A (short) summary on B and case studies on protocols

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Summary on the tools and on the notations

- ✓ Atelier B supports the classical B method: MACHINE, OPERATIONS, REFINEMENT, PROOFS,
- ✓ The event-based B method is not yet supported by a tool
- ✓ Atelier B and B4free are used to generate proof obligations for the invariant preservation
- ✓ Additional proof obligations are added in the invariant
- ✓ An Events System Model: events which are triggered
- ✓ An Abstract Machine: operations which are called

Summary: Set-theoretical notations

Name	Syntax	Definition
Binary Relation	$s \leftrightarrow t$	$\mathcal{P}(s \times t)$
Composition of relations	$r_1 ; r_2$	$\{x, y \mid x \in a \quad \wedge \quad y \in b \quad \wedge$ $\exists z.(z \in c \quad \wedge \quad x, z \in r_1 \quad \wedge \quad z, y \in r_2)\}$
Inverse relation	r^{-1}	$\{x, y \mid x \in \mathcal{P}(a) \quad \wedge \quad y \in \mathcal{P}(b) \quad \wedge \quad y, x \in r\}$
Domain	$\mathrm{dom}(r)$	$\{a \mid a \in s \quad \wedge \quad \exists b.(b \in t \quad \wedge \quad a \mapsto b \in r)\}$
Range	$\mathrm{ran}(r)$	$\mathrm{dom}(r^{-1})$
Identity	$\mathrm{id}(s)$	$\{x, y \mid x \in s \quad \wedge \quad y \in s \quad \wedge \quad x = y\}$
Restriction	$s \lhd r$	$\mathrm{id}(s); r$
Co-restriction	$r \rhd s$	$r; \mathrm{id}(s)$
Anti-restriction	$s \lhd\!\!\!- r$	$(\mathrm{dom}(r) - s) \lhd r$
Anti-co-restriction	$r \rhd\!\!\!- s$	$r \rhd (\mathrm{ran}(r) - s)$
Image	$r[w]$	$\mathrm{ran}(w \lhd r)$
Overriding	$q \lhd\!\!\!+ r$	$(\mathrm{dom}(r) \lhd\!\!\!- q) \cup r$
Partial Function	$s \mathrel{+\!\!\!\rightarrow} t$	$\{r \mid r \in s \leftrightarrow t \quad \wedge \quad (r^{-1}; r) \subseteq \mathrm{id}(t)\}$

Summary: events

Event : E	Before-After Predicate
BEGIN $x : P(x_0, x)$ **END**	$P(x, x')$
SELECT $G(x)$ **THEN** $x : P(x_0, x)$ **END**	$G(x) \quad \wedge \quad P(x, x')$
ANY t **WHERE** $G(t, x)$ **THEN** $x : P(x_0, x, t)$ **END**	$\exists\, t \cdot (\, G(t, x) \quad \wedge \quad P(x, x', t)\,)$

Summary: tools

- ✓ Proving lemmas or theorems from definitions of mathematical structures (constants and properties)
- ✓ **Stop for the demo**
- ✓ Proving proof obligations generated from an events system model

Summary: Sequent Calculus

- ✓ Sequent:

$$HYP \vdash P$$

- ✓ Rules using sequents:

$$\frac{\begin{array}{c} HYP_1 \vdash P_1 \\ \bullet \\ \bullet \\ \bullet \\ HYP_n \vdash P_n \end{array}}{HYP \vdash P}$$

Notations

	Antecedents	Consequent
name	$\begin{cases} HYP_1 \vdash P_1 \\ \vdots \\ HYP_n \vdash P_n \end{cases}$	$HYP \vdash P$

$$Predicate \wedge Predicate$$

$$Predicate \Rightarrow Predicate$$

$$\neg Predicate$$

Basic rules

	Antecedents	Consequent
BR1		$P \vdash P$
BR2	$\begin{cases} HYP \vdash P \\ HYP \subseteq HYP' \end{cases}$	$HYP' \vdash P$
BR3	$P \in HYP$	$HYP \vdash P$
BR4	$\begin{cases} HYP \vdash P \\ HYP, P \vdash Q \end{cases}$	$HYP \vdash Q$

Basic rules for $\wedge$

	Antecedents	Consequent
R1	$\begin{cases} HYP \vdash P \\ HYP \vdash Q \end{cases}$	$HYP \vdash P \wedge Q$
$R2$ $R2'$	HYP $\vdash P \wedge Q$	$\begin{cases} HYP \vdash P \\ HYP \vdash Q \end{cases}$

Basic rules for $\Rightarrow$

	Antecedents	Consequent
DED	$HYP, P \vdash Q$	$HYP \vdash P \Rightarrow Q$
R4	$HYP \vdash P \Rightarrow Q$	$HYP, P \vdash Q$
MP	$\begin{cases} HYP \vdash P \\ HYP \vdash P \Rightarrow Q \end{cases}$	$HYP \vdash Q$

DED (R3) = DEDUCTION RULE

MP = Modus Ponens (BR4 + R4)

Basic rules for $\neg$

	Antecedents	Consequent
R5	$\begin{cases} HYP, \neg Q \vdash P \\ HYP, \neg Q \vdash \neg P \end{cases}$	HYP $\vdash Q$
R6	$\begin{cases} HYP, Q \vdash P \\ HYP, Q \vdash \neg P \end{cases}$	$HYP \vdash \neg Q$

RULES OF CONTRADICTION “reductio ad absurdum”)

A proof

$HYP \vdash\ P \ \Rightarrow\ \neg\neg P$ Sequent 1

Apply DED (R3)

$HYP\,,\, P \ \vdash\ \neg\neg P$ Sequent 2

Apply R6 with X

$HYP\,,\, P\,,\, \neg P\ \vdash X$

$HYP\,,\, P\,,\, \neg P\ \vdash \neg X$

X is P and we use BR3

$HYP\,,\, P\,,\, \neg P\ \vdash P$ Sequent 3

$HYP\,,\, P\,,\, \neg P\ \vdash \neg P$ Sequent 4

Automating proofs obligations checking

- B tools provide automatic proof procedure for sequent calculus
- pr
- pp (predicate prover)
- Atelier B (ClearSy): `http://www.atelierb.societe.com/`
- Click'n'Prove: `http://www.loria.fr/~cansell/cnp.html`
- B4free (ClearSy): `http://www.b4free.com/`

Constructing a protocol

- Paradigms: parachutist, myope, ...
- A protocol is a process which manages a communication between partners
- The main purpose is to ensure the communication of an item or items from an agent to another agent
- Finding a first very abstract model with main events
- Refining abstract model to take into account assumptions on communications
- Localization of variables.

Protocols

- FIFO Protocol without loss of messages, duplication and reordering
- FIFO Protocol without loss of messages and duplication and with reordering
- Data Transfer Protocol of Stenning and more . . .

FIFO Protocol: no loss, duplication and reordering

MACHINE

$fifoprotocol0$

SETS

DATA

CONSTANTS

NN, *FILE*

PROPERTIES

$NN \in \mathbb{N} \wedge$

$NN \neq 0 \wedge$

$FILE \in 1\,..\,NN \longrightarrow DATA$

VARIABLES

file

INVARIANT

$file \in 1..NN \nrightarrow DATA$

INITIALISATION

$file := \emptyset$

OPERATIONS

$TRANSMISSION$ $=$ **BEGIN** $file := FILE$ **END**;

$sndd$ $=$ **BEGIN** $file \in (file \in 1..NN$
$DATA)$ **END**;

$rcvd$ $=$ **BEGIN** $file \in (file \in 1..NN$
$DATA)$ **END**

END

FIFO Protocol: no loss, duplication and reordering

REFINEMENT $fifoprotocol1$

REFINES $fifoprotocol0$

VARIABLES *ss*, *rr*, *file*, *dch*

INVARIANT

$$ss \in \mathbb{N} \wedge rr \in \mathbb{N} \wedge$$

$$file \in 1..NN \nrightarrow DATA \wedge$$

$$dch \in 1..NN \nrightarrow DATA \wedge$$

$$rr \leq ss \wedge$$

$$file = (1..rr) \lhd FILE \wedge$$

$$dch \subseteq (1..ss-1) \lhd FILE \wedge$$

$$file \subseteq dch \wedge ss \leq NN+1 \wedge$$

$$ss \geq 1 \wedge rr \leq NN$$

ASSERTIONS

$$rr = NN \Rightarrow file = FILE$$

INITIALISATION

$$file := \emptyset \parallel ss := 1 \parallel rr := 0 \parallel dch := \emptyset$$

OPERATIONS

$TRANSMISSION = $ **SELECT** rr NN **THEN** SKIP **END**;

$sndd =$ **SELECT** $ss \leq NN$

THEN $dch(ss) := FILE(ss) \parallel$

$ss := ss+1$ **END**;

$rcvd =$ **SELECT** $rr+1 \in \mathrm{DOM}(dch)$

THEN $file(rr+1) := dch(rr+1) \parallel$

$rr := rr+1$ **END**

END

FIFO Protocol: no loss and duplication and with re-ordering

MACHINE

$reliable0$

SETS

DATA

CONSTANTS

NN, *FILE*

PROPERTIES

$NN \in \mathbb{N} \wedge$

$NN \neq 0 \wedge$

$FILE \in 1..NN \longrightarrow DATA$

VARIABLES

file

INVARIANT

$file \in 1..NN \nrightarrow DATA$

INITIALISATION

$file := \emptyset$

OPERATIONS

$TRANSMISSION =$ **BEGIN** $file := FILE$ **END**;

$sndd =$ **BEGIN** $file \in (file \in 1..NN$ -
$DATA)$ **END**;

$rcvd =$ **BEGIN** $file \in (file \in 1..NN$ -
$DATA)$ **END**

END

FIFO Protocol: no loss and duplication and with re-ordering

REFINEMENT $reliable1$

REFINES $reliable0$

VARIABLES

$ss, file, dch$

INVARIANT

$file \in 1..NN \twoheadrightarrow DATA \wedge$

$dch \subseteq (1..NN) \times DATA \wedge$

$ss \in \mathbb{N} \wedge$

$ss \geq 1 \wedge$

$ss \leq NN{+}1 \wedge$

$dch \cup file = (1..(ss-1)) \lhd FILE$

INITIALISATION

$file := \emptyset \parallel dch := \emptyset \parallel ss := 1$

OPERATIONS

$TRANSMISSION =$ **SELECT** ss $NN{+}1 \wedge dch = \emptyset$ **THEN** SKIP **END**;

$sndd =$ **SELECT** $ss \leq NN$

THEN $dch := dch \cup \{(ss$ $FILE(ss))\} \parallel$

$ss := ss + 1$ **END**;

$rcvd =$ **ANY** rr, mes

WHERE $rr \in \mathbb{N} \wedge mes \in DATA \wedge rr$ $mes \in dch$

THEN $file(rr) := mes \parallel$

$dch := dch - \{rr \mapsto mes\}$ **END**

END

Data transfer protocol Stenning

- Goal: transfer data file from an agent to another agent
- an agent sender and an agent receiver
- Both agents **S** and **R** are localized on two differents sites

First model

◇ The file is a total function over $\{1, \ldots, n\}$ into $DATA$.

$n \in \mathbb{N} \wedge$	size of file
FILE $\in 1..n \longrightarrow$ *DATA*	file to send

◇ The result $file$ is a partial function from $\{1, \ldots, n\}$ into $DATA$.

VARIABLES

file

INVARIANT

file $\in 1..n \longrightarrow$ *DATA*

First model

$$transmission = \textbf{BEGIN}\ file := FILE\ \textbf{END}$$

Second model: refining the data transfer

- Data are sent one by one and one assumes that no datum is lost

- Two new variables are introduced for controling the sending and the receiving actions.

VARIABLES

s, r	control of messages traffic

- $FILE$ is progressively transmitted and $file$ contains a part of $FILE$, during the protocol.

INVARIANT

$$\textit{file} = (1..r) \lhd \textit{FILE}$$

Second model: refining the data transfer

◇ Communication channel between S and R is modelles by a variable dch and a confirmation channel between R and S allows us to synchronise sent data.

VARIABLES

dch, *ach*

INVARIANT

$$\mathit{dch} \subseteq (1..s) \lhd \mathit{FILE} \wedge$$

$$\mathit{ach} \subseteq 1..r$$

Events

◇ Initial state:

INITIALISATION

$$\textit{file} := \emptyset \parallel \textit{ss} := 1 \parallel \textit{rr} := 0 \parallel \textit{dch} := \emptyset \parallel \textit{ach} := \emptyset$$

◇ Events $sndd$, $rcvd$, $snda$, $rcva$are new and model the effective communications.

◇ Event $TRANSMISSION$ models the end of the transmission.

Events for sending data

$$\mathit{sndd} \;=\; \textbf{SELECT}\ \mathit{ss} \leq \mathit{NN}\ \textbf{THEN}\ \mathit{dch}(\mathit{ss}) := \mathit{FILE}(\mathit{ss})\ \textbf{END};$$

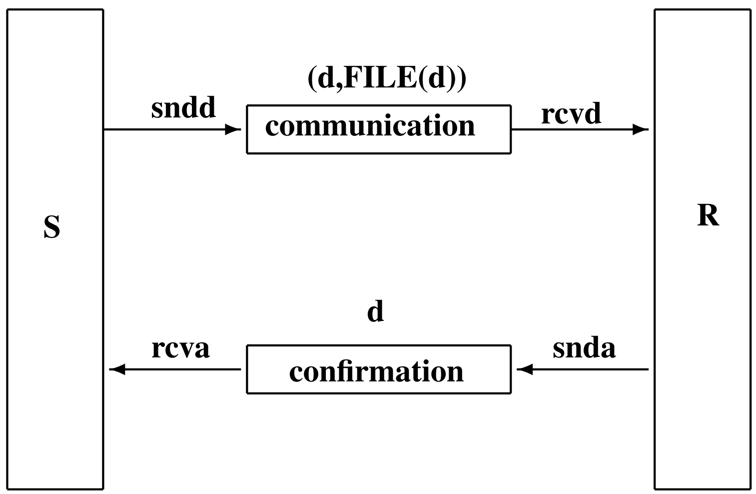
$$\begin{aligned}\mathit{rcvd} \;=\; & \textbf{SELECT}\ \mathit{rr}+1 \in \mathrm{DOM}(\mathit{dch}) \\ & \textbf{THEN}\ \mathit{file}(\mathit{rr}+1) := \mathit{dch}(\mathit{rr}+1) \parallel \mathit{rr} := \mathit{rr}+1\ \textbf{END};\end{aligned}$$

Events for sending acknowledgments

$$snda \;=\; \textbf{SELECT}\; rr \neq 0 \;\textbf{THEN}\; ach \;:=\; ach \cup \{rr\} \;\textbf{END};$$

$$rcva \;=\; \textbf{SELECT}\; ss \in ach \;\textbf{THEN}\; ss \;:=\; ss + 1 \;\textbf{END};$$

View of the second model



Second refinement and third model

- Channels are not reliable!

- Data may be reordered, duplicated and destroyed!
  - A channel is a set: duplication
  - Daemons remove data

Daemons

$$rmvd \;=\; \textbf{ANY}\; ii, dd\; \textbf{WHERE}\; ii \mapsto dd \in dch\; \textbf{THEN}\; dch := dch - \{ ii \mapsto dd \}\; \textbf{END};$$

$$rmva \;=\; \textbf{ANY}\; ii\; \textbf{WHERE}\; ii \in ach\; \textbf{THEN}\; ach := ach - \{ ii \}\; \textbf{END}$$

Invariant of the final model

INVARIANT

$$
\begin{array}{l}
ss \ \in \mathbb{N} \land \\
rr \ \in \mathbb{N} \land \\
file \in 1..NN \twoheadrightarrow \!\!\!\!\!+\ DATA \land \\
dch \in 1..NN \twoheadrightarrow \!\!\!\!\!+\ DATA \land \\
rr \leq ss \land \\
ss \leq rr + 1 \land \\
file = (1..rr) \lhd FILE \land \\
dch \subseteq (1..ss) \lhd FILE \land \\
ach \subseteq 1..rr \land \\
ss \leq NN + 1 \land \\
ss \geq 1 \land \\
rr \leq NN
\end{array}
$$

Safety properties of the last model

ASSERTIONS

$$rr = NN \Rightarrow file = FILE;$$

$$rr \leq ss\,;$$

$$ss \leq rr + 1\,;$$

$$rr = ss \Rightarrow rr \neq 0;$$

$$\forall\,(mm, kk).(kk \mapsto mm \in dch \Rightarrow kk \leq ss);$$

$$\forall\,kk.(kk \in 1..NN \land kk \in ach \Rightarrow kk \leq rr)$$

Questions

- How to get the alternating bit protocol?
- How to model protocols related to WEB services?
- What about probablilistic algorithms?
- **Next the leader election**