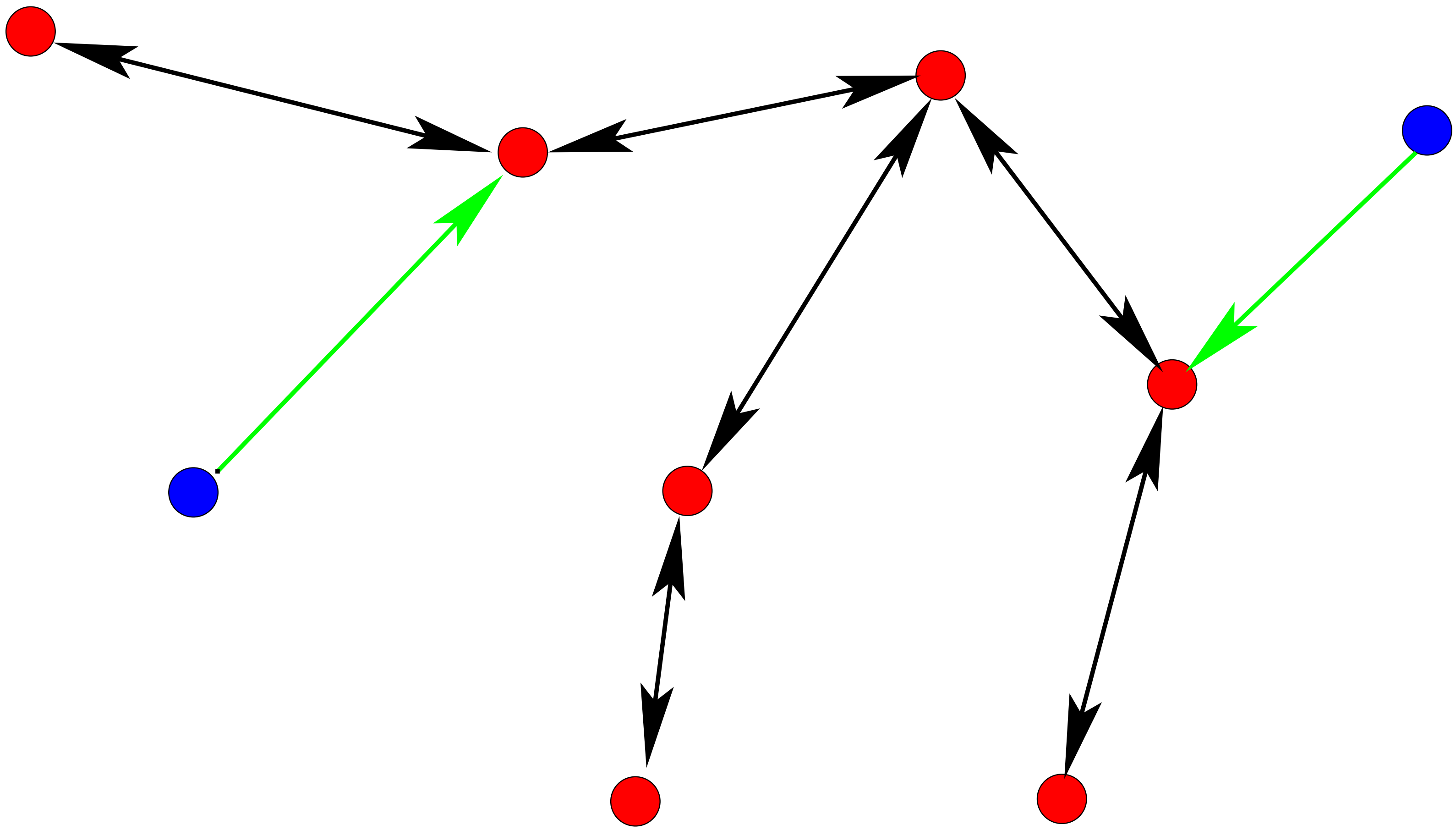


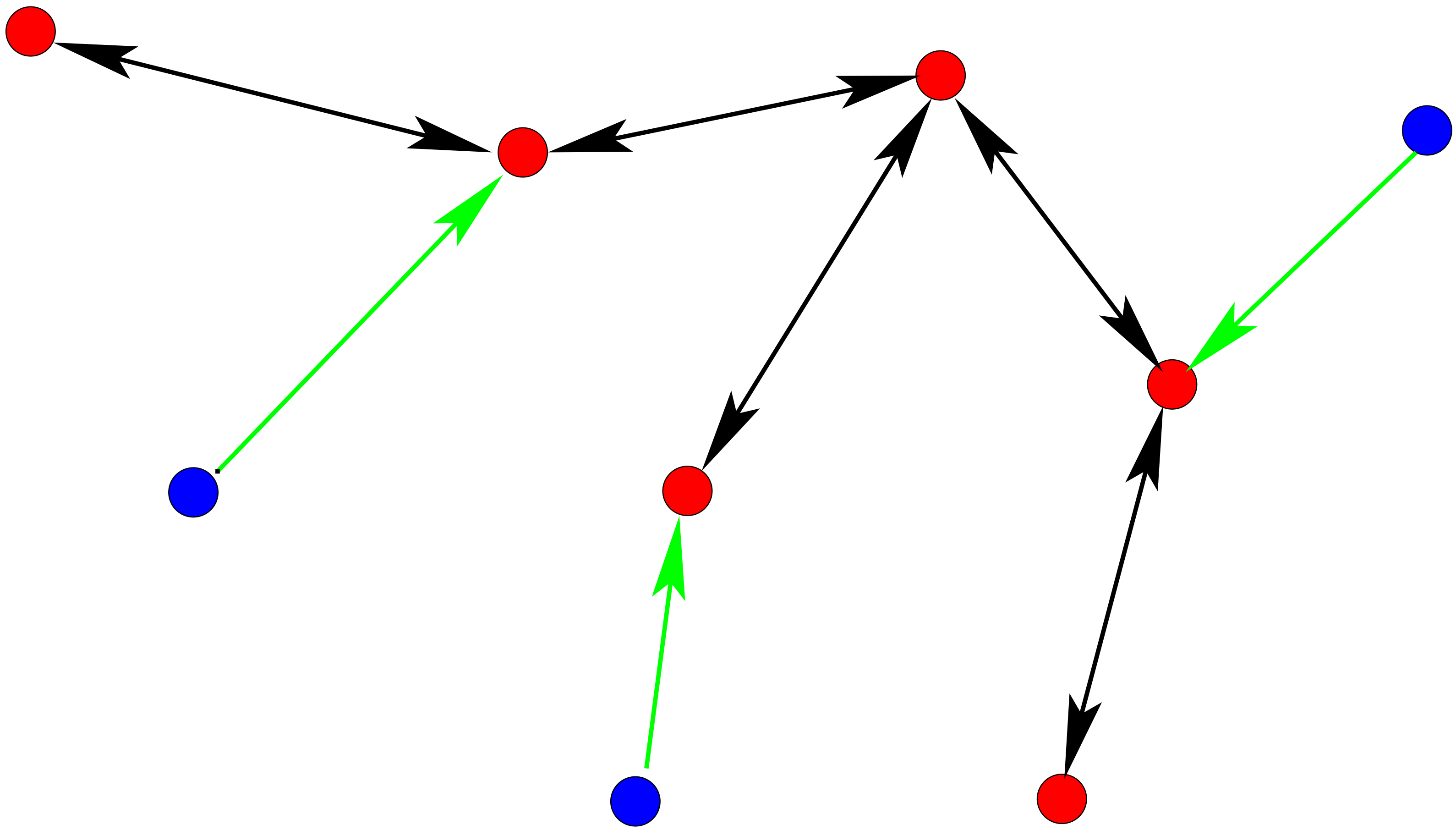


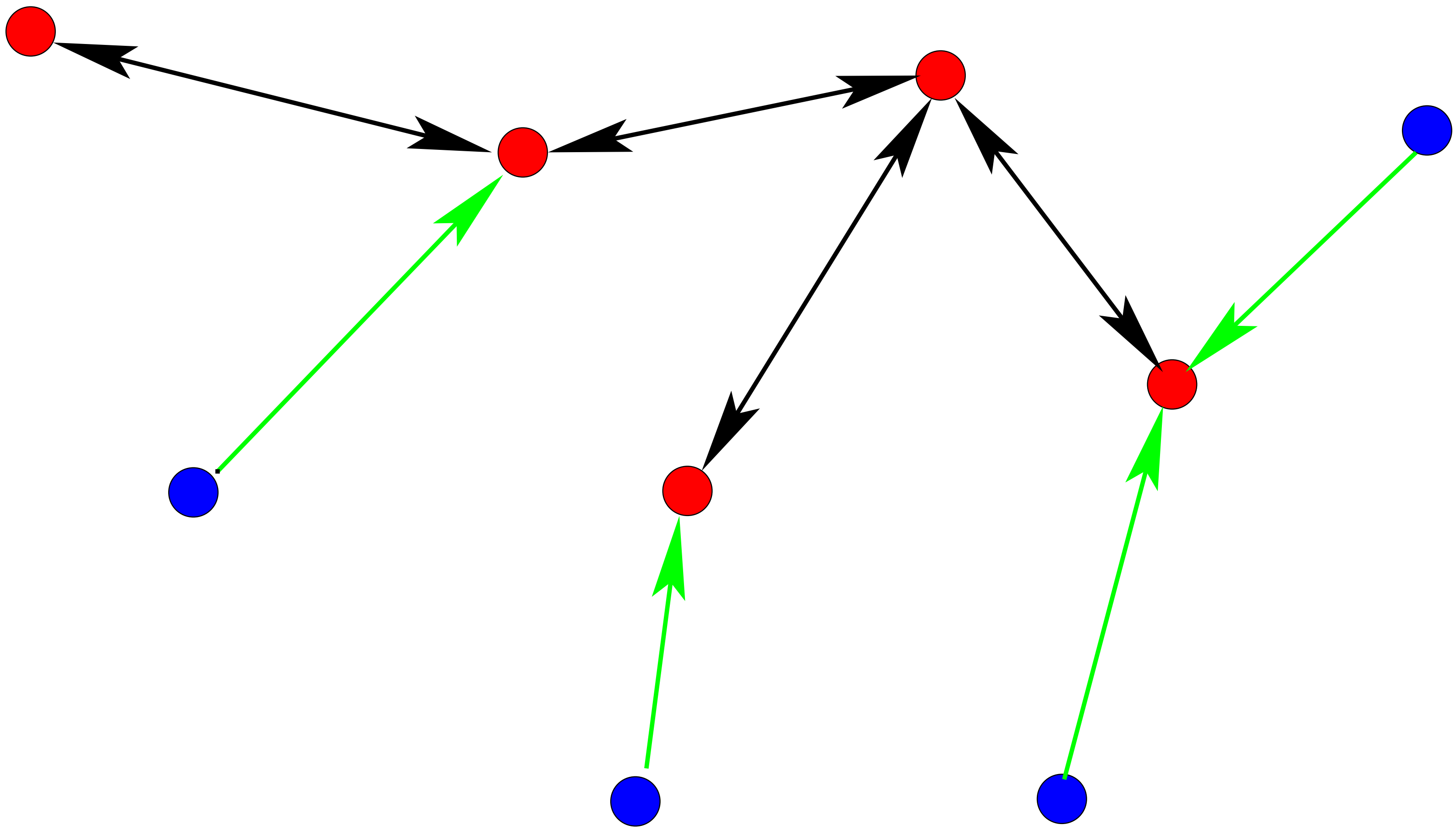


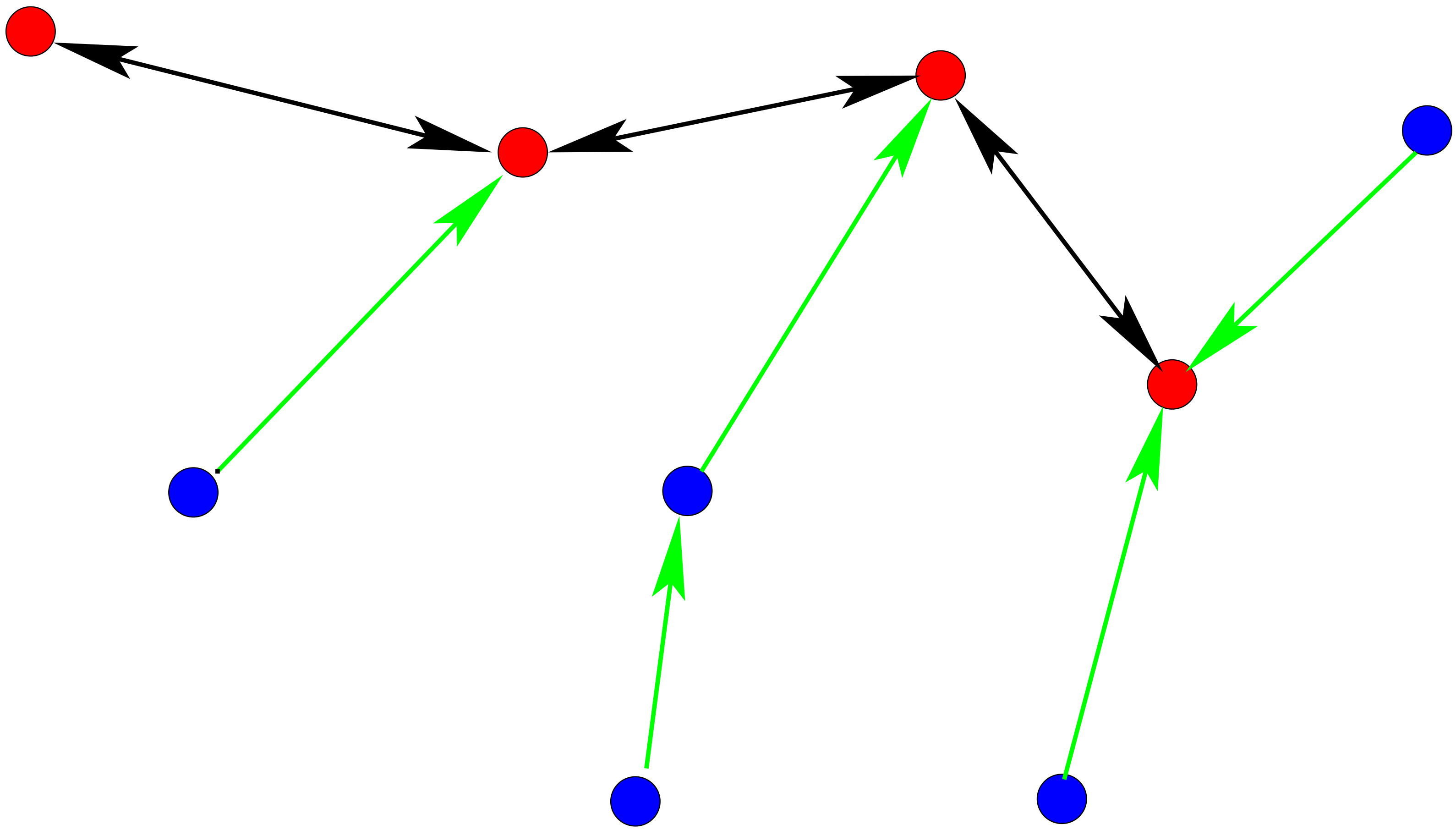


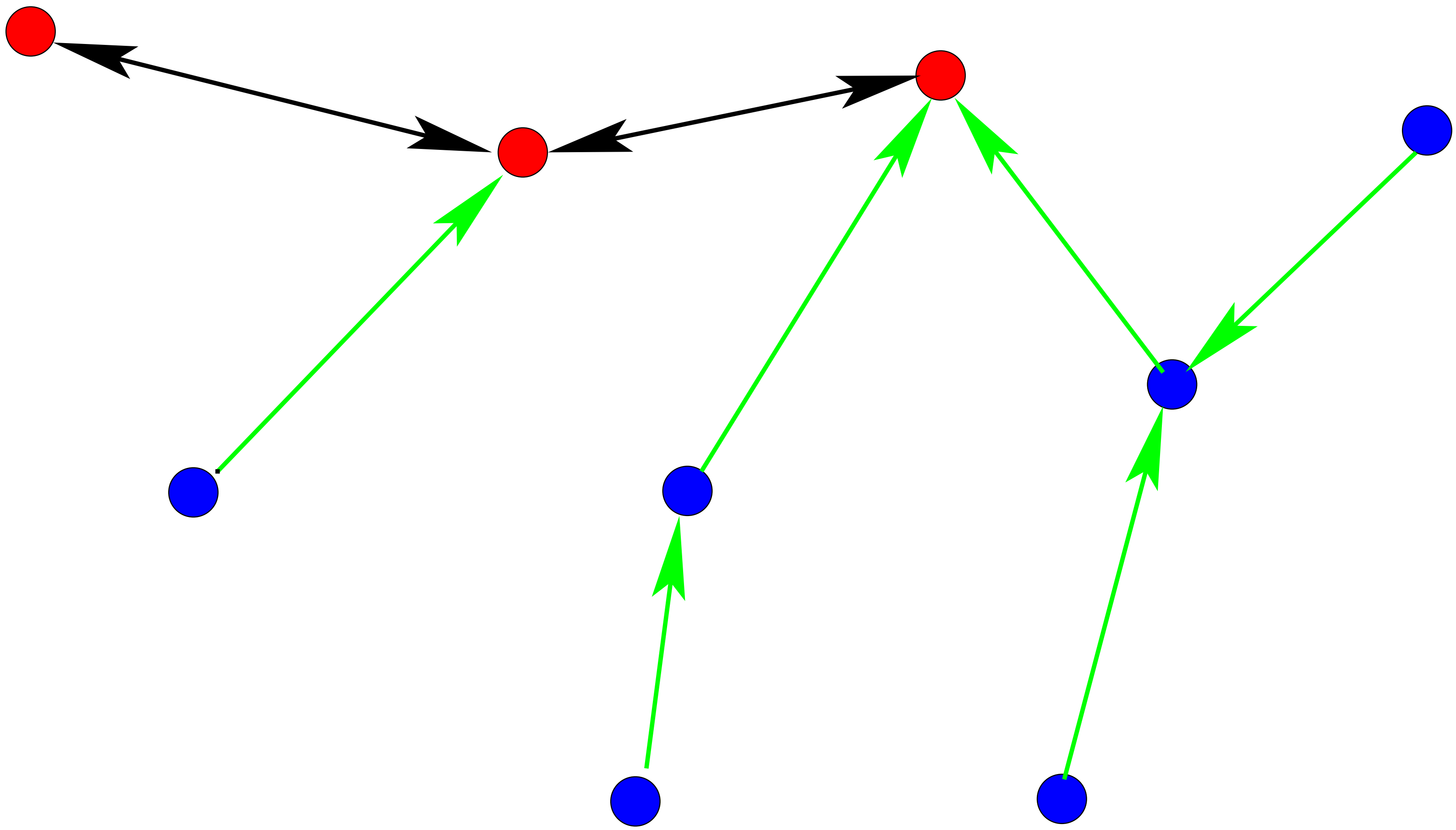


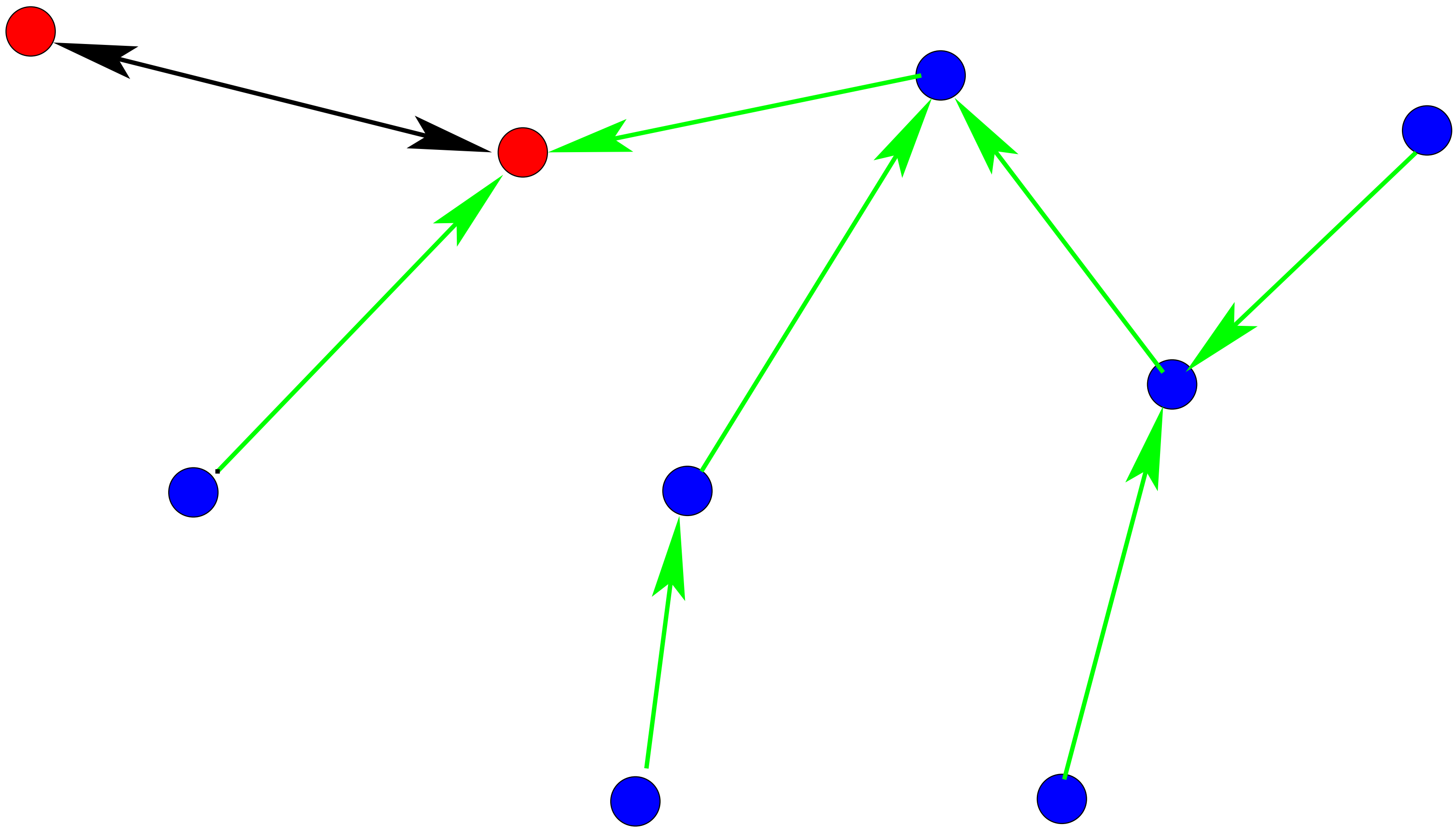


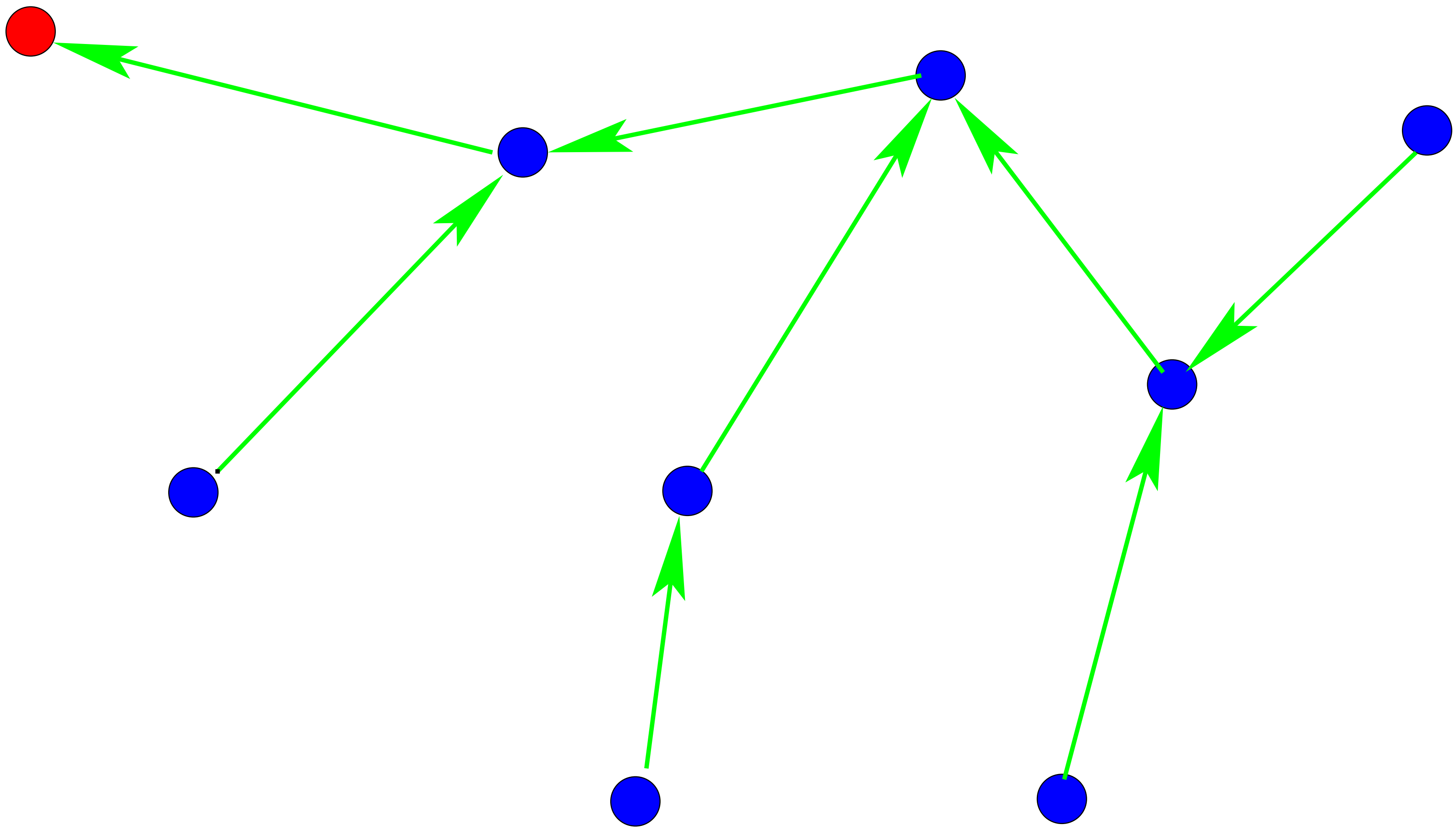












# Second Refinement

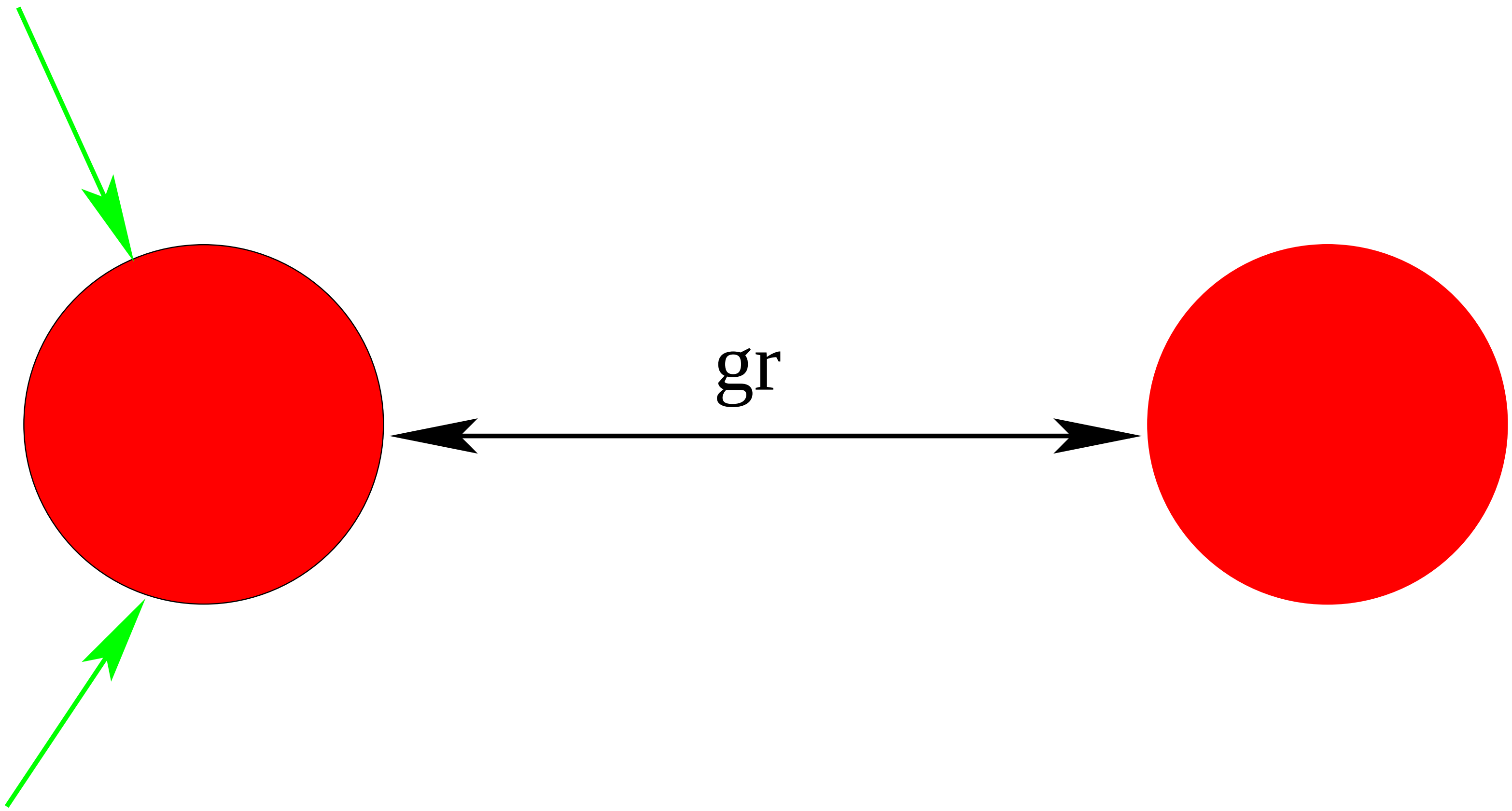
- Nodes are communicating with their neighbors

- This is done by means of messages

- Messages are acknowledged

- Acknowledgements are confirmed

- Next is a local animation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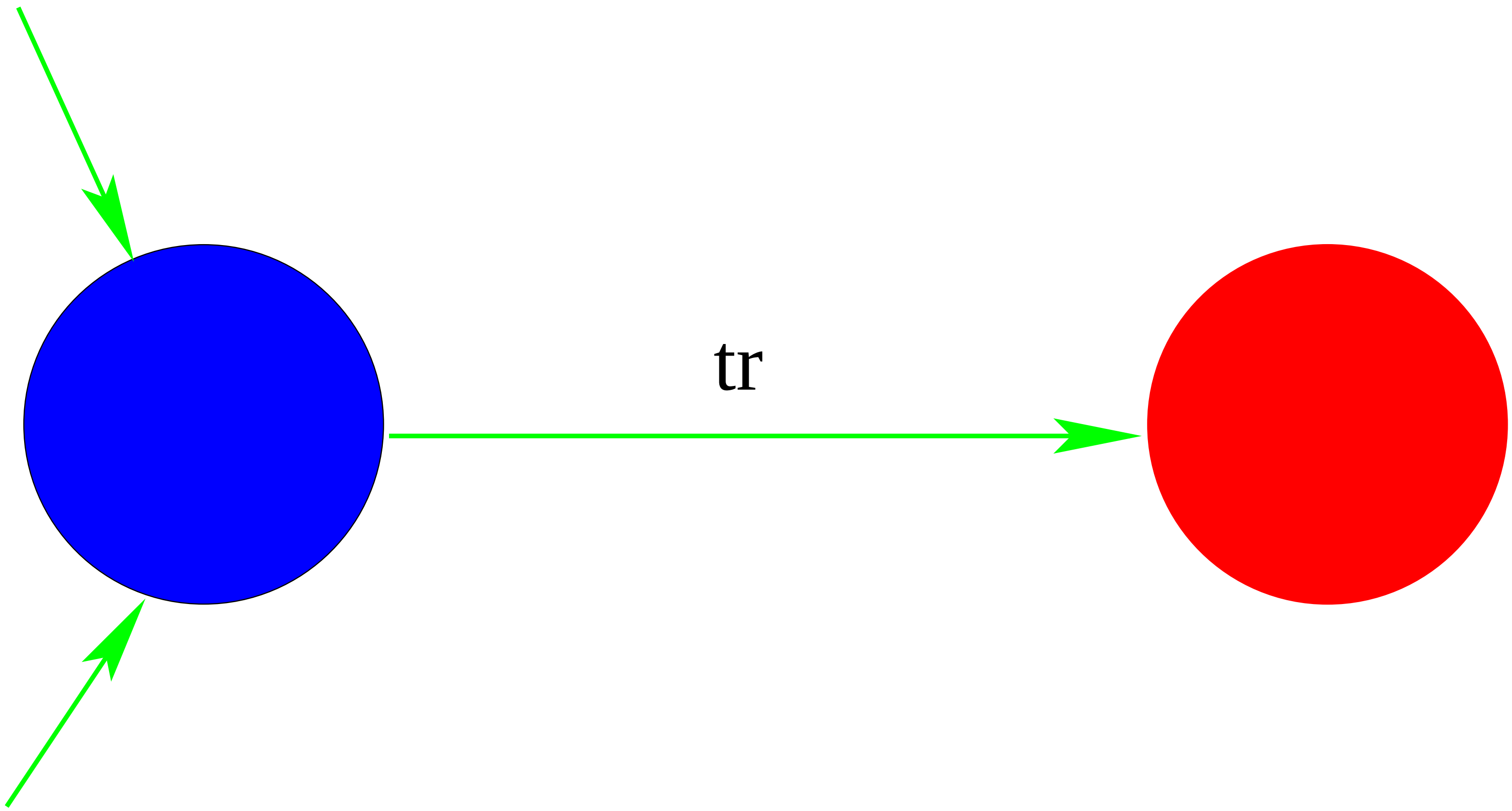
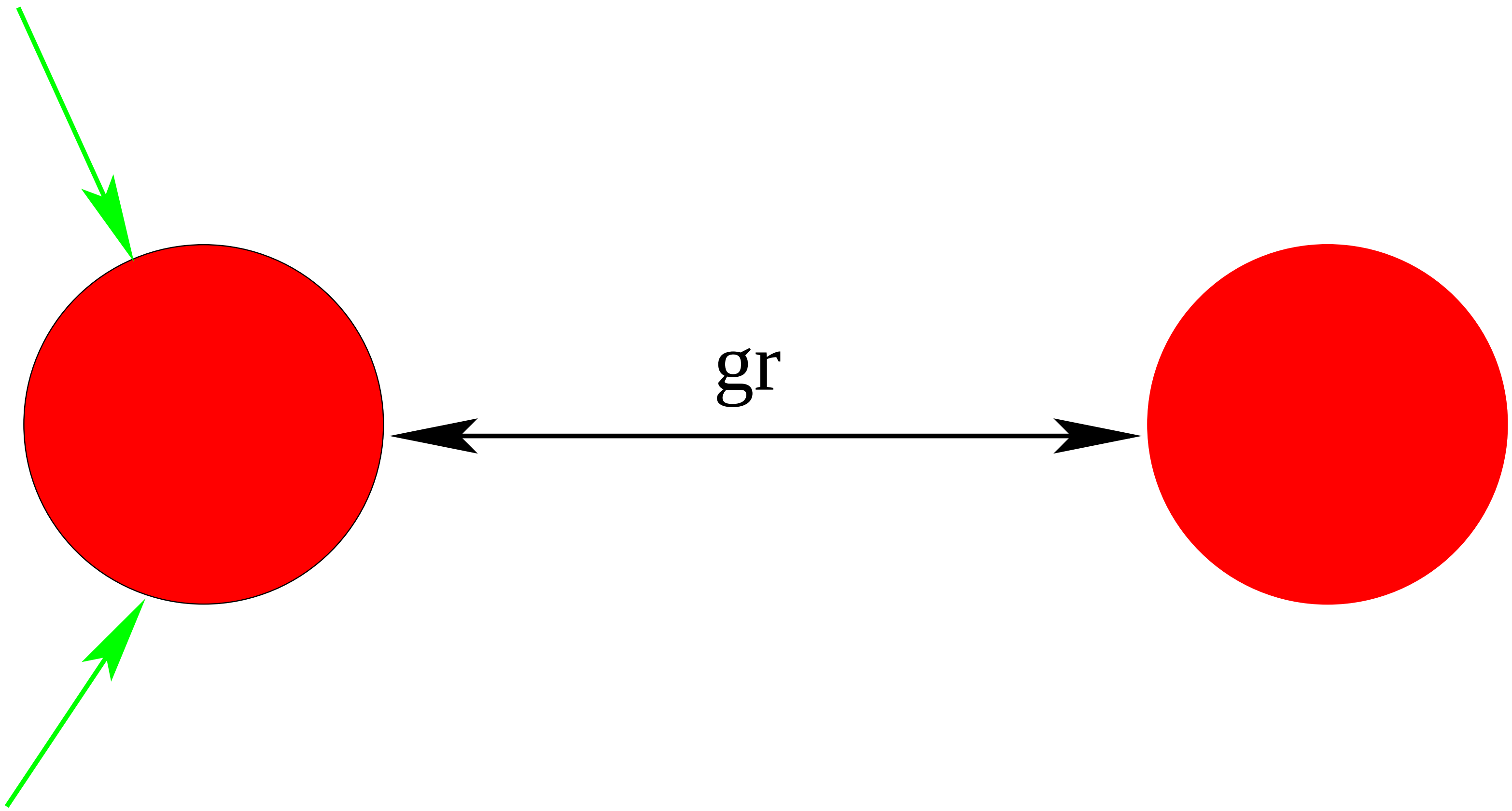
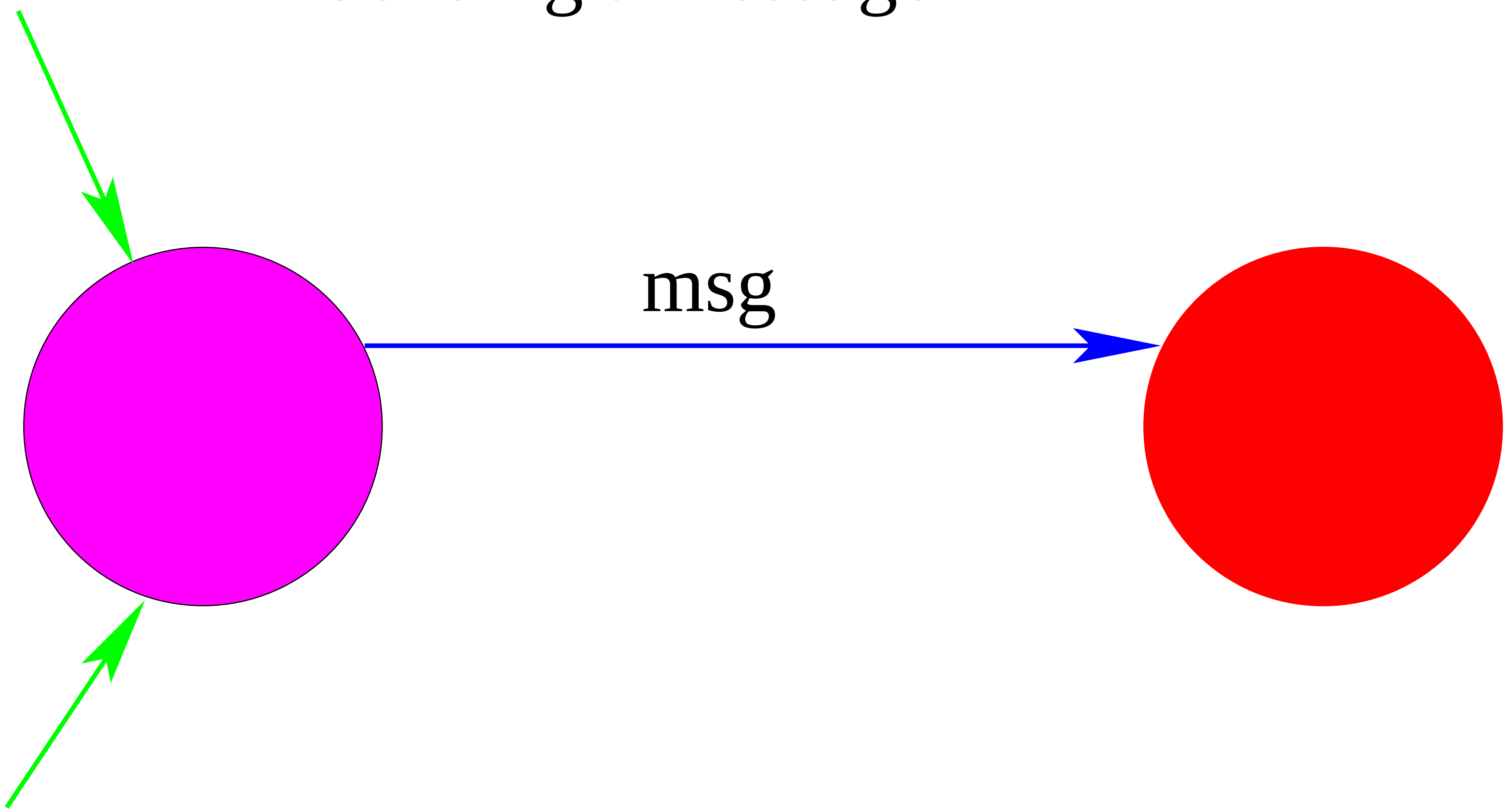
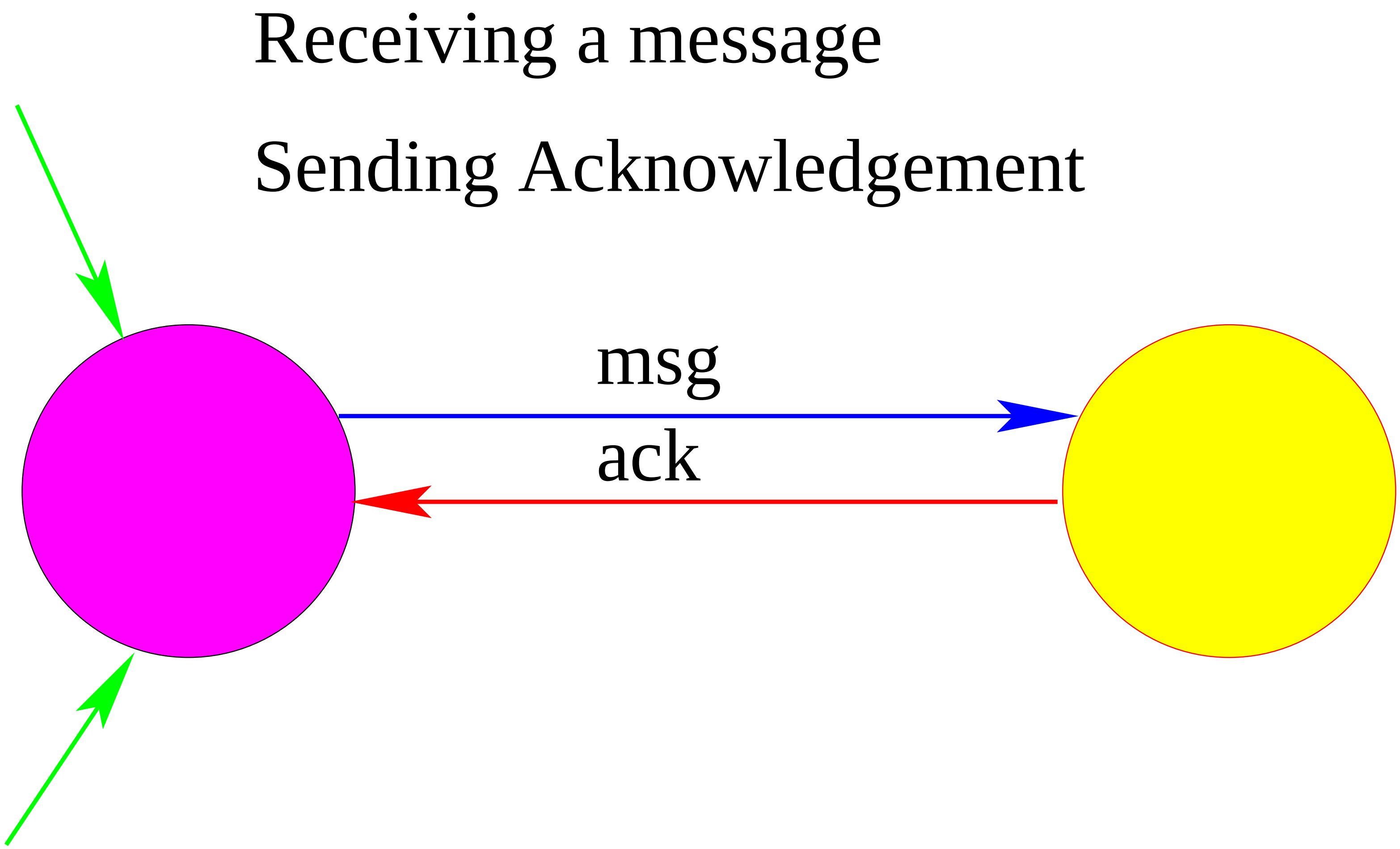
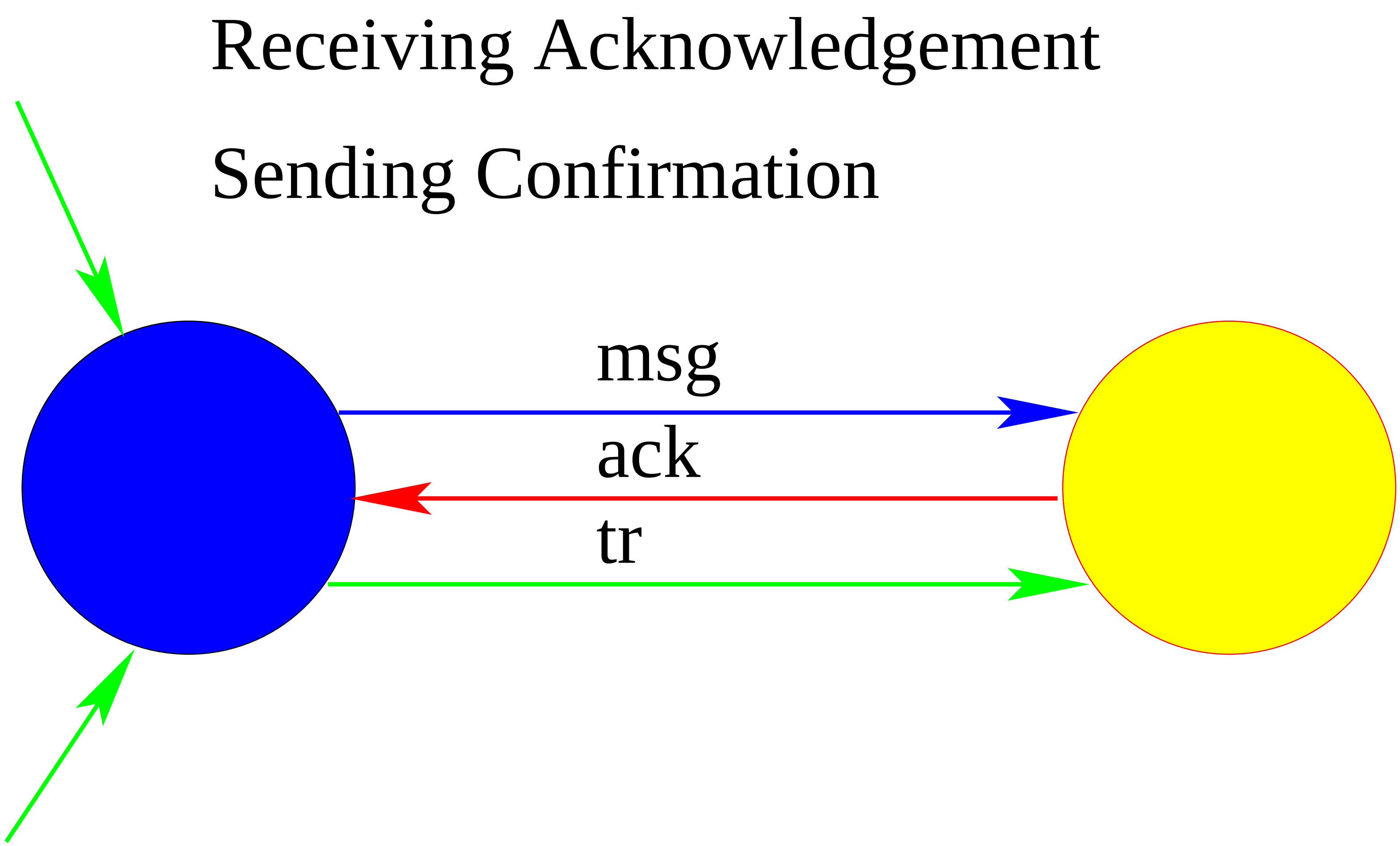

tr


gr

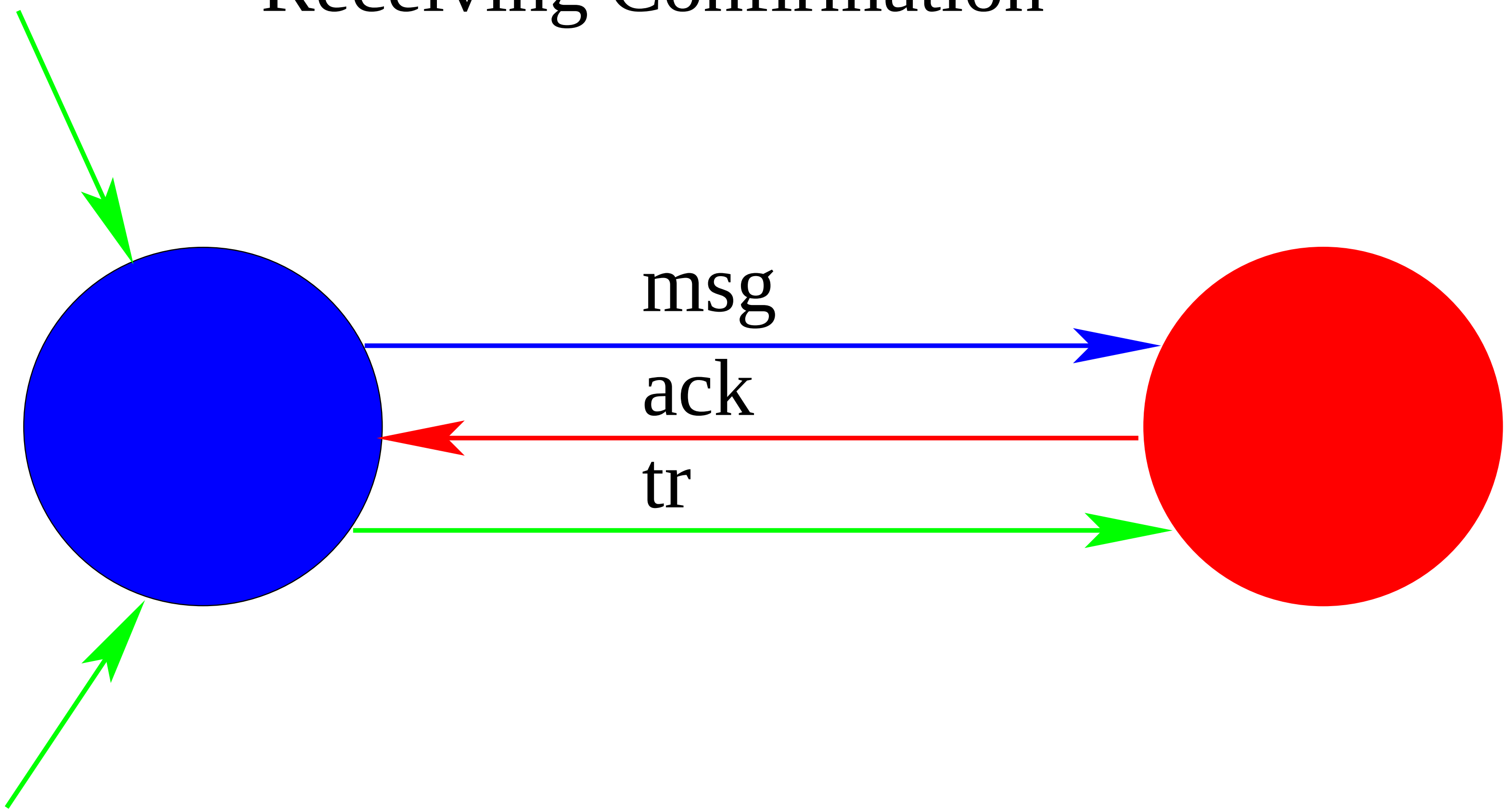
# Sending a message




Receiving a message
Sending Acknowledgement
msg
ack


Receiving Acknowledgement
Sending Confirmation
msg
ack
tr

# Receiving Confirmation



# Invariant (1)

$$msg \in ND \pfun ND$$

$$ack \in ND \pfun ND$$

$$tr \;\subseteq\; ack \;\subseteq\; msg \;\subseteq\; gr$$

# Node $x$ sends a message to node $y$

$$
\begin{array}{l}
\text{send\_msg} \quad \widehat{=} \\
\quad \textbf{ANY } x, y \textbf{ WHERE} \\
\quad\quad x, y \in gr \ \wedge \\
\quad\quad x \notin \mathrm{dom}\,(tr) \ \wedge \\
\quad\quad y, x \notin tr \ \wedge \\
\quad\quad gr[\{x\}] = tr^{-1}[\{x\}] \cup \{y\} \ \wedge \\
\quad\quad y, x \notin ack \ \wedge \\
\quad\quad x \notin \mathrm{dom}\,(msg) \\
\quad \textbf{THEN} \\
\quad\quad msg := msg \cup \{x \mapsto y\} \\
\quad \textbf{END}
\end{array}
$$

## Node $y$ sends an acknowledgement to node $x$

send_ack $\mathrel{\widehat{=}}$

**ANY** $x, y$ **WHERE**

$x, y \in msg - ack \ \wedge$

$y \notin \mathrm{dom}\,(msg)$

**THEN**

$ack := ack \cup \{x \mapsto y\}$

**END**

## Node $x$ sends a confirmation to node $y$

progress $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in ack \;\wedge$

$x \notin \mathrm{dom}\,(tr)$

**THEN**

$tr := tr \cup \{x \mapsto y\}$

**END**

# Invariant (2)

$$\forall\,(x,y)\cdot\left(\begin{array}{l}\quad x,y \in msg{-}ack \\ \Rightarrow \\ \quad x,y \in gr \quad\quad \land \\ \quad x \notin \mathrm{dom}\,(tr) \;\land\; y \notin \mathrm{dom}\,(tr) \;\land \\ \quad gr[\{x\}] \;=\; tr^{-1}[\{x\}] \cup \{y\}\end{array}\right)$$

$$\forall\,(x,y)\cdot\left(\begin{array}{l}\quad x,y \in ack \quad\quad \land \\ \quad x \notin \mathrm{dom}\,(tr) \\ \Rightarrow \\ \quad x,y \in gr \quad\quad \land \\ \quad y \notin \mathrm{dom}\,(tr) \;\land \\ \quad gr[\{x\}] \;=\; tr^{-1}[\{x\}] \cup \{y\}\end{array}\right)$$

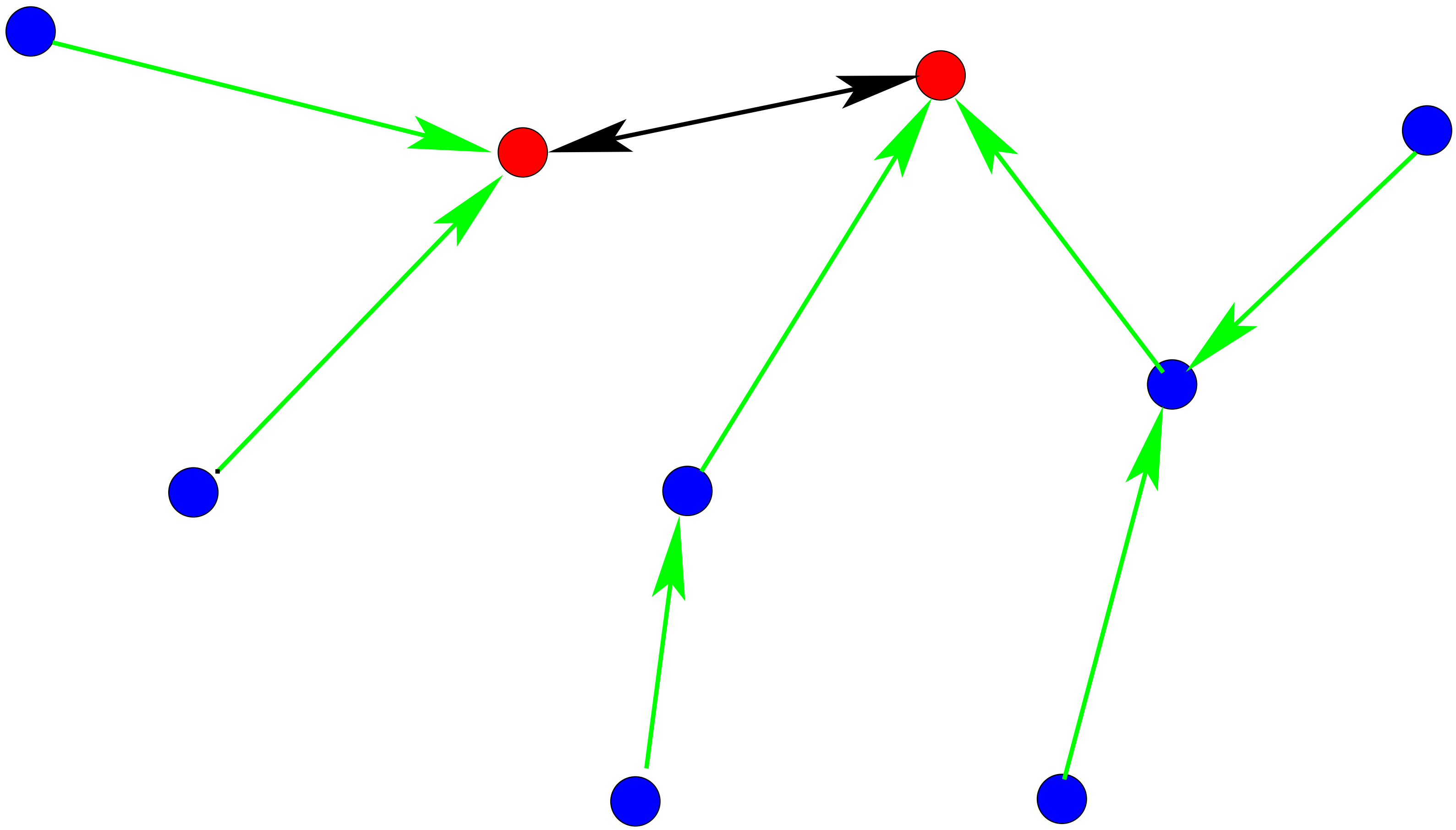
## Second Refinement: The problem of contention

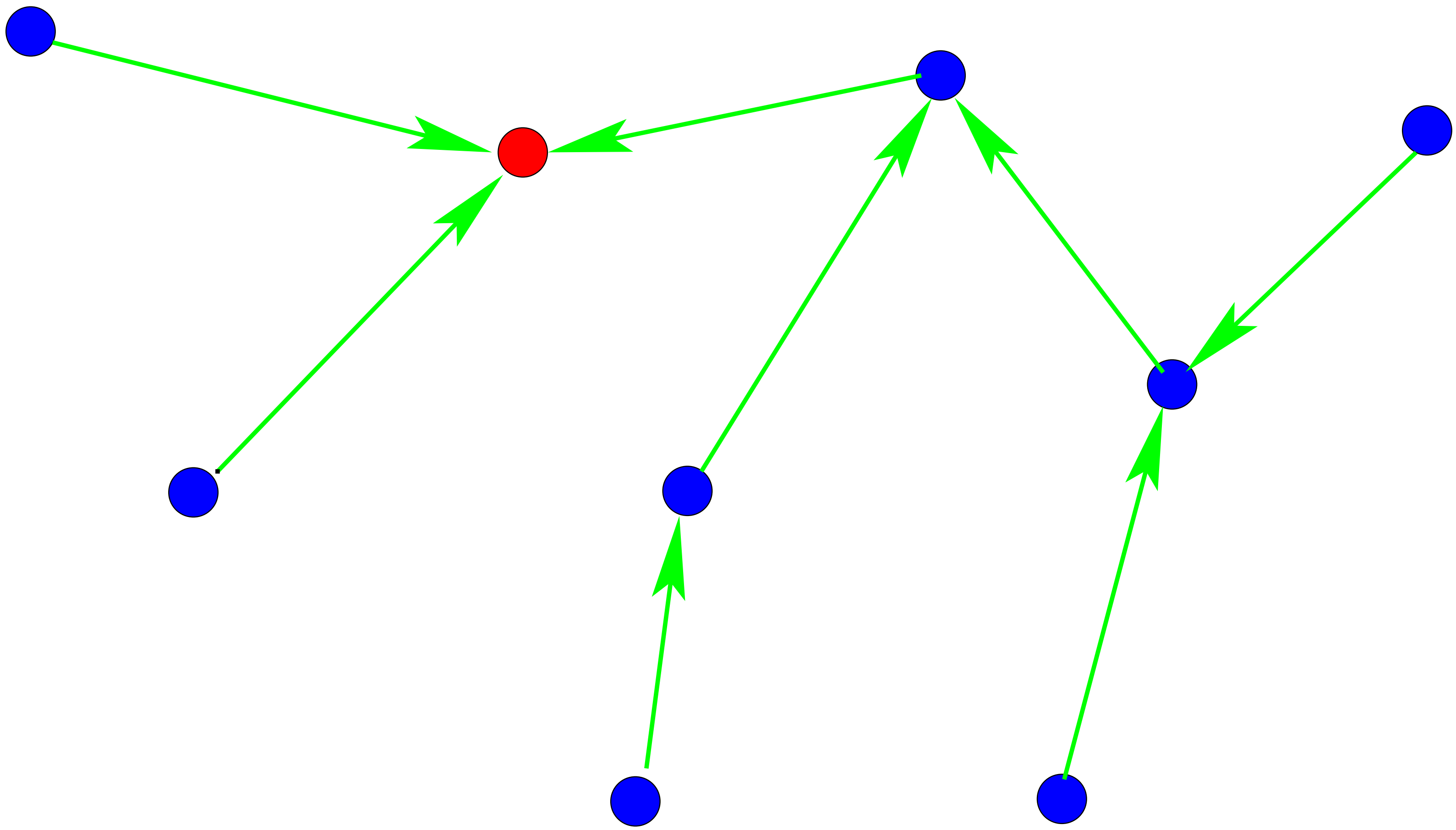
- Explaining the problem

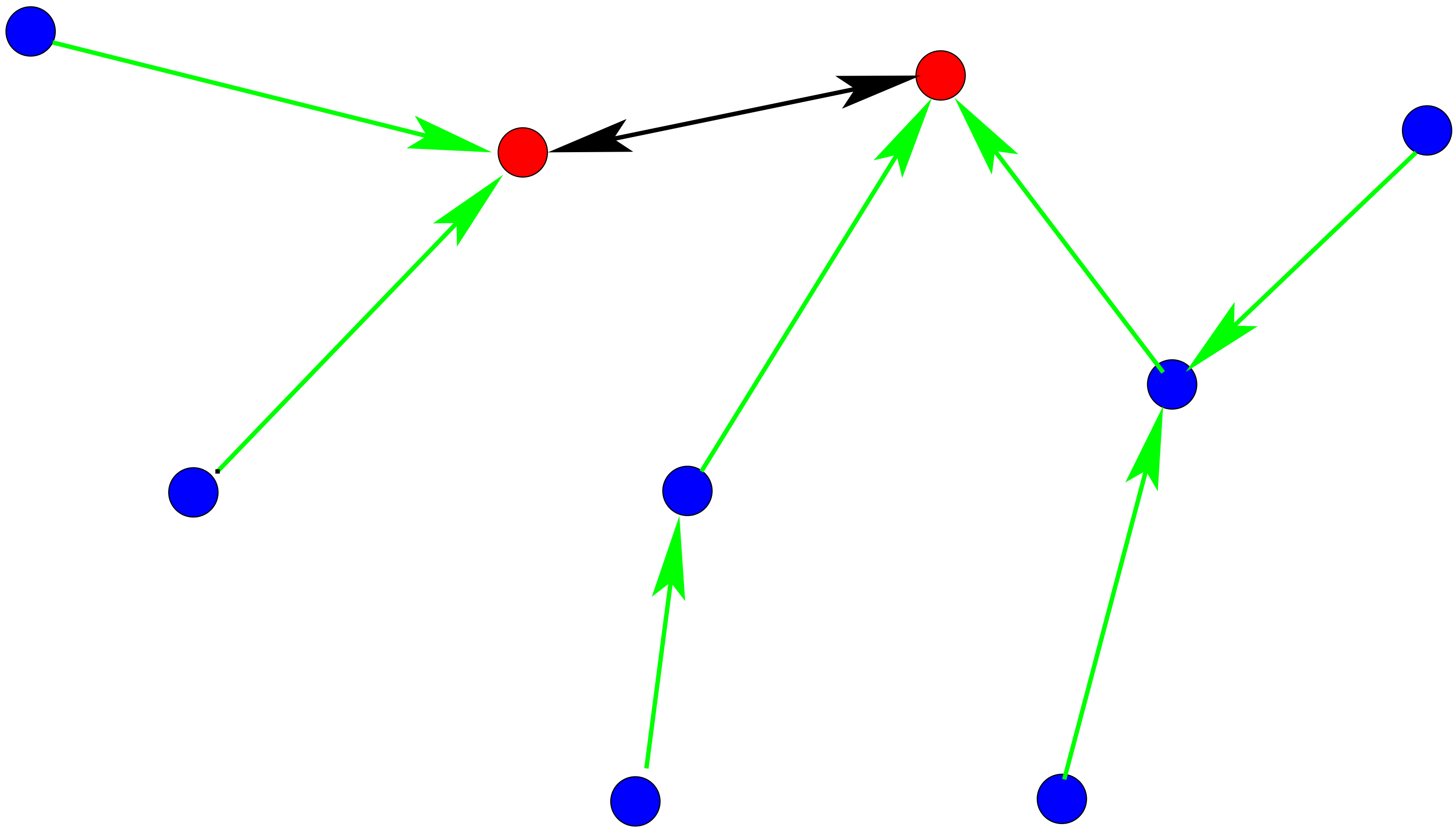
- Proposing a partial solution

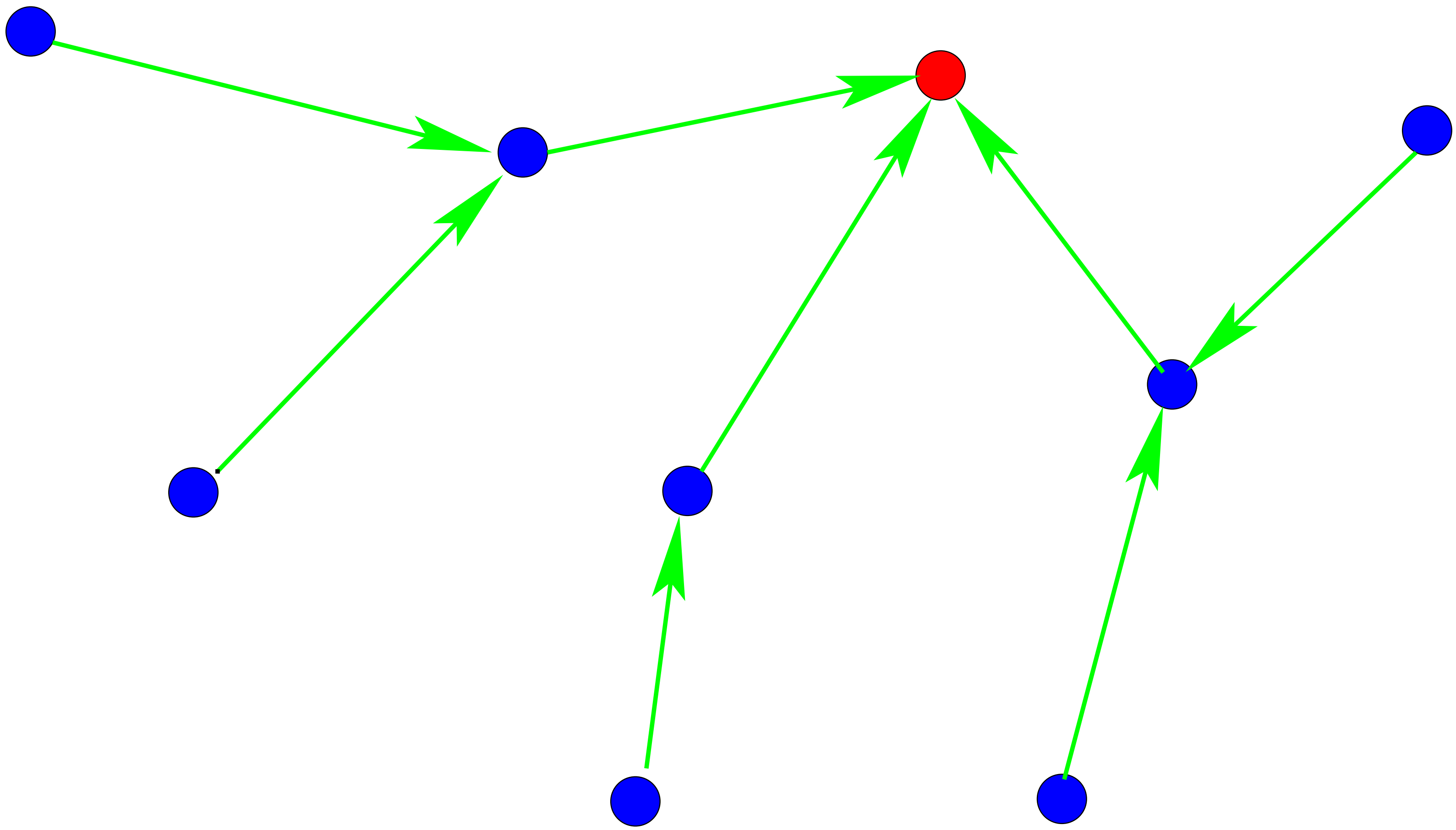
- Towards a better treatment

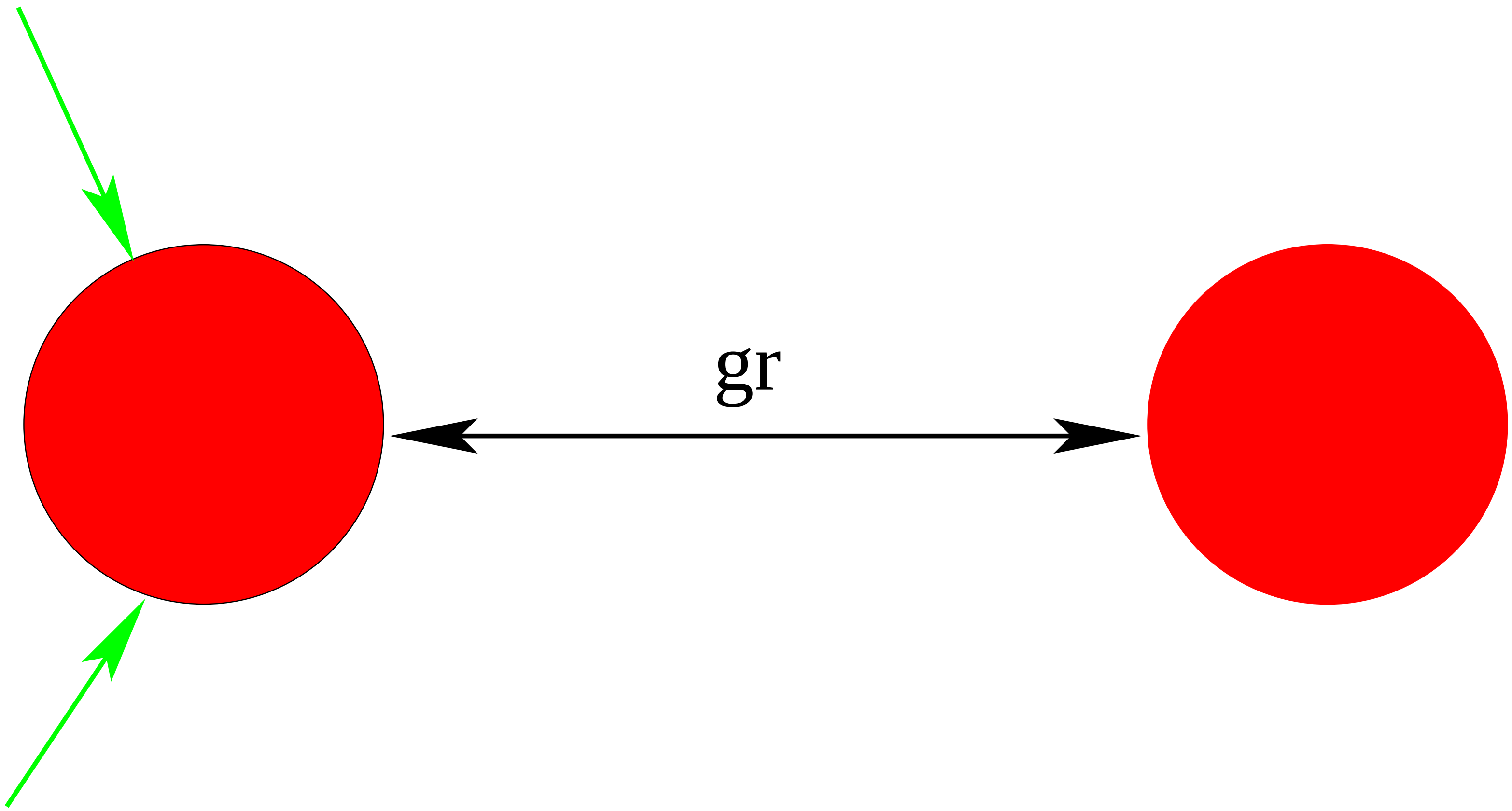
- Back to the local animation



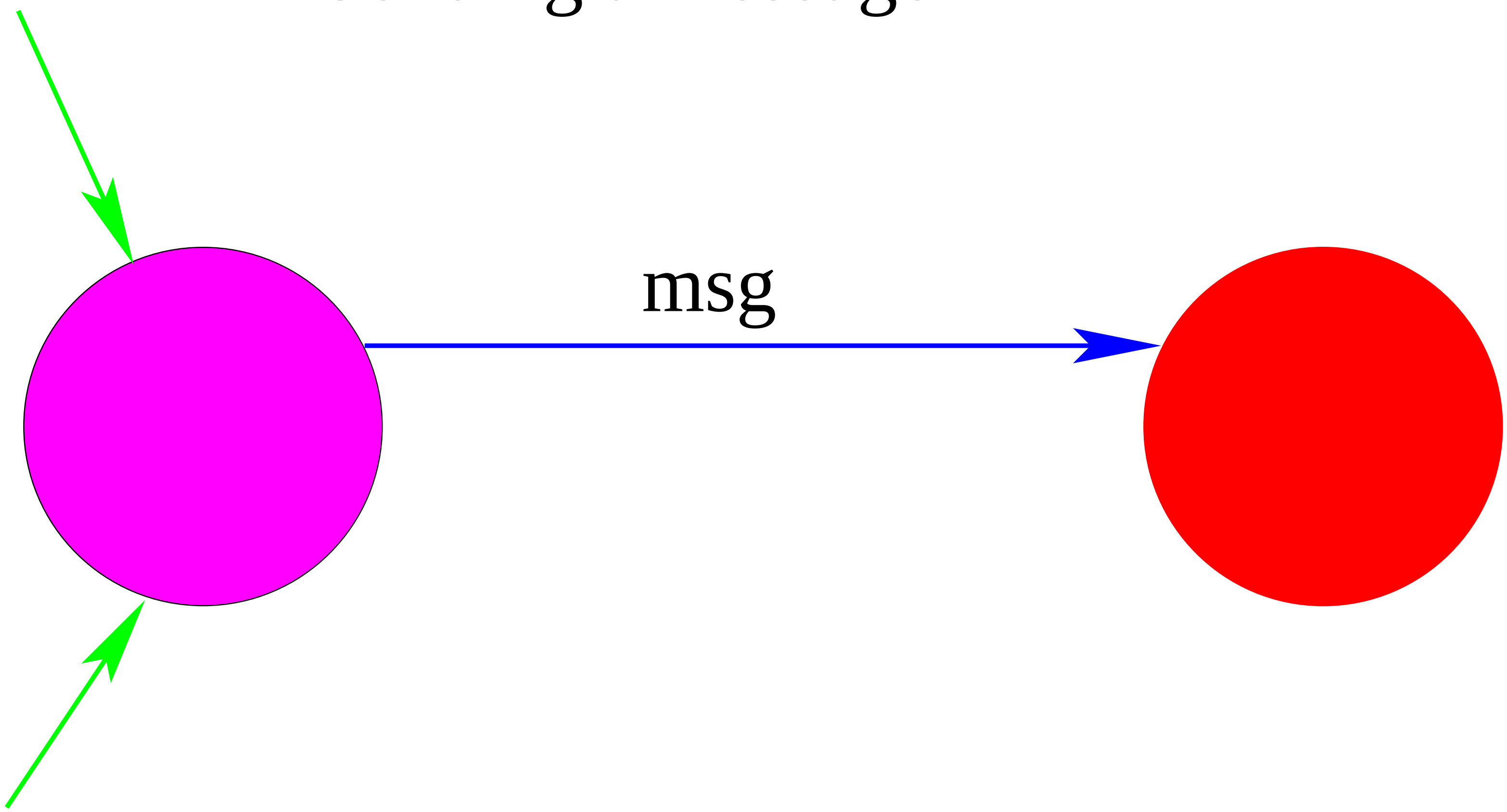




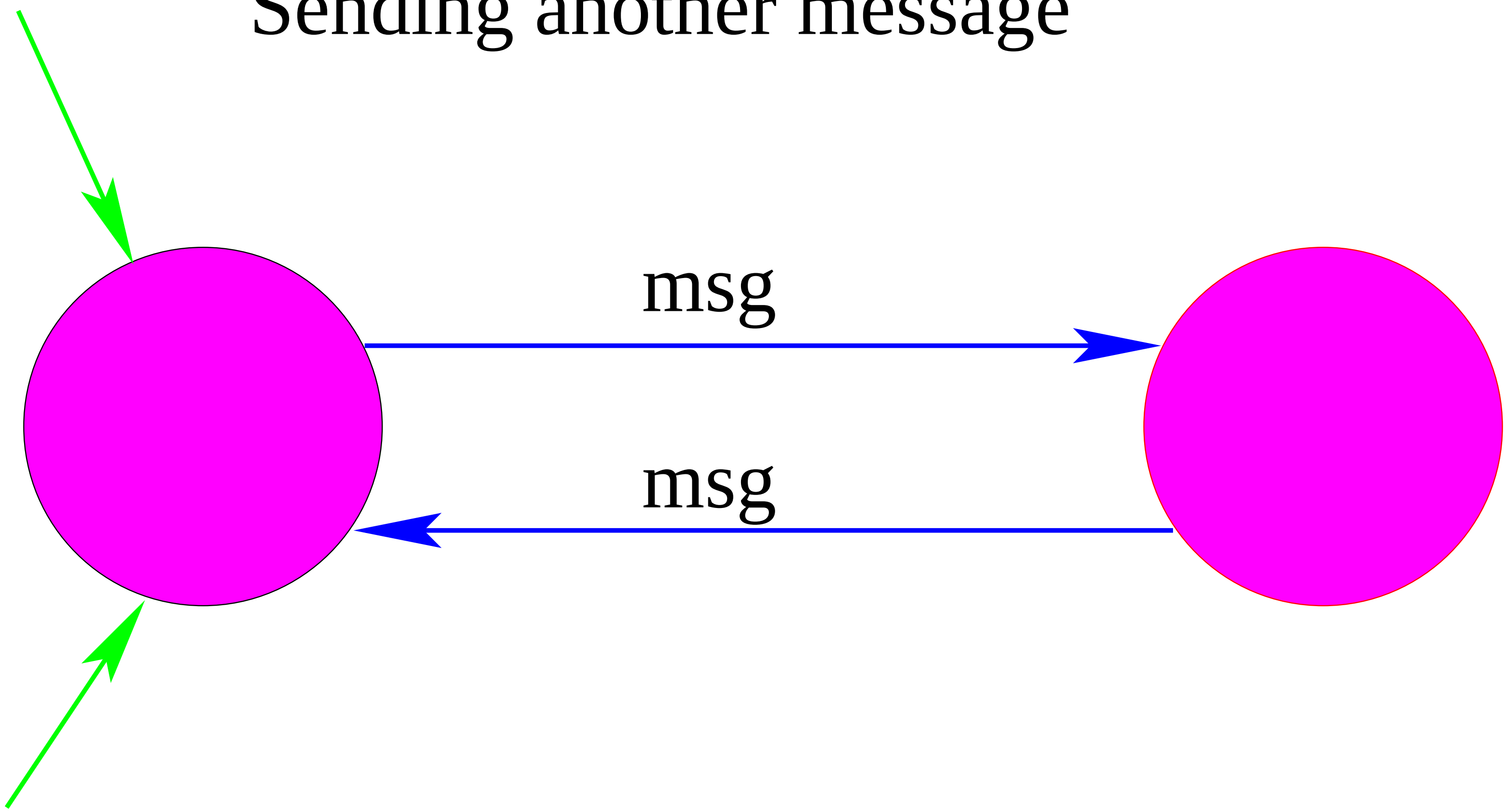



gr

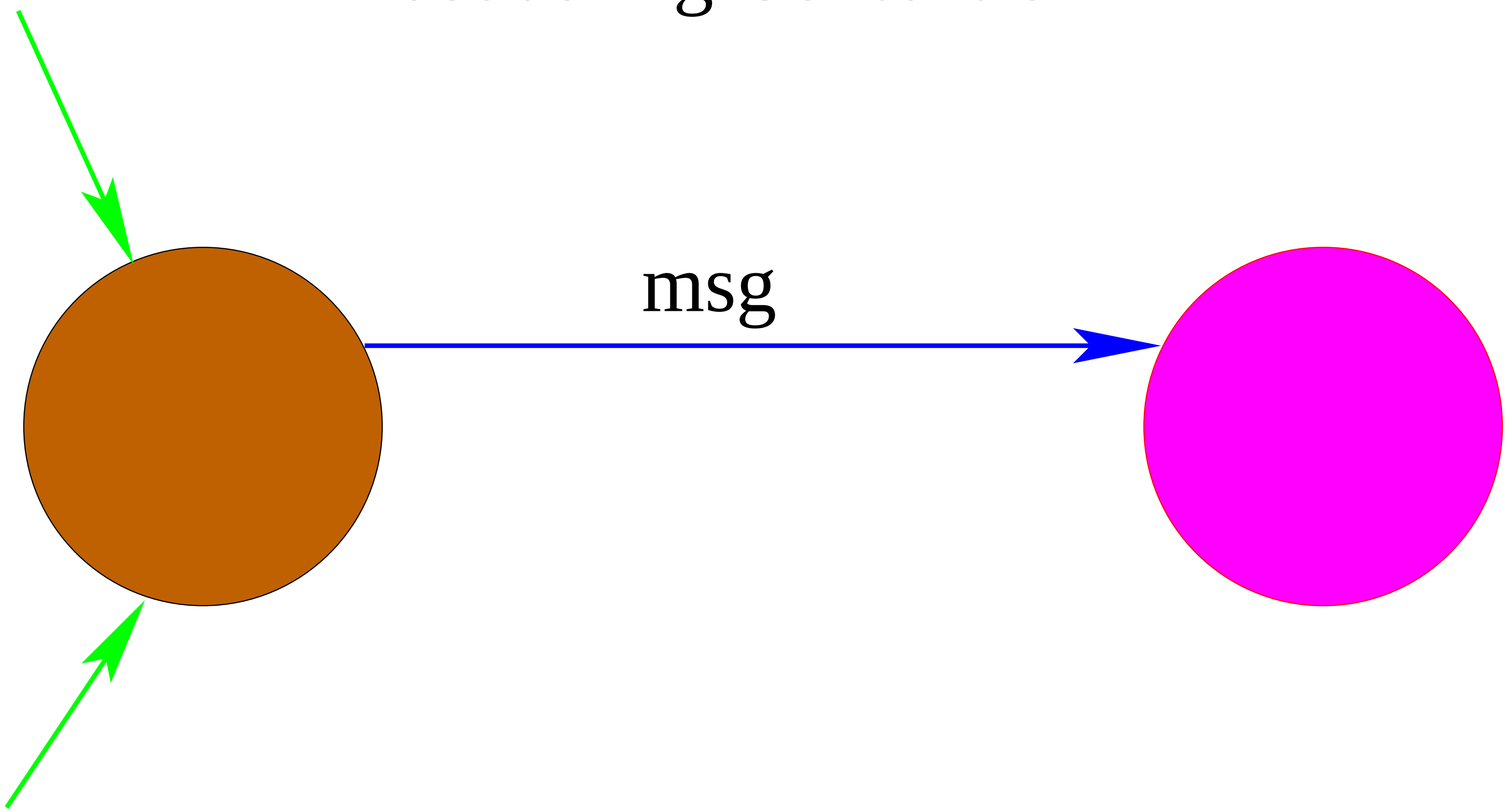
Sending a message



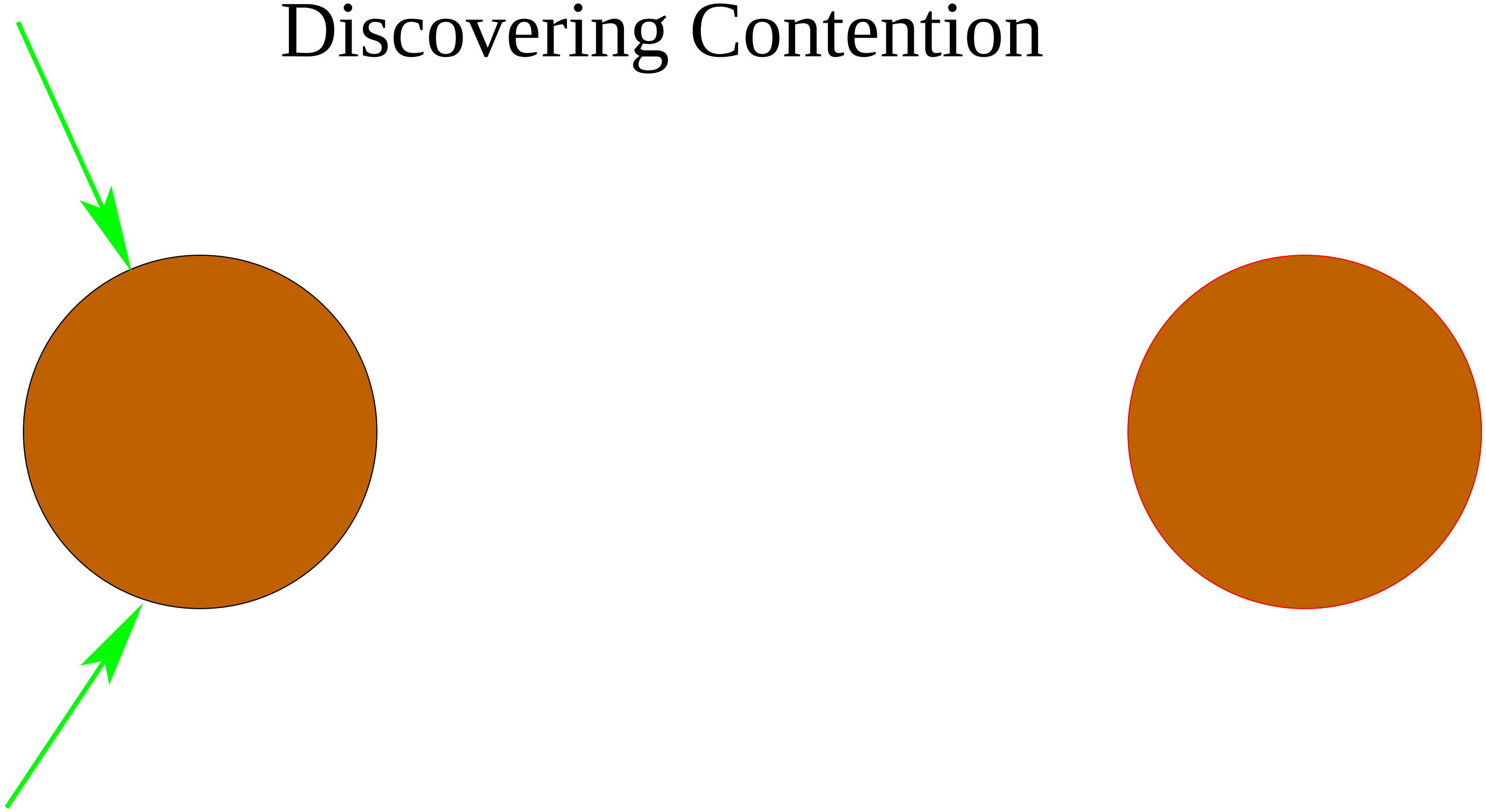
# Sending another message



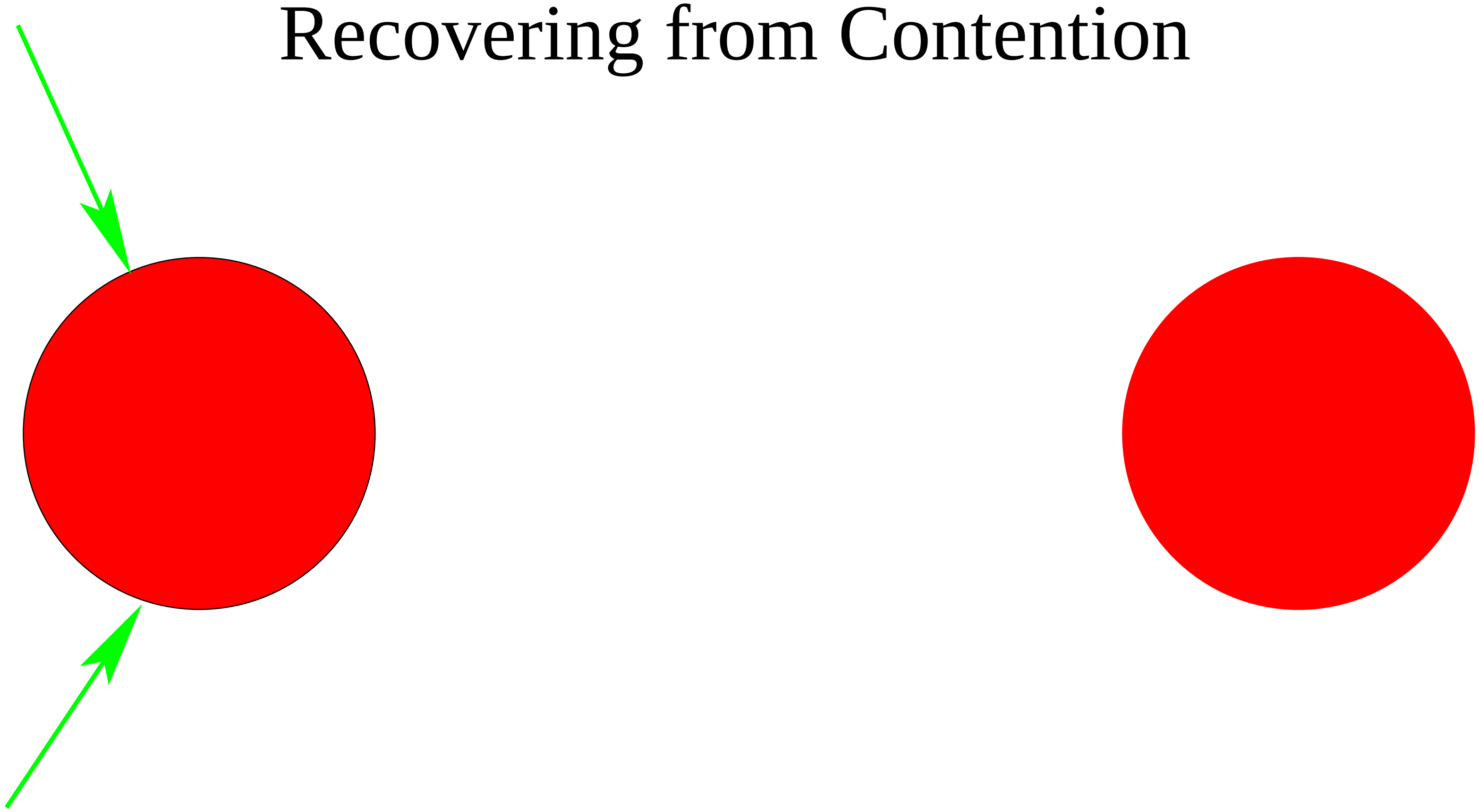
# Discovering Contention



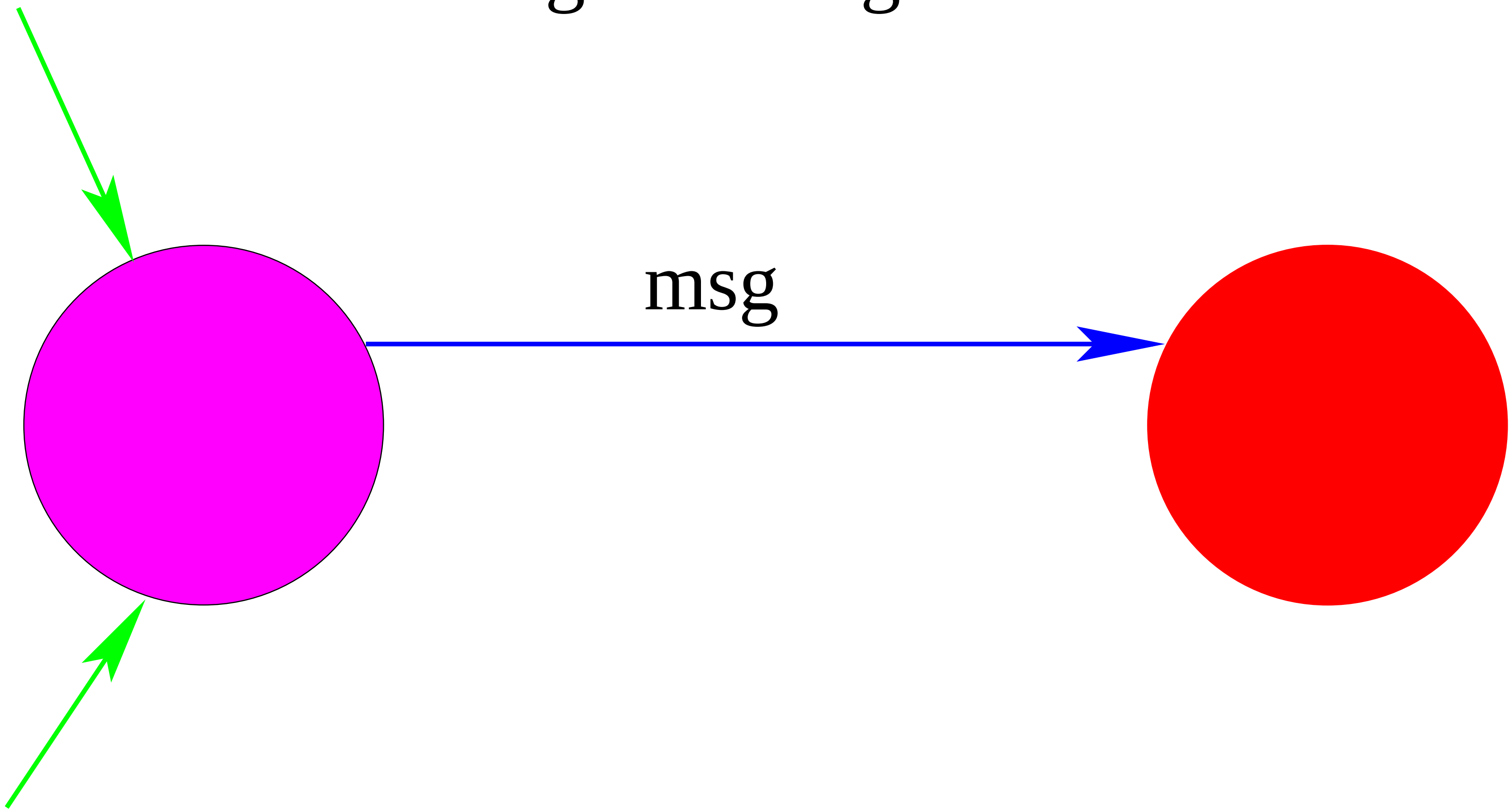
# Discovering Contention



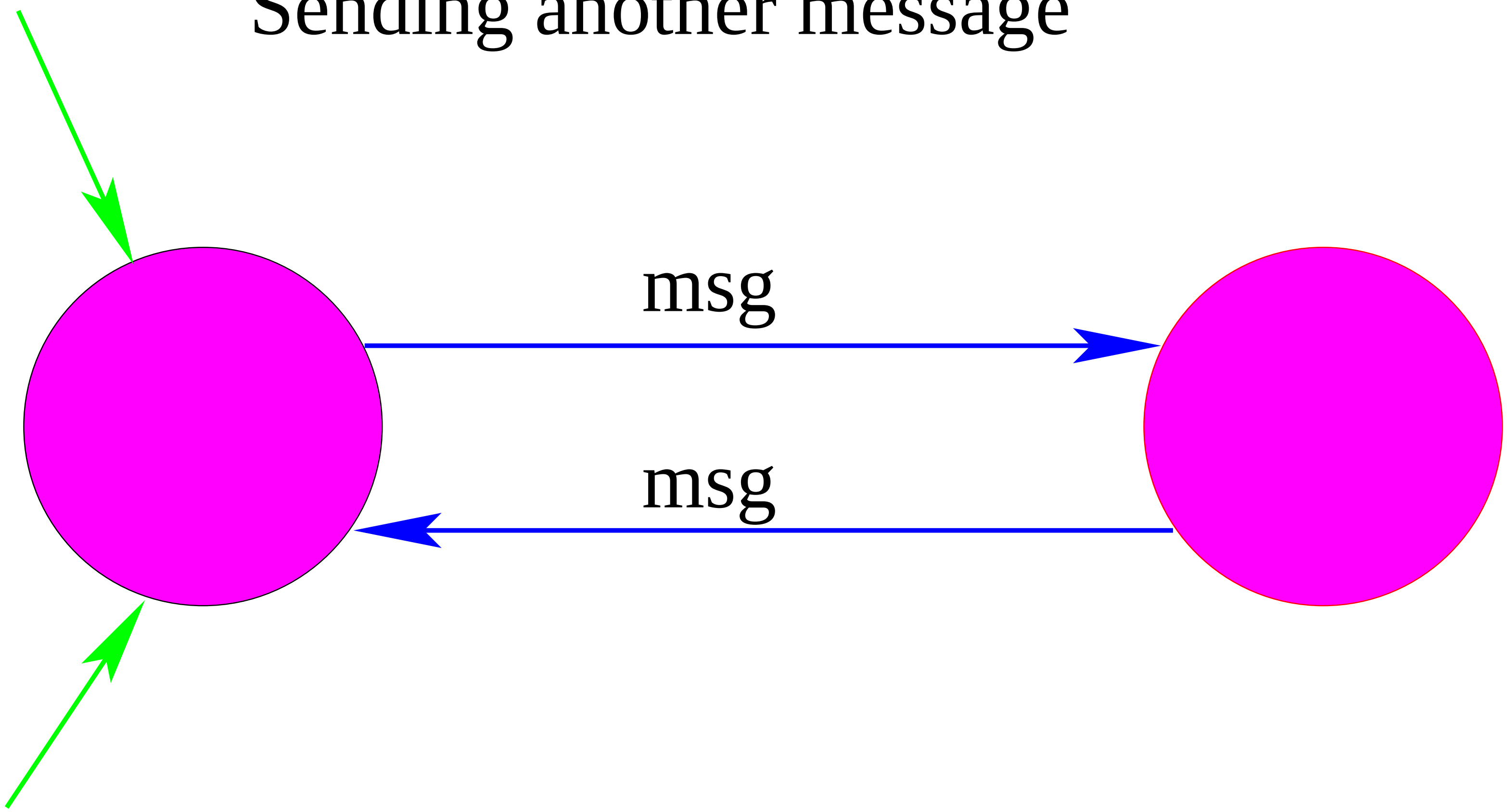
# Recovering from Contention



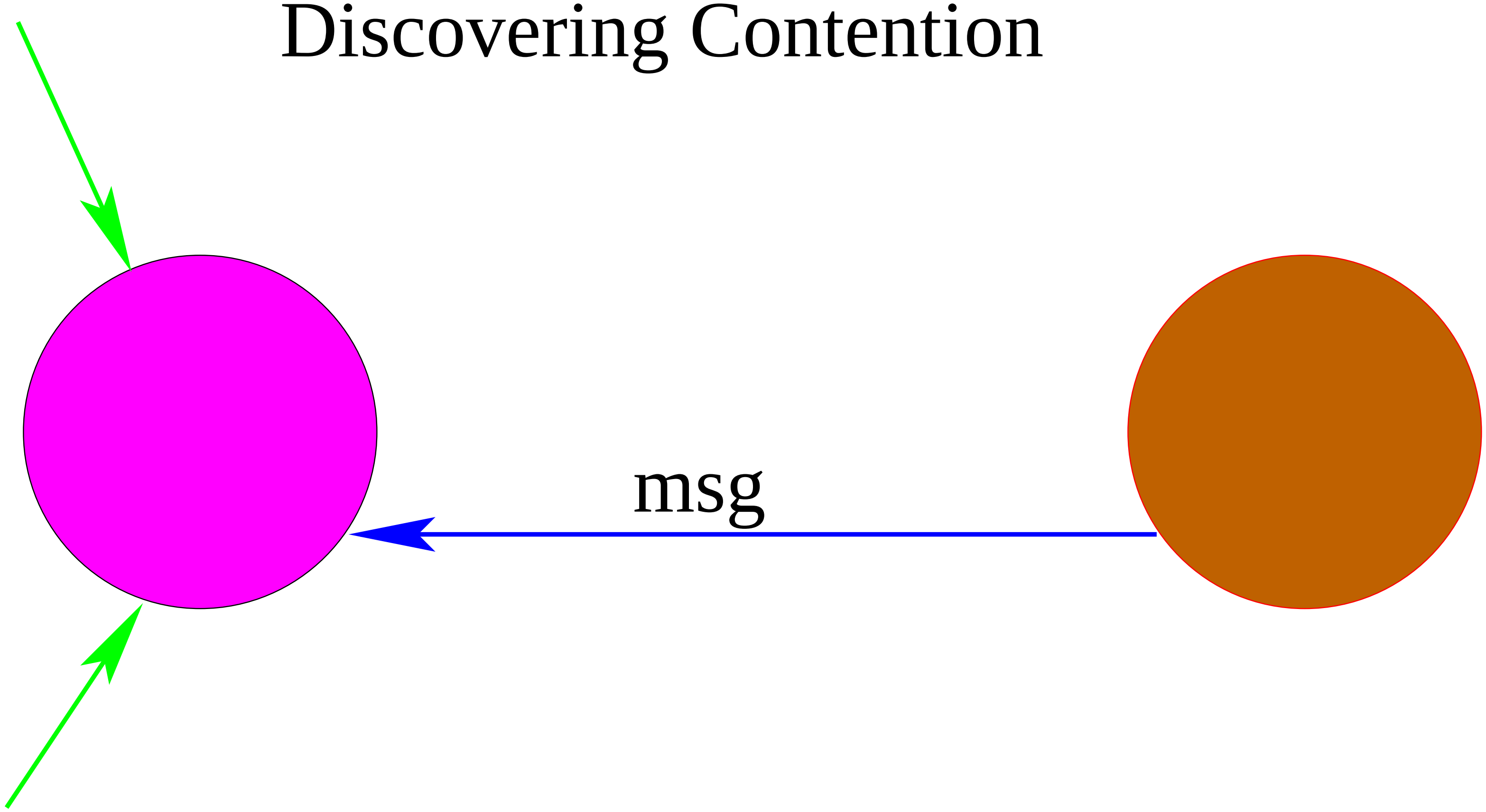
# Sending a message



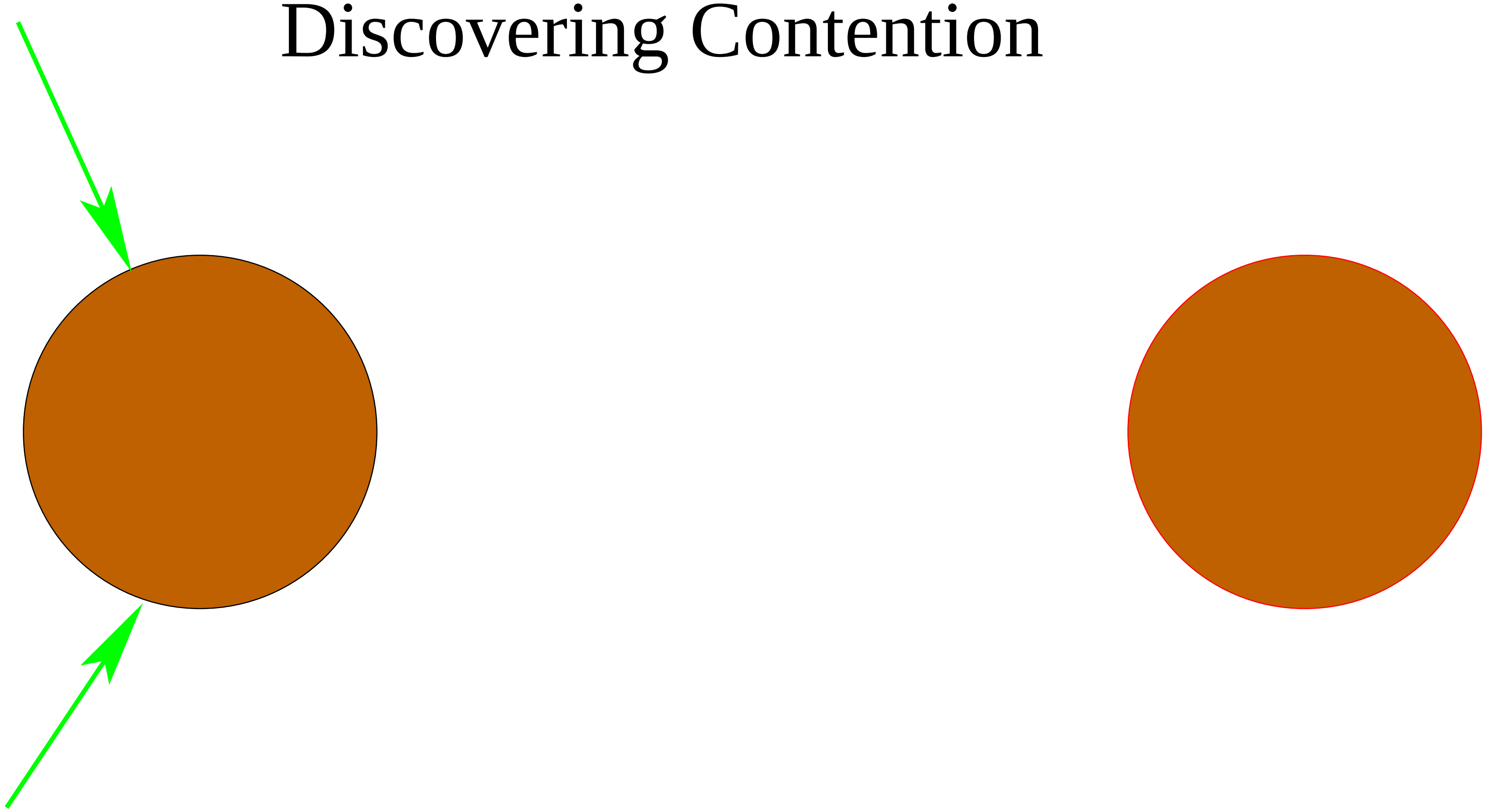
# Sending another message



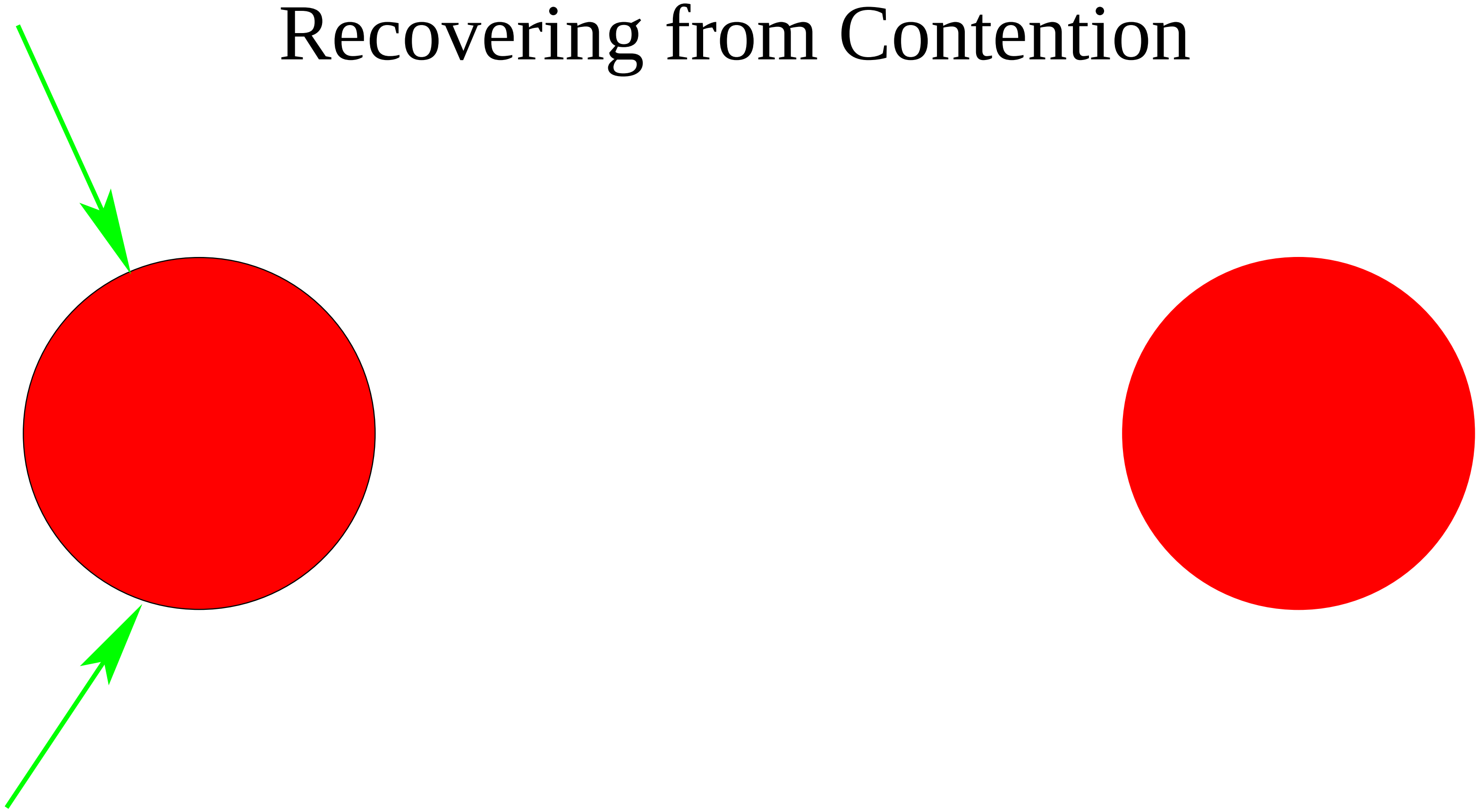
# Discovering Contention



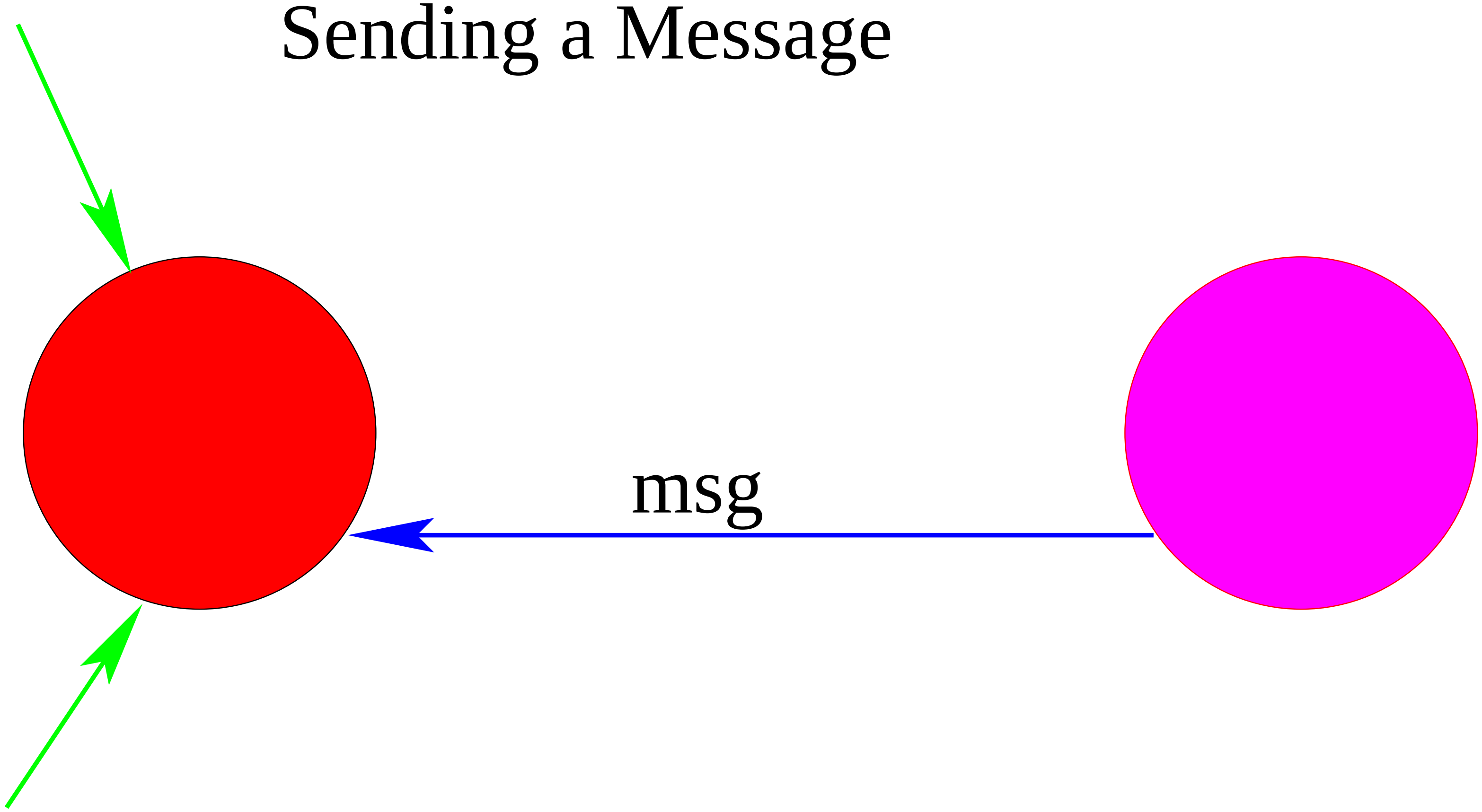
# Discovering Contention



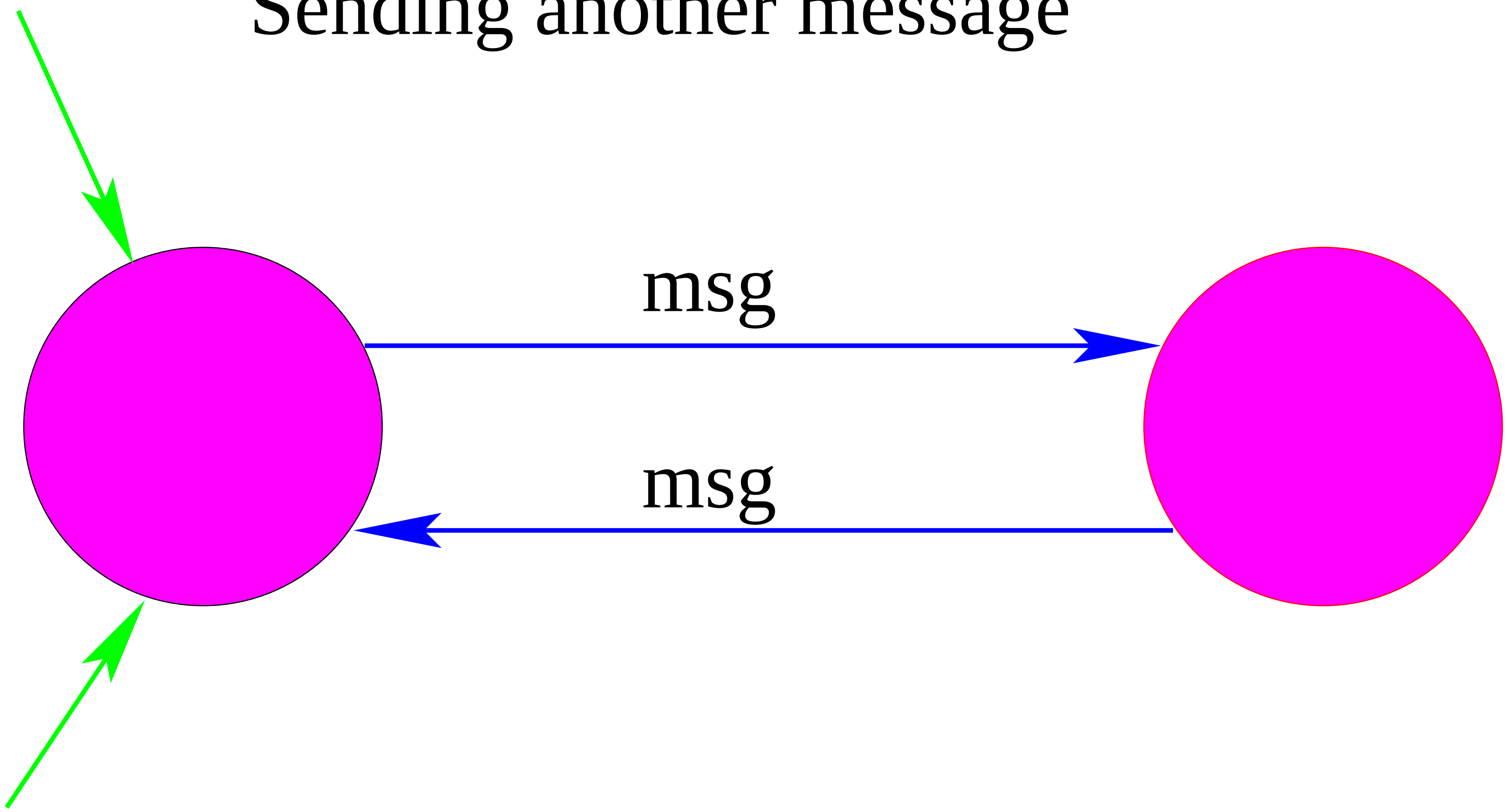
# Recovering from Contention



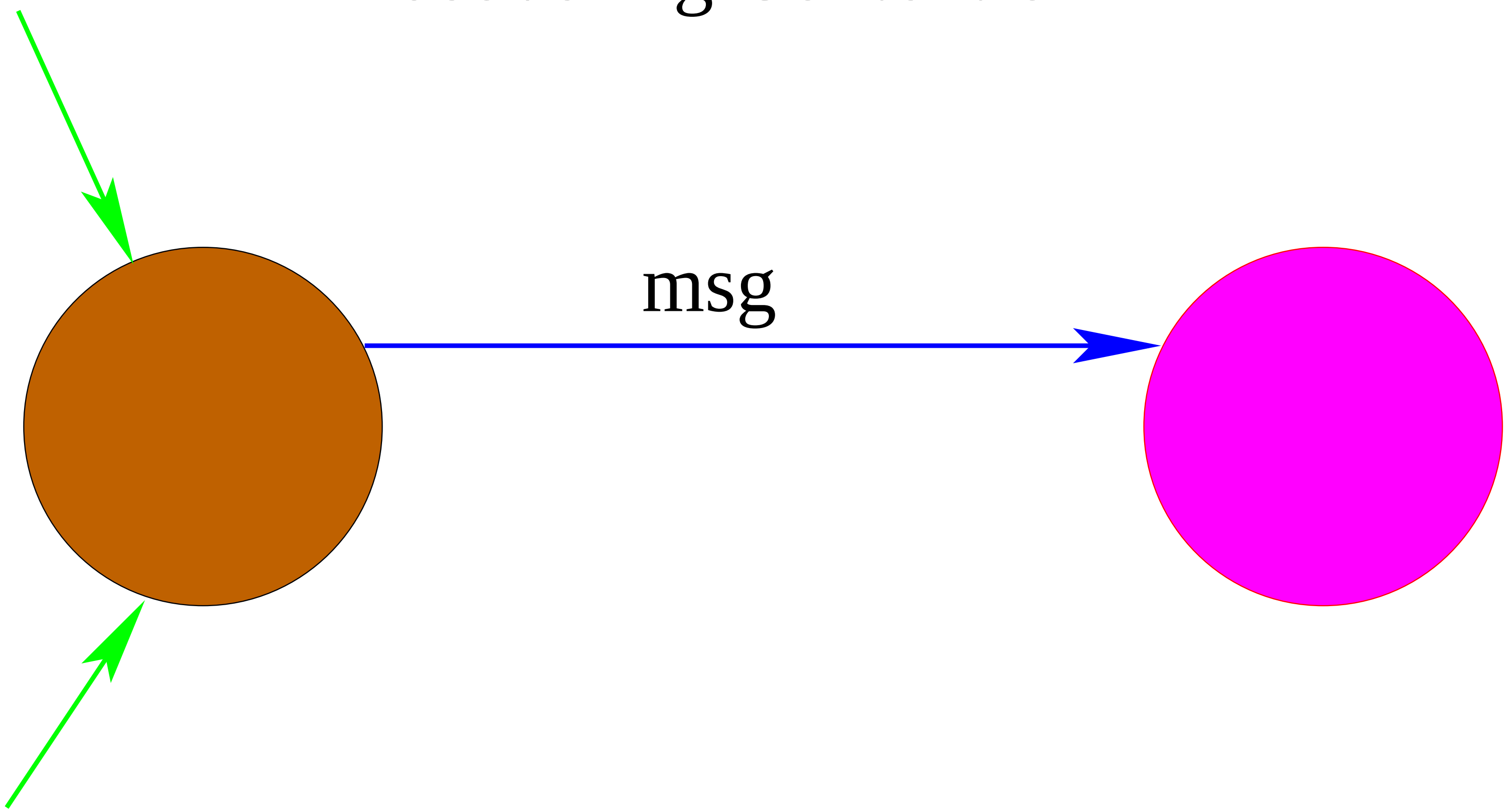
# Sending a Message



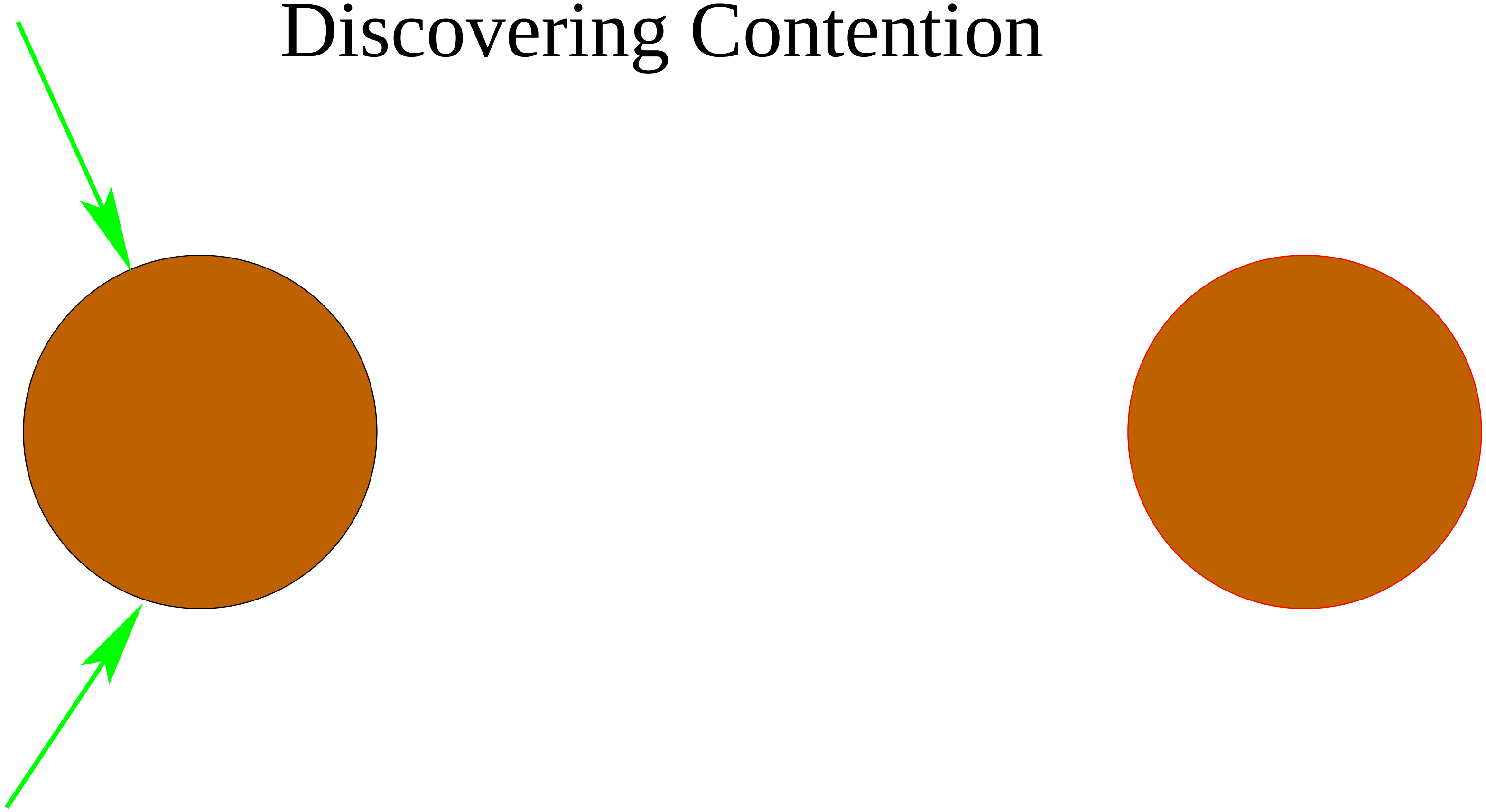
Sending another message



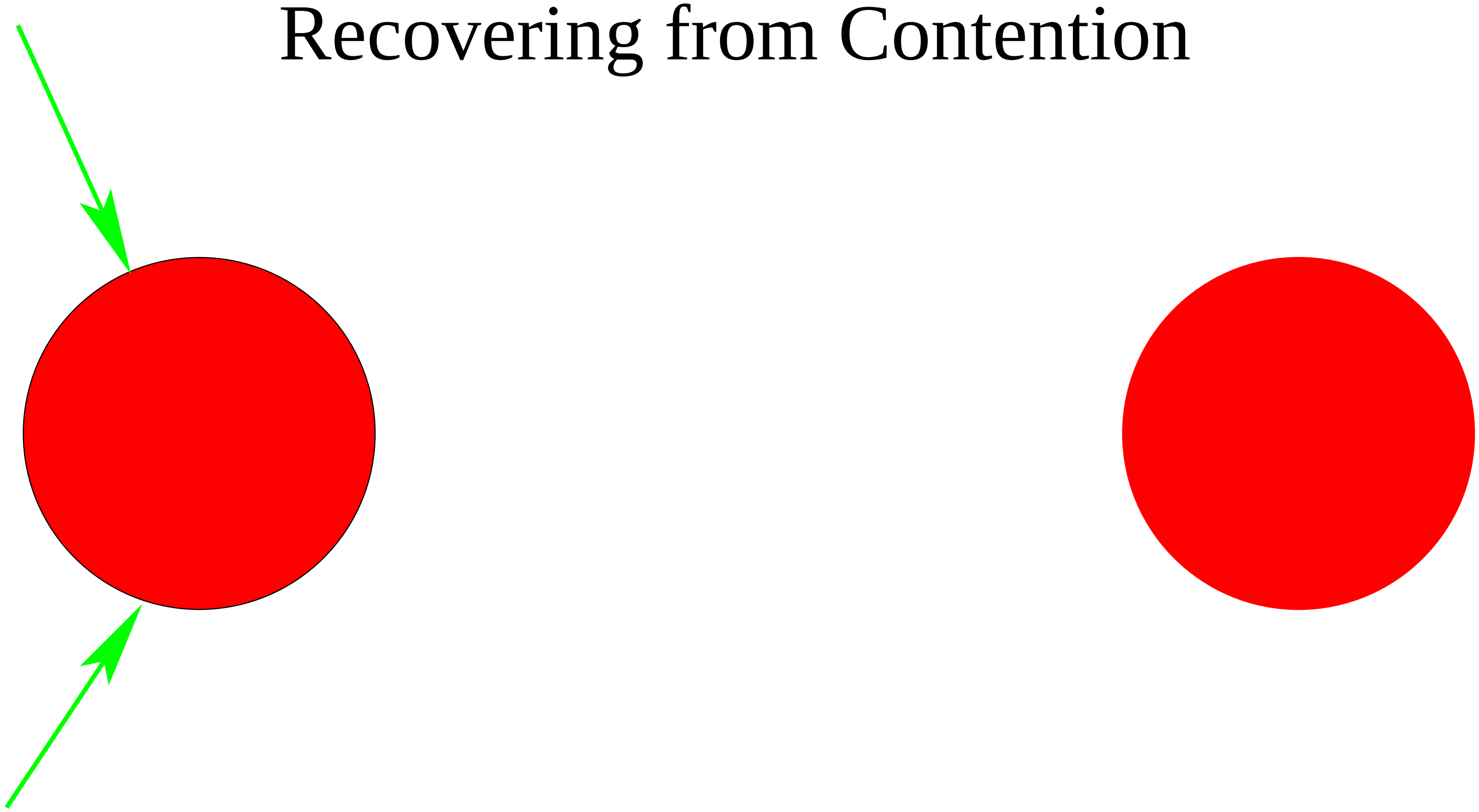
# Discovering Contention



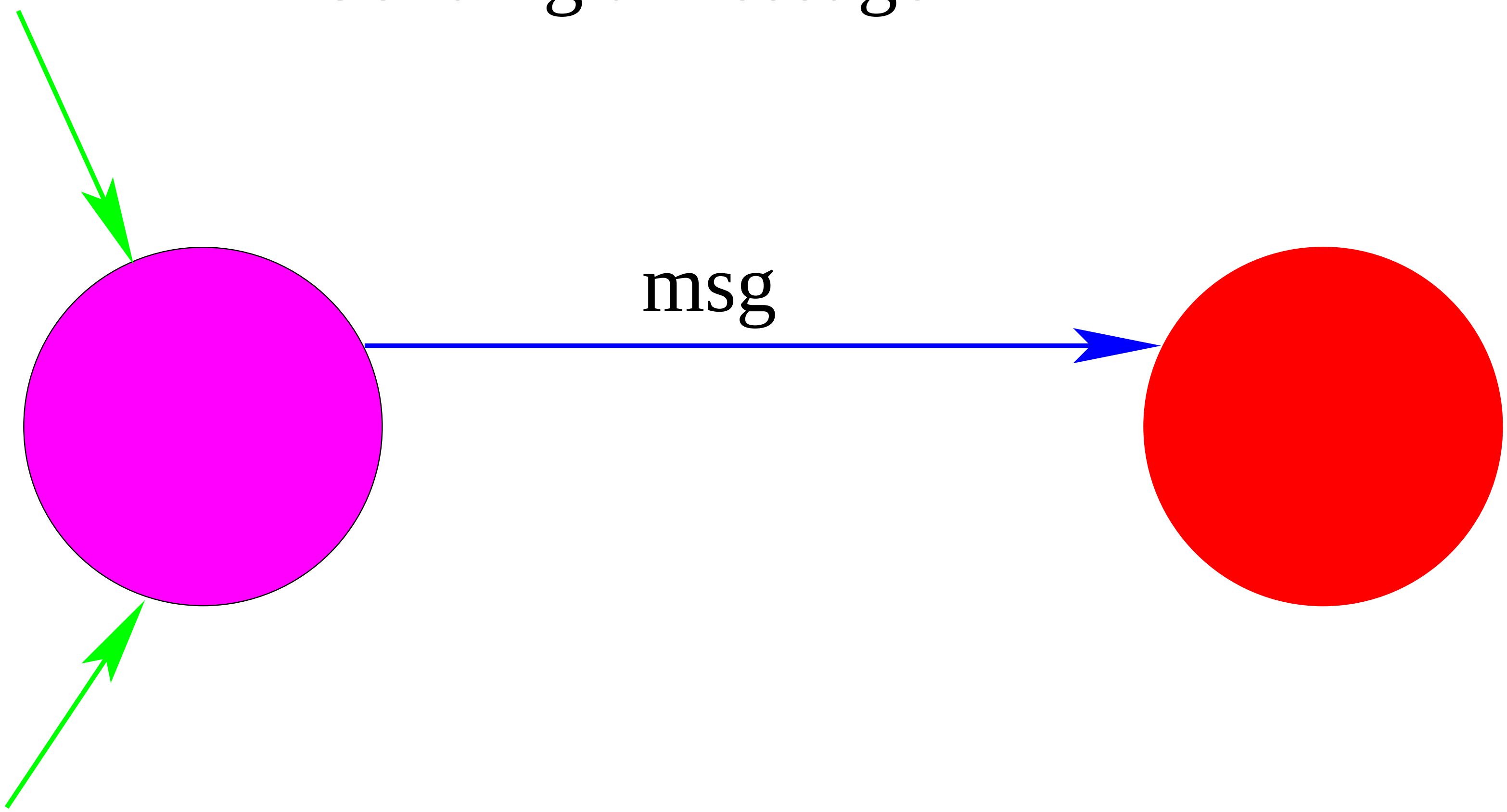
# Discovering Contention



# Recovering from Contention

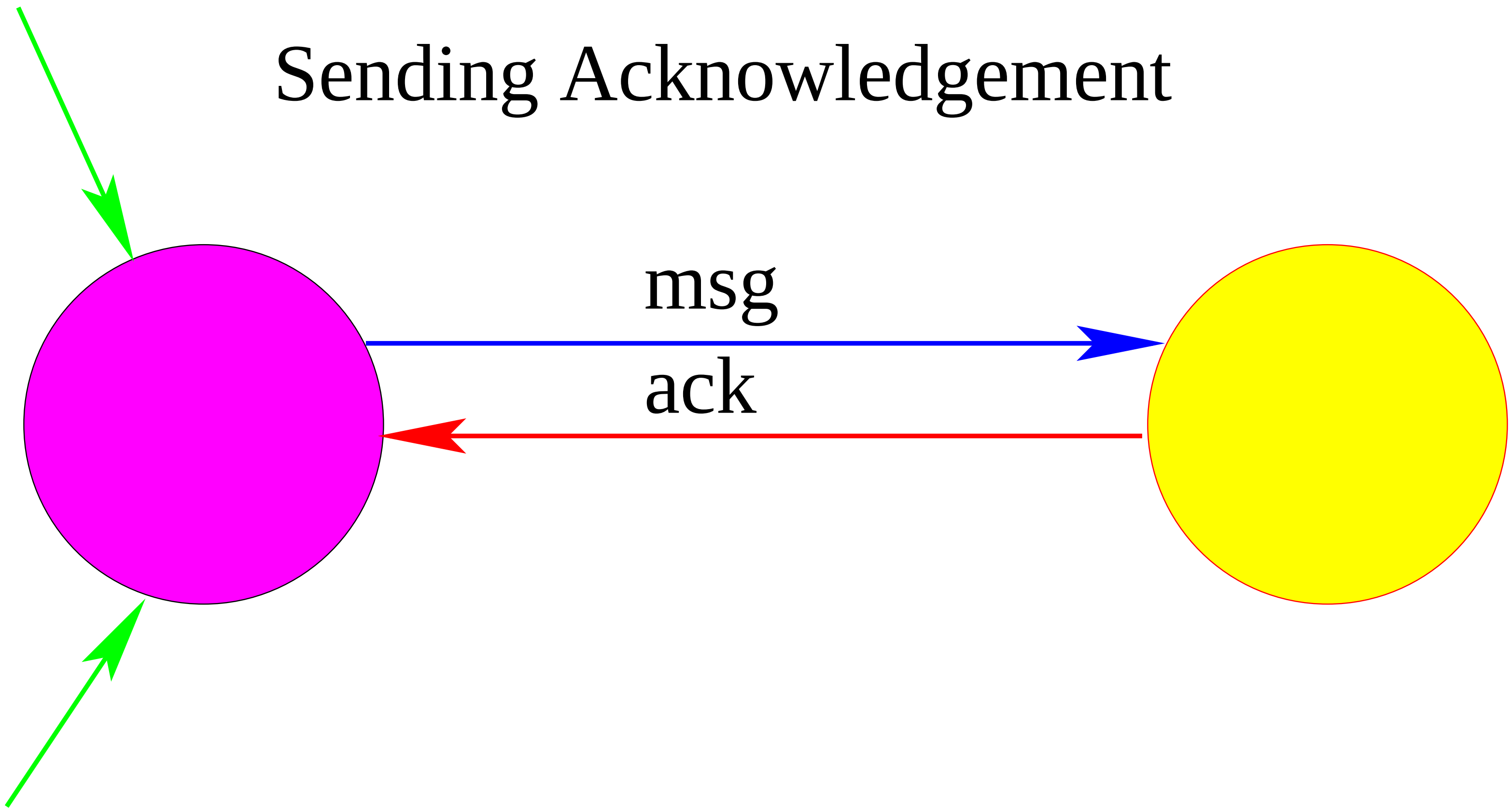


# Sending a message



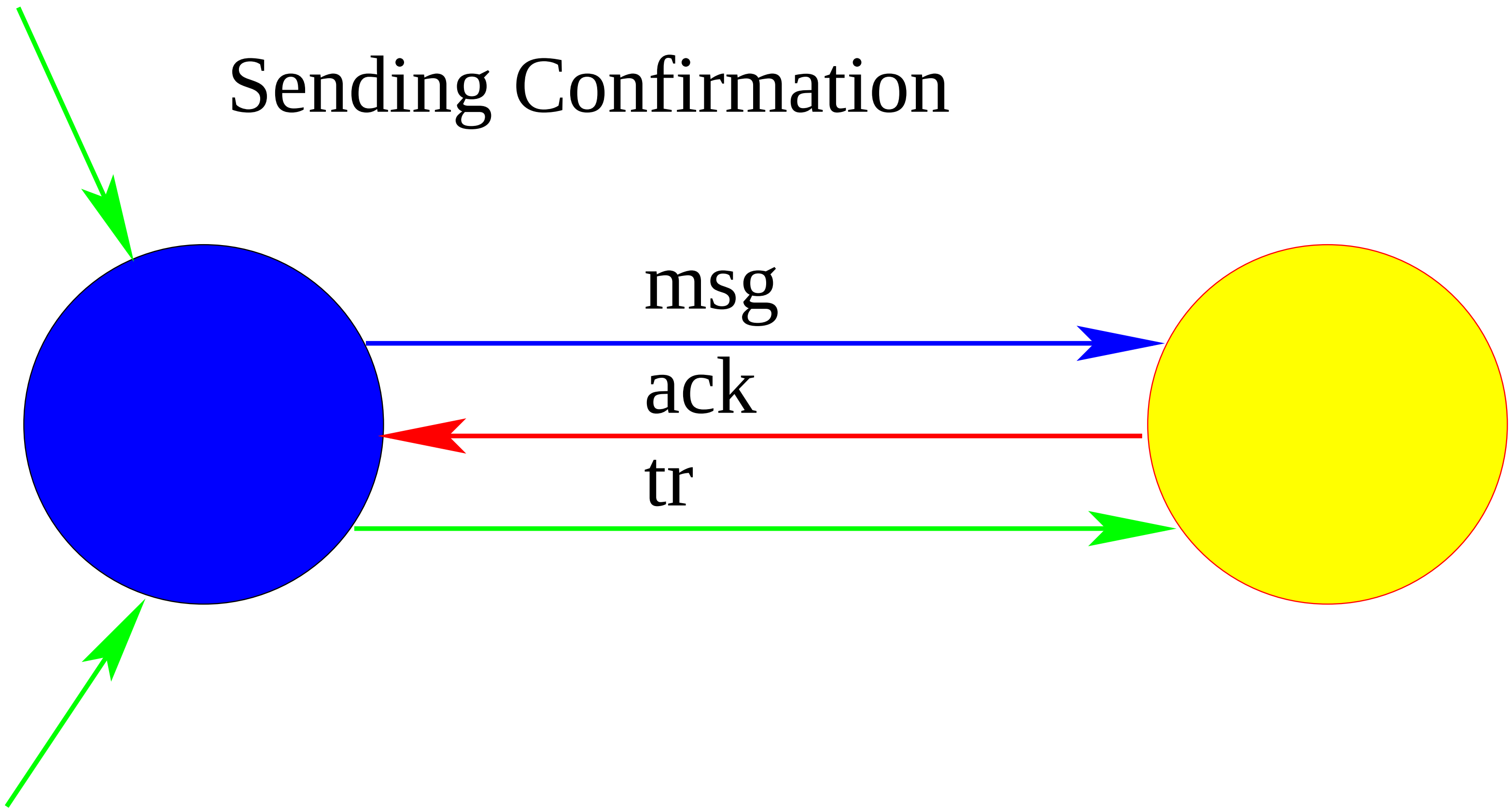
Receiving a message

Sending Acknowledgement

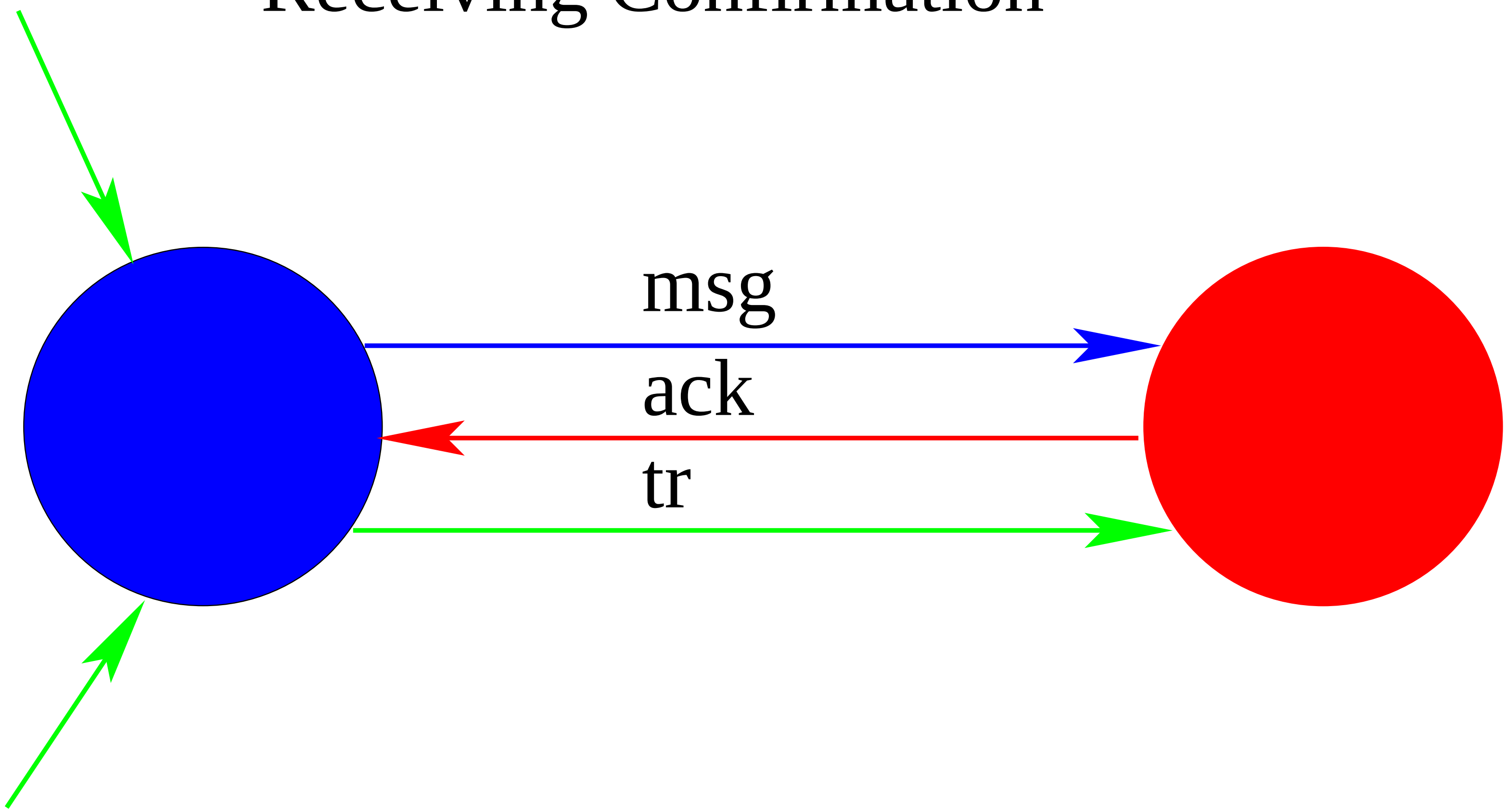


Receiving Acknowledgement

Sending Confirmation



# Receiving Confirmation



# Discovering the Contention (1)

- Node $y$ discovers the contention with node $x$ because:
  - It has sent a message to node $x$
  - It has not yet received a response from node $x$
  - It receives instead a message from node $x$

## Discovering the Contention (2)

- Node $x$ also discovers the contention with node $y$

- Assumption: The time between both discoveries

  IS SUPPOSED TO BE BOUNDED

  BY $\tau$ ms

- The time $\tau$ is the maximum transmission time

  between 2 connected nodes

## A Partial Solution

- Each node waits for $\tau$ ms after its own discovery

- After this, each node thus knows that the other has also discovered the contention

- Each node then retries immediately

- PROBLEM: This may continue for ever

# A Better Solution (1)

- Each node waits for $\tau$ ms after its own discovery

- Each node then choses with equal probability:
  - either to wait for a short delay
  - or to wait for a large delay

- Each node then retries

# A Better Solution (2)

- Question: Does this solves the problem ?

- Are we sure to eventually have one node winning ?

- Answer: Listen carefully to Caroll Morgan’s lectures

# Node $y$ discovers a contention with node $x$

$$
\begin{array}{l}
\text{send\_ack} \quad \widehat{=} \\
\quad \textbf{ANY } x, y \textbf{ WHERE} \\
\quad\quad x, y \in msg - ack \ \wedge \\
\quad\quad y \notin \mathrm{dom}\,(msg) \\
\quad \textbf{THEN} \\
\quad\quad ack := ack \cup \{x \mapsto y\} \\
\quad \textbf{END}
\end{array}
$$

$$
\begin{array}{l}
\text{contention} \quad \widehat{=} \\
\quad \textbf{ANY } x, y \textbf{ WHERE} \\
\quad\quad x, y \in msg - ack \ \wedge \\
\quad\quad y \in \mathrm{dom}\,(msg) \\
\quad \textbf{THEN} \\
\quad\quad cnt := cnt \cup \{x \mapsto y\} \\
\quad \textbf{END}
\end{array}
$$

- Introducing a dummy contention channel: $cnt$

$$cnt \in ND \leftrightarrow ND$$

$$cnt \subseteq msg$$

$$ack \cap cnt = \emptyset$$

# Solving the contention (simulating the $\tau$ delay)

solve_contention $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in cnt \cup cnt^{-1}$

**THEN**

$msg := msg - cnt \quad ||$

$cnt := \emptyset$

**END**

## Summary of Second Refinement

- 73 proofs

- Among which 34 were interactive

# Third Refinement: Localization

- The representation of the graph $gr$ is modified

- The representation of the tree $tr$ is modified

- Other data structures are localized

# Localization (1)

The graph $gr$ and the tree $tr$ are now localized

$$nb \in ND \rightarrow \mathbb{P}(ND)$$

$$\forall x \cdot (\, x \in ND \Rightarrow nb(x) = gr[\{x\}] \,)$$

$$sn \in ND \rightarrow \mathbb{P}(ND)$$

$$\forall x \cdot (\, x \in ND \Rightarrow sn(x) \subseteq tr^{-1}[\{x\}] \,)$$

# Localization (2)

$$bm \subseteq ND$$

$$bm = \mathsf{dom}\,(msg)$$

$$bt \subseteq ND$$

$$bt = \mathsf{dom}\,(tr)$$

$$ba \in ND \rightarrow \mathbb{P}(ND)$$

$$\forall x \cdot (\, x \in ND \Rightarrow ba(x) = ack^{-1}[\{x\}]\,)$$

- Node $x$ is elected the leader

$$
\begin{array}{l}
\text{elect} \quad \widehat{=} \\
\quad \textbf{ANY } x \textbf{ WHERE} \\
\qquad x \in ND \ \wedge \\
\qquad nb(x) = sn(x) \\
\quad \textbf{THEN} \\
\qquad rt := x \\
\quad \textbf{END}
\end{array}
$$

- Node $x$ sends a message to node $y$ ($y$ is unique)

$$
\begin{array}{l}
\text{send\_msg} \;\; \widehat{=} \\
\quad \textbf{ANY } x, y \textbf{ WHERE} \\
\qquad x \in ND - bm \;\; \wedge \\
\qquad y \in ND - (ba(x) \cup sn(x)) \;\; \wedge \\
\qquad nb(x) = sn(x) \cup \{y\} \\
\quad \textbf{THEN} \\
\qquad msg := msg \cup \{x \mapsto y\} \quad \| \\
\qquad bm := bm \cup \{x\} \\
\quad \textbf{END}
\end{array}
$$

- Node $y$ sends an acknowledgement to node $x$

send_ack $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in msg \;\; \wedge$

$x \notin ba(y) \;\; \wedge$

$y \notin bm$

**THEN**

$ack := ack \cup \{x \mapsto y\} \quad \|$

$ba(y) := ba(y) \cup \{x\}$

**END**

- Node $x$ sends a confirmation to node $y$

progress $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in ack \ \wedge$

$x \notin bt$

**THEN**

$tr := tr \cup \{x \mapsto y\} \quad \|$

$bt := bt \cup \{x\}$

**END**

- Node $y$ receives confirmation from node $x$

rcv_cnf $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in tr \ \wedge$

$x \notin sn(y)$

**THEN**

$sn(y) := sn(y) \cup \{x\}$

**END**

contention $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in cnt \cup cnt^{-1} \ \wedge$

$x \notin ba(y) \ \wedge$

$y \in bm$

**THEN**

$cnt := cnt \cup \{x \mapsto y\}$

**END**

solve_contention $\widehat{=}$

**ANY** $x, y$ **WHERE**

$x, y \in cnt \cup cnt^{-1}$

**THEN**

$msg := msg - cnt \quad \|$

$bm := bm - \mathrm{dom}\,(cnt) \quad \|$

$cnt := \emptyset$

**END**

# Summary of Third Refinement

- 29 proofs

- Among which 19 were interactive

# Main Summary

- 119 proofs

- Among which 63 were interactive

# Conclusion: a Systematic Approach to Distribution

- Establishing the mathematical framework

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- Establishing the mathematical framework

- Resolving the mathematical problem in one shot

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- Resolving the same problem on a step by step basis

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- Resolving the same problem on a step by step basis

- Involving communication by means of messages

- Towards the localization of data structures